PROCEEDINGS of the
24th International Congress on Acoustics
October 24 to 28, 2022, in Gyeongju, Korea

Extended abstract

ABS-0051

Hybrid analytic model between the transfer-matrix method and the modal decomposition to design Acoustic Metamaterials: Application to degenerate resonators

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ABSTRACT

Plane waves are usually assumed to design acoustic metamaterials. This assumption is most of the times satisfying, but prevents from accounting for the possible evanescent coupling between the elements of the acoustic metamaterial. In this talk, we present a hybrid analytical model between the transfer-matrix method inside the structured material and the modal decomposition in the surrounding domains. We apply this method to the analysis of the scattering properties of a mirror-symmetric absorber made of several slits loaded by Helmholtz resonators. The transfer-matrix model is used to describe the propagation within each slit, while the modal decomposition is used to described the acoustic field in the surrounding domains. Modal decomposition assumes Bloch waves; thus, it accounts for the possible evanescent coupling between the slits. The domains are matched through the continuity conditions between the exterior media and each slit. This method is coupled to an optimization procedure to design single frequency perfect mirror symmetric absorbers. The results are numerically and experimentally validated showing good agreement with the proposed hybrid model.

Keywords: Sound, Insulation, Transmission

1. INTRODUCTION

Metamaterials made of Helmholtz resonators (HRs) have been exploited during the last year to control acoustic waves [1-4]. Different applications have been developed to attenuate [5], to control the directivity [6, 7] or to absorb acoustic waves [8-11]. In most of the cases the resulting structures are periodic to simplify its modeling. Usually, the transfer matrix method is used to analytically describe the behavior of these systems, i.e., assuming propagation of plane waves and neglecting the higher orders. Moreover, the periodicity implies the existence of Bloch waves that can also produce possible evanescent coupling between the building blocks of the metamaterials. Sometimes, and in some regions of frequencies these Bloch waves can be neglected, but in general and in reality, they produce couplings that make the modeling by the transfer matrix method wrong.

In this work we show an anytically a hybrid model combining transfer matrix and modal decomposition methods used to model a metascreen consisting of a periodic array of slits loaded by different HRs. The transfer matrix model [12] is used to describe the propagation within each slit while the modal decomposition is used to obtain the propagation in the exterior domain considering the Bloch waves accounting for the possible evanescent coupling between the slits. The two domains are coupled through the continuity conditions between the exterior medium and each slit. A full-wave numerical simulation is used to validate the analytical model. A design of a metascreen made of mirror-symmetric resonant building blocks possessing degenerate resonances is used to show the dramatic effect of the evanescent coupling on the acoustic perfect absorption. The metascreen is experimentally tested in a square cross-sectional impedance tube showing very good agreement with the analytical and numerical predictions.

2. THE SETUP

The schematic view of the metascreen design that we analyze in this work, shown in Fig. 1(a), is a combination of slits loaded by one and two resonators [11]. The HRs are made by combining two different elements playing the role of the neck and the cavity. Due to the symmetry of the system, the scattering is reciprocal and symmetric. The metascreen is radiated by a plane wave from the top to the bottom in the medium $\Theta^{(0+)}$. Viscothermal losses are accounted for by using the Stinson's equivalent expressions for the complex and frequency dependent of the density and wavenumber in the slits, necks and cavities of the structure [13].

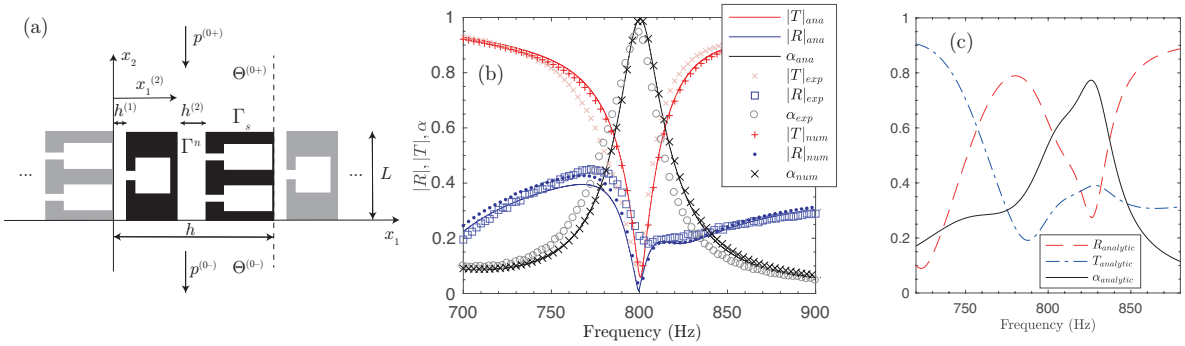


Figure 1. (a) Schematic view of the analyzed metascreen made of two slits loaded by Helmholtz resonators. (b) Analytical, numerical and experimental scattering coefficients of the metascreen. (c) Scattering coefficients considering only the plane wave approximation, i.e., the transfer matrix method.

3. THEORETICAL MODELING

The acoustic field in the exterior domains ($\Theta^{(0+)}$ and $\Theta^{(0-)}$ as defined in Fig. 1(a)), when the system is excited by a plane wave $\vec{\mathrm{k}}=(\mathrm{k}_1,\mathrm{k}_2)$ can be written as

$$p^{0+}=\sum_q\left[e^{-\imath k_{2q}^0(x_2-L)}\delta_q+R_q e^{\imath k_{2q}^0(x_2-L)}\right]e^{\imath k_{1q}^0 x_1} \quad \text{in} \quad \Theta^{0+}$$

$$p^{(0-)}=\sum_q T_q e^{\imath k_{1q}^0 x_1-\imath k_{2q}^0 x_2} \quad in \quad \Theta^{0-},$$

where the subindex q indicates the order of the Bloch wave, with $k_{1q}^0=2q\pi/2d_0+k_1^i$ and $k_{2q}^0=\sqrt{\left(\left|\overrightarrow{k^0}\right|\right)^2-\left(k_{1q}^0\right)^2}$, with $Re\left(k_{2q}^0\right)\geq 0$, R_q and T_q are the reflection and transmission coefficients of the q-th Bloch wave, δ_q is the Kronecker's delta and L is the thickness of the acoustic metascreen.

Wave propagation in each slit is modeled by the transfer matrix method (TMM), in which the HRs are considered as 1D point scatterers. Matrices describing the propagation in the slit, to the side branch resonators and to the section mismatch are assembled giving the total transfer matrix. The TMM assumes the plane-wave propagation in each element, which is valid in the low frequency regime. The radiation corrections are included in the impedances of the resonators to mimic the effect of the higher order modes. Therefore, the wave propagation through the slit *n* can be modeled as

$$\begin{bmatrix} p^{(n)} \\ v_2^{(n)} \end{bmatrix}_{x_2=L} = \begin{bmatrix} T_{11}^{(n)} & T_{12}^{(n)} \\ T_{21}^{(n)} & T_{22}^{(n)} \end{bmatrix} \begin{bmatrix} p^{(n)} \\ v_2^{(n)} \end{bmatrix}_{x_2=0},$$

where the final transfer matrix is given by the product of the transfer matrices of the *N* elements in the *n*-th slit.

Once the acoustic fields inside each domain of the problem are defined, the continuity boundary conditions must be applied. These conditions read as

$$p^{(0+)} = p^n \ on \ \Gamma^n \ at \ x_2 = L,$$
$$p^{(0-)} = p^n \ on \ \Gamma^n \ at \ x_2 = 0,$$
$$v_2^{(0+)} = v_2^n \ on \ \Gamma^n \ at \ x_2 = L,$$
$$v_2^{(0+)} = 0 \ on \ \Gamma_s \ at \ x_2 = L,$$
$$v_2^{(0-)} = v_2^n \ on \ \Gamma^n \ at \ x_2 = 0,$$
$$v_2^{(0-)} = 0 \ on \ \Gamma_s \ at \ x_2 = 0,$$

where Γ_s represents the rigid interfaces on the two faces of the metascreen, and Γ^n is the interface between the *n*-th slit and the surrounding media $\Theta^{(0^+)}$ and $\Theta^{(0^-)}$. These conditions lead to a system of equations after projection on the adequate orthogonal modes. The system is solved for the scattering coefficients of the system.

In order to validate this model, we have used a finite element method (FEM) simulation. A plane wave impinges the system and the complete geometry is considered using the radiation conditions that simulate the Sommerfeld conditions at the limits of the numerical domain.

4. RESULTS

The model is used to calculate the absorption coefficient of the designed metascreen. Figure 1(b) shows the scattering coefficients of the full transmission problem. The agreement between the model, the numerical simulations and the experimental results is very good. Note here that the presence of the Bloch waves is crucial in this kind of systems: if only plane waves had been accounted for in the system, the model would not have reproduced the whole wave process because the evanescent coupling would have been neglected as shown in Fig. 1(c). When the Bloch waves are considered, a perfect absorption peak is observed at 800 Hz as analytically, numerically and experimentally observed. It is worth noting here that perfect absorption is very sensible to the geometry of the resonators, and this would explain the slight discrepancies between the analytical or numerical predictions and the experimental results.

5. CONCLUSIONS

The resonant scattering produced by the metascreen is analytically studied by a hybrid model mixing the transfer matrix method and the modal decomposition accounting for the Bloch waves in order to consider the possible coupling between the slits. Note that this coupling is particularly important for the present topology. If no higher order Bloch waves had been accounted for, i.e., if only the plane wave had been accounted for outside the metascreen, the analytical model would not have been in agreement with neither the full-wave numerical simulation nor the experimental results. The structure is experimentally tested in a square impedance tube showing good agreement with the analytical and numerical predictions. The Physics behind this work will motivate the development of broadband and perfect absorption for ventilation problems using cascade processes similar to the case of the rainbow trapping for the unidirectional absorption.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the ANR-RGC METARoom (ANR-18-CE08-0021) project. This publication is part of the R&D&I Project/Grant No. PID2020-112759GB-I00 funded by MCIN/AEI/10.13039/501100011033/.

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