Characterization of Viscoelastic Media Combining Ultrasound and Magnetic-Force Induced Vibrations on an Embedded Soft Magnetic Sphere

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Abstract— We report a method to locally assess the complex shear modulus of a viscoelastic medium. The proposed approach is based on the application of a magnetic force to a millimeter-sized steel sphere embedded in the medium and the subsequent monitoring of its dynamical response. A coil is used to create a magnetic field inducing the displacement of the sphere located inside a gelatin phantom. Then, a phased-array system using 3 MHz ultrasound probe operating in pulse-echo mode is used to track the displacement of the sphere. Experiments were conducted on several samples and repeated as a function of phantom temperature. The dynamic response of the sphere measured experimentally is in good agreement with Kelvin–Voigt theory. Since the magnetic force is not affected by weak diamagnetic media, our proposal results in an accurate estimation of the force acting on the inclusion. Consequently, the estimated viscoelastic parameters show excellent robustness and the elastic modulus agrees with the measurements using a quasi-static indentation method, obtaining errors below 10% in the whole temperature range. The use of the macroscopic inclusion limits the direct application of this method in a biomedical context, but it provides a robust estimation of the elastic modulus that can be used for material characterization in industrial applications.

Index Terms— Magnetic force, magneto-motive, shear modulus, ultrasound, viscoelasticity.

I. INTRODUCTION

The assessment of the mechanical properties of viscoelastic media is of great interest in medicine, where biomechanical properties of tissue have been proved to be often correlated with their physiological state [1]–[3], and in industry, where the knowledge of the mechanical properties of the medium allow to predict the properties, appearance, processing, and performance of polymers [4], [5], concrete [6], [7], asphalt [8], or food [9], [10].

The mechanical response of a solid sphere subjected to the effect of an external force can be used to determine the viscoelastic properties of the surrounding medium. This relation was first explored theoretically by Oestreich considering a Kelvin–Voigt rheological model in a medium with equal stiffness and viscosity [11]. Later, Ilinskii et al. [12] obtained the static and transient displacement responses of a sphere and a bubble embedded in an elastic medium, and Aglyamov et al. [13] extended the work to viscoelastic media. Finally, Urban et al. [14] proposed a theoretical development to describe the generalized embedded sphere response both in time and frequency domains so that any viscoelastic rheological model can be used.

From these theoretical works, several studies have been carried out to locally characterize tissue-like phantoms by exciting a sphere using the acoustic radiation force (ARF) produced at the focus of an ultrasonic beam. The group of Prof. Greenleaf presented a quantitative model for a sphere vibrated by two ultrasound beams of nearby frequencies able to estimate the material properties of the medium surrounding a sphere [15]. Later, the group of Prof. Emelianov locally assessed the shear modulus of a medium by using the ARF generated at the focus of a focused ultrasound beam excited with a short pulse [16]. In addition, the motion of the media due to magnetomotive-generated shear waves can also be measured [17], but to our knowledge, this method has not been applied to estimate the viscoelastic parameters of the medium.

In this article, we propose a new method to estimate the viscoelastic parameters of soft-solids. We use a classical magneto-motive ultrasound (MMUS) experimental setup [18]–[22] to track the dynamics of a macroscopic steel sphere embedded in a gelatin phantom excited by an external magnetic force, as sketched in Fig. 1. First, a coil is excited with an electrical pulse. Second, the generated magnetic field produces a transient attracting force on the soft magnetic inclusion. Third, using a 64-channel...
phased-array system, the position of the inclusion is dynamically tracked, and by using cross correlation methods the time-varying displacement waveform of the sphere is obtained. Finally, the elastic parameters of the surrounding viscoelastic medium are estimated from the dynamics of the sphere.

The procedure is similar to the one used in previous works that make use of ARF [15], [16], but, in this case, we use a magnetic force acting on a soft magnetic inclusion. A common problem in ARF-based techniques is that it is difficult to accurately measure or estimate the magnitude of the applied force. This is mainly caused because ultrasound waves attenuate as they propagate into heterogeneous tissues, the magnitude of the ARF depends on the geometry and properties of the object itself and tissue absorption, and in addition weakly nonlinear effects can be relevant to the estimation of the ARF. The uncertainty in the estimation of the ARF directly has an impact on the accuracy of the estimated viscoelastic parameters. In contrast, since the magnetic force is almost not affected by tissues which are, in general, weak diamagnetic media, our proposal results in an accurate estimation of the force acting on the sphere. In addition, ARF should induce motion not only at the bead but on the viscoelastic media due to ultrasound attenuation and scattering at heterogeneities. Using a magnetic force, only the sphere is pulled. Therefore, it increases the robustness of the estimation of the viscoelastic parameters of the surrounding medium.

It is important to remark that while the setup is similar to MMUS techniques that track the motion of a soft-solid produced by magnetic nanoparticles [18]–[22], we make use of a millimeter-size steel sphere. As we will demonstrate, the use of this inclusion allows an accurate estimation of the force exerted on it, enabling the quantitative estimation of the viscoelastic parameters.

The article is structured as follows: Section II presents the theoretical model to describe the dynamics of a solid sphere embedded in a viscoelastic medium and to evaluate the magnetic force on the sphere. Section III shows the experimental setup and methods. Section IV provides the experimental results and their comparison using indentation techniques. Finally, the concluding remarks are given in Section V.

II. MODEL

A. Dynamics of a Solid Sphere Embedded in Viscoelastic Media

The bulk modulus of most soft-tissues and tissue phantoms is several orders of magnitude higher than their shear moduli. Therefore, to model low-frequency shear deformations these media can be considered an incompressible viscoelastic medium, whose equations of motion for a Kelvin–Voigt-type medium are given by Aglyamov et al. [13]

\[
\rho \ddot{\mathbf{u}} = -\nabla p + \mu \nabla^2 \mathbf{u} + \eta \nabla^2 \dot{\mathbf{u}}
\]

(1)

where \( \mathbf{u} = \mathbf{u}(t, \mathbf{r}) \) is the displacement vector, \( p = p(\mathbf{r}) \) is the internal pressure, \( \mu \) and \( \eta \) are the shear elastic and shear viscous coefficients, \( \rho \) is the density of the medium and \( t \) is the time. Assuming a Fourier convention of \( e^{-i\omega t} \), the equations of motion in the frequency domain are written as

\[
-\nabla P + G \nabla^2 \mathbf{U} + \rho \omega^2 \mathbf{U} = 0
\]

(2)

where \( P \) and \( \mathbf{U} \) are the Fourier transforms of \( p \) and \( \mathbf{u} \), \( \omega \) is the angular frequency and \( G = (\mu - i\omega\eta) \) is the complex shear modulus. We consider a rigid sphere embedded in an infinite and homogeneous viscoelastic medium submitted to a transient external force \( F_z^{ext}(t) \), acting on direction \( z \). In the frequency domain, and for small displacements, the \( z \) component of the displacement of the sphere \( U_z(\omega) \) and the Fourier transform of the external force \( F_z^{ext}(\omega) \) are linearly related by Iliniok et al. [12]

\[
F_z^{ext}(\omega) = \left[ -M\omega^2 + 6\pi GR \left( 1 - ikR - \frac{1}{9}k^2R^2 \right) \right] U_z(\omega)
\]

(3)

where \( M = 4\pi \rho_s R^3 / 3 \) is the mass, \( \rho_s \) the density, \( R \) is the radius of the solid sphere, and \( k \) is the complex wavenumber in the viscoelastic medium given by the dispersion relation \( k^2 = \rho \omega^2 / (\mu - i\omega\eta) \). Then, the displacement of the
sphere \( u_z(t) \) in time domain can be recovered using the inverse Fourier transform as
\[
F_z(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U_z(\omega) e^{-i\omega t} d\omega.
\] (4)

In our study, we submit the sphere to a pulsed magnetic force. In practice, obtaining an ideal rectangular waveform for the current is not possible due to the inductance of the coil. Therefore, we assume that the temporal evolution of the magnetic force is given by a trapezoidal pulse with a finite rise time, as shown in Fig. 1(c), as
\[
F_z^{*\text{trap}}(t) = \begin{cases} 
-F_0 \left( \frac{t}{t_r} \right) & 0 \leq t \leq t_r \\
-F_0 & t_r \leq t \leq t_0 \\
-F_0 \left( 1 - \frac{t - t_0}{t_r} \right) & t_0 \leq t \leq t_0 + t_r \\
0 & t > t_0 + t_r 
\end{cases}
\] (5)

where \( t_0 \) is the pulse duration, \( F_0 \) is the amplitude of the pulse, and \( t_r \) is the rise time. The Fourier transform of the trapezoidal pulsed force results in
\[
F_z^{*\text{trap}}(\omega) = \frac{F_0}{t_r \omega} (e^{i\omega t_0} - 1)(e^{i\omega t_r} - 1).
\] (6)

Combining (3) and (6) and applying the inverse Fourier transform (4), the displacement of the solid sphere in a viscoelastic medium is written in the time domain as
\[
u_z^{\text{trap}}(t) = -\frac{F_0}{12\pi^2 R_0} \int_{-\infty}^{\infty} \left( e^{i\omega t_0} - 1 \right) \left( e^{i\omega t_r} - 1 \right) e^{-i\omega t} \frac{1}{\omega^2 (\mu - i\omega \eta)(1 - ik R - k^2 R^2(1 + 2\beta)/9)} d\omega 
\] (7)

where \( \beta = \rho_r/\rho \) is the normalized density of the sphere with respect to the medium density, in analogy with the expression obtained in [12] for a rectangular pulsed excitation. Equation (7) is integrated numerically to obtain the theoretical estimation of the displacement of the sphere. Note the displacement of the sphere for a long pulse converge to a constant value \[12\] of \( u_0 = F_0/6\pi \mu R \), as shown in the example in normalized scale in Fig. 1(c).

**B. Magnetic Force Exerted on the Sphere**

The magnetic force acting on a small ellipsoid of volume \( V \) due to the presence of the magnetic flux density \( B_0 \) along the \( z \)-axis is given by Schenck [23]
\[
F_z^{\text{ext}}(z, t) = \frac{\mu_0 V}{\mu_0} B_0(z, t) \frac{\partial B_z(z, t)}{\partial z} \left[ \frac{\cos^2 \theta}{1 + \chi D_{\alpha}} + \frac{\sin^2 \theta}{1 + \chi D_{\gamma}} \right]
\] (8)

where \( \chi \) is the volumetric magnetic susceptibility of the particle (neglecting the magnetic susceptibility of the background), \( \mu_0 \) is the magnetic permeability in vacuum, \( D_{\alpha} \) is the demagnetizing factor along the axis of symmetry, \( D_{\gamma} \) is the radial demagnetizing factor, and \( \theta \) is the angle between the symmetry axis of the particle (\( x \)-direction) and the magnetic field direction (\( z \)-axis). For a spherical particle, \( D_{\alpha} = D_{\gamma} = 1/3 \), other relations can be found for other geometries [23].

The soft magnetic sphere used in this experiment is made of a low-alloy chrome steel (AISI 52100), having a relative magnetic permeability \( \mu_r > 300 \), therefore, the volumetric magnetic susceptibility \( \chi \gg 1 \), thus, it can be considered as a soft magnetic material [24]. As such, and considering that all experiments in this study are performed at relatively low values of magnetic field (around 100 mT), insufficient to saturate the material, both coercivity and hysteresis are relatively small and are neglected for this material in our experiments [25]. Note the magnetic saturation of the sphere can be roughly estimated as around 2 T for a low-alloy steel with less than 2% of chromium [26]. Therefore, and taking into consideration, the spherical geometry of the bead aligned with the magnetic field, at the \( z \)-axis of the axisymmetric system, the force acting in the soft magnetic sphere of radius \( R \) can be reduced to [23]
\[
F_z^{\text{ext}}(z, t) = -\frac{4\pi R^3}{\mu_0} B_z(z, t) \frac{\partial B_z(z, t)}{\partial z}.
\] (9)

Note that for a soft magnetic sphere the ratio between the magnetic field strength, \( H \), and the magnetization of the material, \( M \), does not depend on the magnetic properties of the material [27] and is given by \( H = -3M \), the induced mechanical force is, therefore, independent of \( \chi \). Also note that (9) can only be applied if the system is axisymmetric, i.e., the sphere must be accurately located at the axis of the ferritic core. To know the magnetic force amplitude as a function of the electrical current on the coil, a finite element method (FEM) simulation of the whole magnetic system was carried on using COMSOL 5.2 software. Numerical simulations were carried out and the magnetic field was calculated at different distances from the center of the ferritic core for different values of electrical current. A phenomenological model was obtained by fitting the simulated data to the power-law
\[
F_0(z, I_0) = I_0^2 \alpha z^\gamma,
\] (10)

where \( I_0 \) is the electrical current on the coil, \( z \) is the distance from the ferritic core to the sphere, and \( \alpha \) and \( \gamma \) are the fitted parameters of the model. This model, as we will show below, was validated using (9) and experimental measurements of the magnetic field, and direct measurements of the force on a soft magnetic material [27]. As such, and considering all experiments in this study are performed at relatively low values of magnetic field (around 100 mT), insufficient to saturate the material, both coercivity and hysteresis are relatively small and are neglected for this material in our experiments [25].
the coil and evaluate the quality of the trapezoidal pulse. We measured a rising time of $t_r = 3$ ms. A three-axis Hall magnetometer (THM1176, Metrolab) is used to characterize the magnetic field of the system at a steady current. The field sensitive volume of the probe is $200 \mu m \times 200 \mu m \times 5 \mu m$, the instrument range is up to $3$ T with a resolution $0.1$ mT, and it provides an accuracy of $\pm 1\%$ T. Finally, a weighing scale (Cent-O-Gram Model 311, OHAUS, Moscow, Russia) is used to experimentally measure the magnetic force exerted on the soft magnetic sphere (AISI 52100 steel) at different heights in the absence of the viscoelastic medium and under a current of $I_0 = 30$ A. In this regard, the original pan of the balance was removed and substituted by few pieces of lead having the same weight as the original pan. Then, a nylon thread was attached to the balance on one side and to the metallic sphere on the other, making sure that every metallic part of the balance scale was placed at a distance far enough to avoid any detectable influence of the pulling force. This allows us to validate the force model given by (9). Note that the magnetic force acting on the sphere depends on the distance to the ferritic core. However, the displacements considered here are smaller than $100 \mu m$ and the force presents spatial variations that are below $1\%$. Therefore, we assume that the magnetic force does not depend on the sphere displacement, only on its position at rest.

The displacement of the sphere is estimated from the temporal shifts of the echoes produced by a phased-array system (Vantage 256, Verasonics, Kirkland, WA, USA) using a 64-channel 3 MHz ultrasound probe (P4-2V, Verasonics, Kirkland, WA, USA) as shown in Fig. 1(b) and (c). The phased-array is set to a pulse repetition frequency of $4$ kHz, a 64-channel 3 MHz ultrasound probe (P4-2V, Verasonics, Kirkland, WA, USA) as shown in Fig. 1(b) and (c). The phased-array is set to a pulse repetition frequency of $4$ kHz, allowing the reception of an echo every $0.25$ ms. The radio frequency (RF) signals from the ultrasound probe are acquired by the phased-array with a total recording duration of $75$ ms. The position of the sphere is given by the cross correlation between the RF signals and a reference signal in which the sphere is at rest, i.e., in its initial state without being magnetically excited [28]. Interpolation up to a sampling frequency of $15.6$ GHz is considered here to increase the accuracy on the estimated displacements up to a resolution of $0.1 \mu m$.

We consider two different gelatin phantoms of cylindrical shape with a diameter of $30$ mm and a volume of $25$ ml. First, “phantom A” was produced mixing water and $200$ Bloom gelatin powder, with a gelatin concentration of $60$ g/l. Second, “phantom B” was produced by mixing water, glycerol ($99.5\%$ purity), and $200$ Bloom gelatin powder, with a concentration of $60$ g/l and a glycerol concentration of $40\%$ of the total volume. The density of both phantoms was measured independently, obtaining slightly different densities for both phantoms, as expected ($\rho_s = 1010$ kg/m$^3$, $\rho_s = 1080$ kg/m$^3$). A $2.000 \pm 0.001$ mm diameter soft magnetic sphere of normalized steel (AISI 52100) was introduced during the manufacturing of both phantoms at about $15$ mm from the bottom surface. The sphere was attached to a nylon thread fastened to a vertical axis and introduced inside the phantoms while they were still liquid, easing a precise positioning of the sphere. The location of the sphere was measured using pulse-echo measurements.

The measurement procedure is as follows: first, the sample is prepared and located in front of the ferritic core as shown in Fig. 1(a). Second, the coil is electrically excited and a pulsed magnetic force is generated on the soft magnetic sphere. Third, the motion of the sphere is tracked by the phased-array system. Finally, the viscoelastic parameters of the medium are obtained by minimization techniques. We use a curve-fitting algorithm to optimize the values of $\mu$ and $\eta$ that minimizes the least-squares error between the experimental displacement waveform and the theoretical one given by (7). Note that the value of the magnetic force is fixed during the minimization process. This procedure is repeated three times in this work for each experiment.

Experiments were carried out at an increasing temperature, taking the samples from a refrigerator and letting them slowly achieve room temperature. Measurements were taken ranging from $16^\circ C$ to $26^\circ C$. The temperature was measured using two thermistor probes (Tinytag Temperature Logger TK 4023). The first thermistor was located inside the phantom at the same height of the sphere on the lateral wall of the sample container while the second probe was placed on the top surface of the phantom.

Finally, indentation tests were performed using a mechanical testing machine calibrated to measure the bloom strength of gelatins (TAXTExpress, Texture Technologies Corporation, Hamilton, MA, USA). We used a flat-ended cylinder indenter of radius $a = 1/4$ in, compressing the samples at a rate of $0.5$ mm/s. A maximum displacement of $1$ mm was set. The shear modulus was calculated by fitting the force, $F(\Delta z)$, to $F(\Delta z) = 8 \mu a \Delta z$ where $\Delta z$ is the indentation depth [29], [30]. Indentation tests were repeated at different temperatures ranging from $20^\circ C$ to $26^\circ C$.

IV. RESULTS

A. Magnetic Field and Force Characterization

We start showing the calibration and validation of the magnetic forces acting on the sphere in the absence of viscoelastic medium. First, the simulated magnetic field in the absence of sphere is shown in Fig. 2(a). As expected, the magnetic field is highly enhanced near the tip of the ferritic core. Second, a direct measurement of the magnetic field of the system in the static regime has been carried out around the area in which the steel sphere will be located, feeding the coil using in this case a continuous current of $20$ A. The value of the $z$ component of the magnetic field was sampled along three horizontal sweeps of $40$ mm in length with a $1$ mm pitch in $x$-direction, at $10$, $15$, and $20$ mm height from the core, as shown in Fig. 2(b). The experimental data is in excellent agreement with the simulated magnetic flux density. Simulations including the sphere show that the small soft magnetic bead do not remarkably disturb the field outside the boundaries of the sphere [see dashed gray lines in Fig. 2(b)]. Finally, the magnetic force as a function of the distance to the coil, i.e., at the axis of symmetry, is shown in Fig. 2(c). First, the magnetic force was estimated using the simulated magnetic field and (9) as a function of the height. This was repeated for different currents (from $5$ to $50$ A), and (10) was fitted in a least-squares sense to the simulated force distribution in the range $5$ mm $< z < 25$ mm, obtaining.
the parameters $\alpha = 2.66 \times 10^{-10} \text{ N/A}^2 \text{ m}^2$ and $\gamma = -2.3$.

To validate this model, the magnetic field was experimentally measured along the axial direction from the tip of the core, from 3 to 24 mm, and using (9) the magnetic force was calculated. In addition, the force was directly measured using a weighing scale with an accuracy of 0.01 g ($\approx 0.1 \text{ mN}$). All measurements, direct and indirect, agree with the simulated force, as shown in Fig. 2(c). The magnitude of the force shows a power-law dependence, decreasing faster near the tip of the ferritic core (note data in Fig. 2(c) is in a logarithmic scale).

For distances around 20 mm and for small displacements of the order of 100 $\mu$m, force variations are small with respect to the force value ($<1\%$). Therefore, we consider constant the magnetic force exerted on the moving sphere around a fixed depth.

For the viscoelastic measurements, the coil was fed with a current pulse of $t_0 = 30 \text{ ms}$ and $I_0 = 30 \text{ A}$. As the coil presents a finite inductance, the current pulse shows a finite rising time, that for the current setup was set to $t_r = 3 \text{ ms}$, as measured experimentally, see Fig. 3. As compared with a flat (ideal) rectangular pulse, the effect of a finite rise time is to smooth the displacement waveform, i.e., the peak velocity of the embedded particle is decreased, and the peak amplitude of its oscillation is reduced. However, as long as the correct shape is introduced in the model using (5)–(7), the viscoelastic parameters can be retrieved. Also, note that if the pulse is long enough a quasi-static displacement value of $u_0 = F_0/6\pi \mu R$ will be also reached using the trapezoidal pulsed force.

B. Estimation of the Viscoelastic Parameters

For the first experiment, the displacement waveforms corresponding to phantoms A and B are shown in Fig. 4 for $I_0 = 30 \text{ A}$ and $t_0 = 30 \text{ ms}$. The distance from the tip of the core to the sphere was estimated using ultrasound echo-impulse measurements, obtaining $z_A = 14.6 \text{ mm}$ for phantom A and $z_B = 15.2 \text{ mm}$ for phantom B. This distance was used to calculate the force acting on the sphere using (10), as given in Table I, and used for the optimization method to calculate the viscoelastic parameters. First, for the phantom A at 24.4 °C we obtain $\mu_A = 1862.6 \text{ Pa} \pm 4.5 \text{ Pa}$, $\eta_A \approx 0.610 \text{ Pa-s} \pm 0.012 \text{ Pa-s}$. The theoretical curve for the displacement using the retrieved parameters fits the experiments for the first 15 ms, as shown in Fig. 4(a). The ratio between the initial slope and the amplitude of the first oscillation, as well as its period is properly fitted by the viscoelastic model. When glycerin is introduced in the phantoms, as in phantom B [Fig. 4(b)], the shear modulus is increased. In addition, the first oscillation is damped proportionally to the viscous modulus. The viscoelastic model correctly fits the experimental data for this
Fig. 4. Dynamics of the sphere embedded in the viscoelastic medium. (a) Displacement waveform for an individual test on the phantom A at $T = 24.4^\circ C$, measured experimentally (circles), using the fitted parameters and the viscoelastic theory (continuous). The data used for fitting was for $t < 6$ ms. (b) Corresponding displacement waveform for the phantom B at $T = 24.5^\circ C$. Velocity waveform calculated from the displacement shown in (a) from experimental displacement data (circles) and using the optimized theoretical expression in (7) for (c) phantom A and (d) phantom B.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Estimated Viscoelastic Parameters, and Measured Sphere Location, Magnetic Force and Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phantom A</td>
<td>1842.6 ± 4.5</td>
</tr>
<tr>
<td>Phantom B</td>
<td>4964.4 ± 25.3</td>
</tr>
</tbody>
</table>

It is worth noting here that theory was developed for an infinite medium. However, in practice, when the inclusion is forced to vibrate, shear waves are generated, and they propagate through the finite-size phantom. When shear waves reach the boundaries of the sample they are reflected back and, thus, the vibration of the particle is influenced by them. Fig. 4(c) and (d) illustrates the velocity signals obtained by taking the derivative of the theoretical and experimental displacement waveforms shown in Fig. 4(a) and (b). In particular, a reflection coming from the sidewalls of the container (located at a distance of $d_w \approx 25$ mm) is observed in Fig. 4(d). By considering the fitted elastic modulus, the shear wave velocity (in the low frequency limit) is $c_{s,B} \approx (\mu/\rho)^{1/2} = 2.1$ m/s. The time-of-flight corresponding to the reflection on the sidewalls is shown in gray lines in Fig. 4(d), and the reflection is visible at the sphere location, validating the calculated shear modulus. On the contrary, the low-frequency limit of the shear wave velocity for phantom A is slower ($c_{s,A} = 1.4$ m/s) and the reflection from the sidewalls arrives when the displacement pulse has already finished (at about $t = 40$ ms). Although one can argue that in this situation the plateau of the experimental displacement pulse should not manifest any deviations from the theoretical one, multiple reflections coming from other boundaries, such as the bottom ($d_b \approx 15$ mm) and top boundaries ($d_t \approx 20$ mm) of the container, prompts a noticeable distortion of the flat part of the pulse. Hence, the time window to the fit was adjusted accordingly in order to capture the dynamical response without shear wave reflections, which, for the conditions of the experiments presented in this work, results in fitting approximately the first 10 ms of the pulse, including the first oscillation.

A validation was performed using the indentation tests. For both phantoms, the measured values using the penetrometer agrees the retrieved using the current method, as shown in Table I. Finally, the temperature inside the gelatin was also measured showing negligible variations between repetitions. This is extremely important in this kind of material because, as we will show in Section IV-C, shear elasticity in gelatins is strongly dependent on temperature.
C. Temperature Dependence of Viscoelastic Properties

In order to test the robustness and consistency of the viscoelastic parameters estimated using the present method, a temperature dependence study was carried out. Using the viscoelastic phantoms A and B, a set of tests were performed while temperature was slowly increased from 16°C to 26°C, i.e., up to room temperature. First, the temperature-dependent viscoelastic parameters are shown in Fig. 5(a) and (b). The estimated parameters are shown in blue (red) markers for phantom A (phantom B). On the one hand, for the real part of the shear modulus, shown in Fig. 5(a), we observe that at low temperatures the experimental data asymptotically converges to a constant value. On the other hand, the gelatin becomes softer as the temperature rises, and the shear elastic modulus decreases. Note that at a temperature around 26°C the gelatin is about to reach a phase transition from soft-solid to viscous fluid and the method is no longer applicable.

Finally, the quasi-static viscoelastic parameters using the indentation tests are shown in Fig. 5 for both samples. The values obtained by penetrometry agree with those obtained by the proposed hybrid method for the range of measured temperatures. Both methods describe the softening of the gelatins as temperature increases and the values of elastic modulus are in good agreement.

V. DISCUSSION AND CONCLUSION

We propose a method to locally assess the complex shear modulus of a viscoelastic medium based on the measurement of the dynamical response of a millimeter-size soft magnetic sphere embedded on it under the action of a known magnetic force. The magnetic force was produced with a classical MMUS setup consisting of a coil and a ferritic steel core. Both, measured magnetic field and the magnetic forces acting on the sphere agree with simulations and theory. Finally, the measured displacement of the sphere inside the phantom was fitted to an incompressible Kelvin–Voigt-type viscoelastic model to obtain the shear elastic and viscous coefficients.

Two kinds of phantoms presenting different viscoelastic properties were considered: 1) water and gelatin powder and 2) water, gelatin powder, and glycerin. Their mechanical properties were monitored as a function of the temperature, showing a decrease in both the shear elastic and the shear viscous coefficients when temperature increases. The shear elastic coefficient of the different phantoms was independently validated using a quasi-static indentation technique, showing good agreement between the results of the indentation test and the proposed method combining ultrasound and magnetic forces. The maximum error was estimated to be around 10% at low temperature, although errors remain well below 5% for most temperatures.

As the generated magnetic field rapidly decays with distance, the penetration depth is limited in the present setup to 1.5 or 2 cm. Note ARF-based techniques are able to focus ultrasound beams at several centimeters. However, as most soft-solids are weak diamagnetic, the present system allows a precise estimation of the force acting on the inclusion. If the distance from the sphere to the ferritic core is measured, this system improves the reliability of the force calculation with respect to the estimated force using ARF techniques. Note the measurement of the location of the sphere can be accurately performed using the pulse-echo system. In ARF-based techniques, the force can vary due to a broad range of factors that include nonlinearities, inhomogeneities, or attenuation in the path between the transducer and the sphere. In contrast, a weak diamagnetic medium does not influence the value of the magnetic force. For this reason, the proposed technique allows a robust characterization of the medium and is able to provide not only the elastic shear coefficient but the viscous shear coefficient. Note that, as occurs with ARF-based techniques, the theory used here for the fitting assumes a homogeneous medium. If shear waves are scattered by an abrupt change of elastic properties or density, the particle vibration will be the superposition of the free oscillation and the motion due to reflected shear waves. Therefore, the method is restricted to homogeneous media. In particular, for soft-solids of shear modulus in the range 1 to 10 kPa, a volume with a radius of 15 mm around the sphere should not create strong shear wave reflections.

It is worth mentioning that MMUS imaging techniques using magnetic nanoparticles excited using an externally
applied magnetic field and then imaged using ultrasound have been proposed in the past [18]–[22]. Using its pulsed modality these techniques allow the visualization of events at molecular and cellular levels [31], [32]. The potential of this technique to assess viscoelastic properties of tissues was also pointed out but specific values were not reported [33]. Moreover, MMUS systems have been used to generate shear acoustic waves for ultrasound elastography imaging [17], [34] or to generate pulsed displacements for optical coherence elastography [35]. MMUS techniques have been applied in vivo [36], [37], and they have been combined with other imaging techniques such as photoacoustics [38], positron emission tomography (PET) or magnetic resonance imaging (MRI) [39]. In the proposed method, we employ a macroscopic soft magnetic inclusion instead of using magnetic nanoparticles. Because of the size of the object, the in vivo application of the method for tissue elastography is limited. However, due to the fact that the force and the induced displacements can be accurately described by analytical methods when using a macroscopic object, the proposed method results in a precise estimation of the viscoelastic parameters, which may have potential applications in viscoelastic material characterization in the industry.

In order to envision potential applications in the medical field using this technique, replace the macroscopic object with magnetic nanoparticles will be mandatory, which will result in a more complicated estimation of the magnetic force due to the unknown distribution of the nanoparticles within the tissue. Other factors, such as a precise estimation of the magnetic susceptibility of the magnetic nanoparticles, as well as the use of larger magnetic fields and their gradients are to be taken into account. In this regard, effort has been made in the research community in order to precisely estimate the magnetic force while applying a magnetic field to magnetic nanoparticles [40], [41], making feasible to extend this technique to a more realistic preclinical/clinical application.

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