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Nonlinear Waves in a Discrete Magnetic Lattice

A. Mehrem[1], N. Jiménez[1], L. Salmerón[1], X. García-Andrés, Ll. García-Raffi[2], R. Picó, V.J. Sánchez-Morcillo[1]

[1] *IGIC, Universitat Politécnica de Valencia, Paranimf 1, 46730 Gandia Spain,*
[2] *IUMPA, Universitat Politécnica de Valencia, Camino de Vera s/n 46022 Valencia, Spain*
ahmeh@epsg.upv.es

Abstract: We investigate the propagation of nonlinear waves in a periodic lattice of magnetically coupled oscillators. Experiments show that higher harmonics of the forcing frequency are generated and propagate along the chain. It is shown that at high frequencies harmonic generation is strongly influenced by the dispersive properties of the chain. Experiments are compared with simulation, with good agreement.

The equations of motion for the one dimensional lattice of identical particles with nearest neighbor interactions (the simplest form of a phononic chain) can be written, when no external forces are present, as

$$m\frac{\mathrm{d}^2 u_n(t)}{\mathrm{d}t^2} = V'\left(u_{n+1} - u_n\right) - V'\left(u_n - u_{n-1}\right) \tag{1}$$

where u_n stands for the displacement of the n−th particle, m the mass particle, V the interaction potential between the n−th and the (n−1)−th particles and V' its derivative. If the interaction force is linear with respect to the displacements u_n i.e. $V'(r) = \kappa r$ where □ is a constant, then Eq. (1) represents a system of coupled harmonic oscillators. Such a system has normal modes (phonons), and its natural motion can be expressed as a superposition of these normal modes. When nonlinearity is introduced, transition of energy between these normal modes is allowed in general and the system might become ergodic. Coupled chains of oscillators have been extensively studied in the past for different types of interaction potentials. The most popular examples are the FPU lattice $V'(r) = \kappa_1 r + \kappa_2 r^2$, the Toda lattice $V'(r) = e^{-r} - 1$ or the granular lattice $V'(r) = r^{3/2}$.

Here we consider a lattice where its elements interact via an inverse power law, $V'(r) = r^{-n}$, which is typical of Coulomb-type interactions. Considering the nearest-neighbor magnetic interactions, the equation of motion takes the form

$$m\frac{\mathrm{d}^2 u_n(t)}{\mathrm{d}t^2} = \beta\left[\frac{1}{\left(x_0 + u_{n+1} - u_n\right)^n} - \frac{1}{\left(x_0 + u_n - u_{n-1}\right)^n}\right] \tag{2}$$

where the exponent n take the value n=4 for magnetic dipole,s as in our experimental setup described below. The physical system under study is an array of cylindrical magnets, forming a nearly one dimensional chain of masses coupled by its magnetic fields. The magnetic moments of neighbor magnets are oriented in the same directions, so they experience repulsive forces dependent on the separation between poles. Each magnet is the mass of a pendulum, and then is forced to oscillate around an equilibrium position.

Our experimental linear array of magnets is shown in Fig. 1. In the equilibrium state, the centers of mass are at rest and spaced by a. When each mass is displaced from its equilibrium position by the effect of a propagating wave, the distance between magnets becomes time-dependent, as $x(t) = a - u_n(t) + u_{n+1}(t)$

For small displacements u_n, we can assume that $\Delta u_n = u_n - u_{n-1} << a$, and use the binomial expansion in Eq. (2), in the quadratic approximation, to get

$$\frac{d^2u_n(t)}{dt^2} = \kappa_1\left(u_{n+1} - 2u_n + u_{n-1}\right) - \kappa_2\left(u_{n+1} - 2u_n + u_{n-1}\right)\left(u_{n+1} - u_{n-1}\right) \tag{3}$$

which is the □-FPU lattice.

Nonlinear waves in lattices have been studied in many previous works, which mostly addressed the problem of the formation of solitons. Recently, problems of this type have been treated also in magnetic lattices [1,2]. The process of harmonic generation in an FPU chain (derived as a limit of the strongly compressed granular chain) has been investigated theoretically in [3]. In this work we have investigated the response of the chain to boundary oscillation at a fixed frequency, and different amplitudes, in the experimental chain of magnets in Fig. 1.

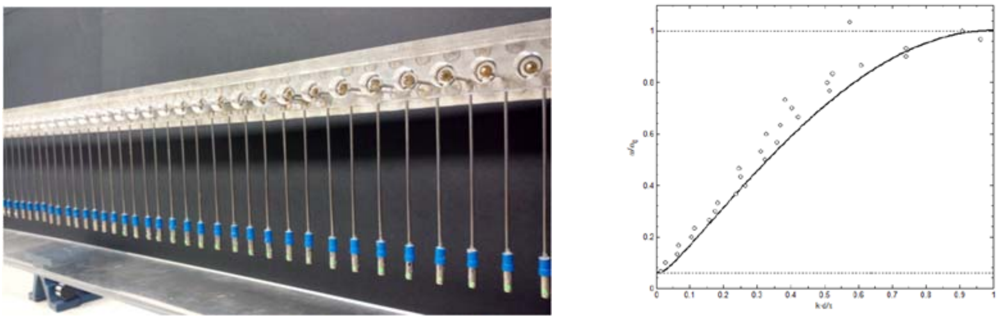

Figure 1. Left - Picture of the experimental chain of magnets. Right - Theoretical (continuous line) and experimentally measured (dots) dispersion of the chain

We report theoretical and experimental evidence of nonlinear waves in the magnetic chain. In particular, we calculated the linear dispersion relation (Fig. 1, right), and its dependence with the forcing amplitude. The generation of harmonics (second and higher order) is also demonstrated (Fig.2). Notably, is shown that harmonics, being forced oscillations, exist even when the corresponding frequency is higher than the linear cutoff frequency. In the case of diatomic chains of alternating masses, also subharmonics of the driving frequency are observed. Finally, evidences of soliton behavior are reported.

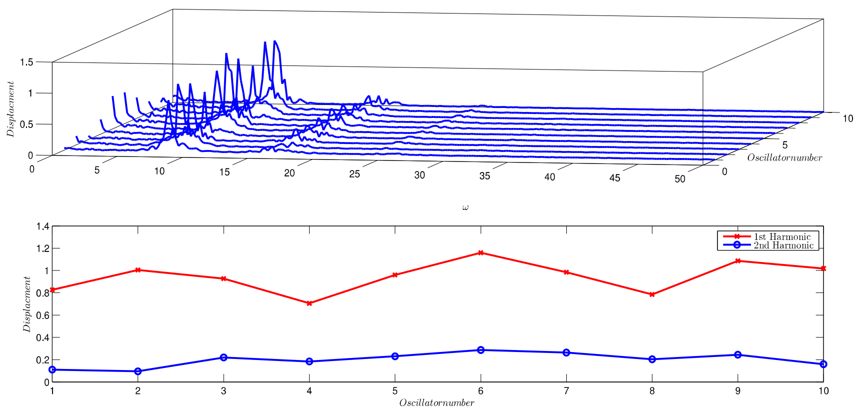

Figure 2. Top: Spectrum of the oscillations measured at different masses from n=1 to 10. Driving frequency ω=8 Hz. Note the growth of second and third harmonics. Bottom: First and second harmonic amplitudes at different distances.

References

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