Second harmonic generation in a chain of magnetic pendulums

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A chain of magnetic pendulums has been demonstrated to be a good analogue model of a crystal lattice [**1**]. We consider an infinite chain of identical magnets with mass m aligned along the x-axis. The equation of motion for every magnet in such chain, considering a magnetic dipole-dipole interaction only between two nearest neighbors , takes the form:

$$m\ddot{u}_n = -\frac{g}{L}u_n + \frac{\epsilon}{(a-u_{n+1}+u_n)^4} - \frac{\epsilon}{(a-u_n+u_{n-1})^4}$$

where u_n represent the displacement of magnet n^{th} measured with respect to its equilibrium position, ϵ is a coupling constant, a is the distance between the center of magnets (lattice constant) and g is the gravitational acceleration. In particular, for small displacements, this equation reduces to the well-known α-FPU equation (quadratic approximation). Then, it is expected to observe the same kind of phenomena that appears in other systems like acoustic layered media or granular chains [**2**] . This discrete medium is called a superlattice and presents dispersion at frequencies close to the cutoff.

In this work, we studied numerically, analytically and experimentally the nonlinear dynamics of this system, in particular, harmonic generation and the balance between nonlinearity and dispersion. In the experimental setup, each magnet is attached to a T-shaped rod with length L and the system presents a very low damping. Three regimes have been explored exciting the chain, forcing the first pendulum with an harmonic force of long wavelength with respect to a and with such amplitude that the dispersion relation still remains valid. We have studied harmonic generation when; (a) the frequencies of the $First$ harmonic (driven force) and the $second$ harmonic generated by the nonlinearity lies into the non-dispersive part of the dispersion relation (almost linear $k(\omega)$). In this case the second harmonic starts to increase while first harmonic decreases. (b) The driven frequency is in the non-dispersive part of the dispersion relation but the second harmonic is evanescent. The amplitude of the second harmonic reaches a constant value along the chain. (c) The frequency of the second harmonic lies in the dispersive part of the dispersion relation. Harmonic components travel with different velocities and beating appears. Analytical results and numerical simulations are in good agreement with the experimental results.

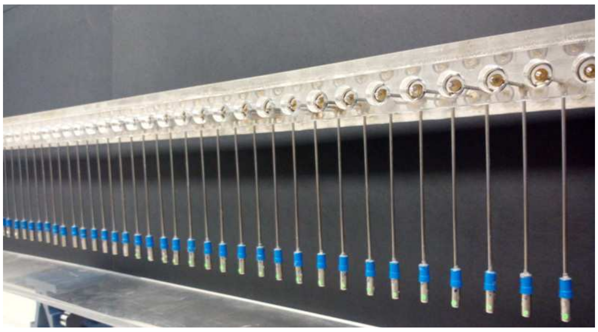

FIGURE 1. Experimental setup

Keywords: FPU, magnetic pendulums, dipole-dipole potential, Nonlinear lattice, Harmonic generation.

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