Chapter 9

Nonlinear Acoustic Radiation Forces in Biological Media

Abstract

In this Chapter we present the finite-amplitude dependence of the acoustic radiation force (ARF) in a soft tissue-like media. The acoustic radiation force is calculated in media with different exponent of the frequency power law attenuation. Numerical results are obtained for one dimensional propagation, where the spatial distribution of the force field is studied. The results obtained show that the nonlinear dependence of the acoustic radiation force sharply depends on the tissue attenuation model for both, weakly and strong nonlinear regime. We also show the underestimation of the momentum transfer in the nonlinear regime by calculating the ARF by using the wave intensity. Considerations due to nonlinear absorption are included and the relation between the momentum transfer and energy deposition is underlined.

9.1 Introduction

Waves carry energy and momentum. Propagation trough lossless homogeneous media implies the absence of interaction between the medium and the wave. However, if scattering or absorption processes are present, the energy and momentum can be transferred from the wave to the medium. In this sense, the radiation force is an universal wave phenomenon related to the momentum transfer from a wave to the medium. It has been demonstrated to exist for electromagnetic (Lebedew, 1901), acoustic waves (Faraday, 1831; Rayleigh, 1902), and also by phonons in crystal lattices (Sorbello, 1972) as those presented in Chapter 1.

In the case of acoustic waves, acoustic radiation forces (ARF) appear when a gradient in the acoustic energy density is produced by scattering or absorption processes. We refer to the critical review of Sarvazyan et al. (2010) for a broad historical references and a clear description of the ARF in the acoustic context. The acoustic radiation force is a time-average second-order process. In the case of a continuous monochromatic wave, steady state acoustic radiation forces appear when the wave is scattered by an object, reflected by a rigid surface or an interface between two media, or absorption is produced in the propagation. In all these situations, some momentum is transferred from the wave to the medium, and the consequence is that the acoustic radiation tensor Beyer (1978) is no longer spatially constant. In these cases, the bulk of the medium is time average stressed. The radiation forces are only one of the consequences of this stresses. In fluids, the acoustic radiation stresses induce time average movements or streams. In solids and elastic media in general, time average stresses induce constant strains, being the material deformed in the direction of the wave but also in the transversal direction. In fluids with inclusions, the scatterers can be statically compressed, deformed, pushed, and also rotated by the induced acoustic radiation stress.

Therefore, is no surprising the huge amount of practical applications that exploit the phenomena related acoustic radiation stresses due to the extremely broad possibilities for interacting with the media using acoustic waves.

In the case of biological media and medical ultrasound applications, the use of acoustic radiation force includes the power calibration of High Intensity Focused Ultrasound (HIFU) devices by radiation force balances (Wood et al., 1927); acoustical tweezers and levitation for particle and cell manipulation (Wu, 1991; Lee et al., 2006; Hultström et al., 2007); stimulation of sensory receptors by ultrasound radiation force or improving targeted drug and gene delivery (Sarvazyan et al., 2010). On the other hand, acoustic streaming can be used for stirring and mixing liquids (Sarvazyan et al., 2009), remote assessment of biological fluids (Nightingale et al., 1995) and microfluidics applications (Laurell et al., 2007; Hultström et al., 2007). It is worth noting here that in some of these techniques the effects of the acoustic radiation force and streaming are closely linked.

One important remark is that the time averaging of the acoustic radiation stresses can be modulated by a slowly varying time function. If the wave amplitude presents a low frequency modulation (compared to the acoustic variations), the radiation stresses are also modulated in time, leading to oscillating radiation forces inside the media. In elastic media, these stresses dynamically deform the bulk of the medium in axial and also in transversal direction. Thus, time varying shear deformations induce shear waves at the modulation frequency, that propagates transversely to the direction of the wave.

A huge amount of ultrasound medical diagnostic techniques have been developed in order to asset the media elasticity by measuring the mechanical response of the tissues under the action of acoustic radiation stresses. In general, the elasticity can be evaluated by measuring in some way the strains induced by the ultrasound field. These techniques include vibroacoustography (VA) (Fatemi et al., 1998; Urban et al., 2011), shear wave elasticity imaging (SWEI) (Sarvazyan et al., 1998), acoustic radiation force impulse imaging (ARFI) (Nightingale et al., 2001; Trahey et al., 2004), supersonic shear imaging (SSI) (Bercoff et al., 2004), harmonic motion imaging (HMI) (Konofagou et al., 2003; Vappou et al., 2009) and real time monitoring of lesion formation in HIFU therapy (Maleke et al., 2008). Also, the induced tissue deformations can be tracked with other imaging techniques as magnetic resonances (Fowlkes et al., 1995; Manduca et al., 2001). Acoustic streaming can be used to assessment and detecting fluid-filled lesions or cysts (Nightingale et al., 1995; Nightingale et al., 1999) or monitoring blood coagulation by sonorheometry (Viola et al., 2004). We refer to the reviews (Sarvazyan et al., 2010; Palmeri et al., 2011; Doherty et al., 2013) and references therein for further details.

Human soft tissues show strong acoustic attenuation processes, but also internal scattering due to micro-inhomogeneity. It is commonly accepted that the contribution to the acoustic radiation force in the bulk of soft tissues is mainly caused by absorption rather than scattering processes (Doherty et al., 2013). In order to obtain high amplitude acoustic radiation force fields and induce detectable deformations, the amplitude of the ultrasonic excitation is commonly high enough to excite nonlinear effects in some degree (Carstensen et al., 1980), even in diagnostic techniques (Sarvazyan et al., 2010). On the other hand, although radiation forces are observed also for small amplitude waves, the acoustic radiation force is a second order process and therefore requires solutions to the nonlinear model equations.

At ultrasound frequencies, soft tissues do not effectively support shear wave propagation. The typical shear wave speed in soft tissue is around two orders of magnitude lower that the longitudinal acoustic wave speed. Shear waves generated by low frequency modulated acoustic radiation forces, present different spatio-temporal scale to the longitudinal waves. The common approach is to model first the acoustic radiation force generated by the ultrasonic longitudinal waves. Then, shear waves and the corresponding tissue strain are treated as uncoupled problem: the time-varying acoustic radiation force field is used as an excitation of some elastic model, commonly a linear elastic or viscoelastic solid model. Therefore, the constitutive equations for nonlinear acoustics are accurate to describe the nonlinear generation of acoustic radiation force, provided that the proper attenuation and dispersion are included in the model as a frequency power law.

The aim of this Chapter is to study the acoustic radiation forces induced by intense waves in soft tissue. Thus, we present the nonlinear dependence of the acoustic radiation force by solving numerically the constitutive equations for nonlinear acoustics in a broad range of frequency power law attenuation media. We show special emphasis in the effects of the power law frequency attenuation.

9.2 One dimensional acoustic radiation force

The problem of accurately solve the acoustic radiation force is associated to the knowledge of the full acoustic field variables: pressure p, density ρ and particle velocity vector $\mathbf{v}$. For plane waves in a lossless homogeneous media, the relation between these quantities is well-known, but in this case no acoustic radiation force appears. The inclusion of some frequency dependent attenuation and its corresponding dispersion induce a phase shift between the acoustic variables and its relationship becomes more complicated.

For the sake of simplicity, we start by review the acoustic radiation force generated by plane waves. We first expand in series the acoustic magnitudes, p, ρ, v as in Refs (Sarvazyan et al., 2010; Doherty et al., 2013). Thus, each term includes a high-order correction to to the steady-state solutions p_0, ρ_0, v_0. For a quiescent homogeneous fluid, $v_0 = 0$ and p_0 and ρ_0 are time independent, p_1, ρ_1 and v_1 are the first order approximations to the acoustic field, and p_2, ρ_2 and v_2 are the second order corrections. Thus, the field can be expressed as

$$p = p_0 + p_1 + p_2 + \cdots \tag{9.1}$$

$$\rho = \rho_0 + \rho_1 + \rho_2 + \cdots \tag{9.2}$$

$$v = 0 + v_1 + v_2 + \cdots \tag{9.3}$$

assuming that each term is small that the preceding one.

The problem of modeling the acoustic radiation force F^V (per unit volume), reduces to obtain the time-average of the change in momentum for a fluid as

$$F^V = -\left\langle \rho \frac{\mathrm{D}v}{\mathrm{D}t} \right\rangle . \tag{9.4}$$

It is important to note that the period average (indicated as $\langle\rangle$) of the first order magnitudes (p_1, ρ_1, v_1) is zero. However, the time average of the momentum does not vanish due to the nonlinearity in material derivative of the momentum Eq.(9.4). Substitution of Eqs.(9.1-9.3) into the momentum equation, and keeping up to second order terms leads to (Doherty et al., 2013)

$$F^V = -\left\langle \rho \frac{\mathrm{D}v}{\mathrm{D}t} \right\rangle = -\left\langle \rho_0 \frac{\partial v_2}{\partial t} + \frac{\partial \rho_1 v_1}{\partial t} + \rho_0 \left(v_1 \nabla \cdot v_1 + v_1 \cdot \nabla v_1 \right) \right\rangle . \tag{9.5}$$

The period average of the first two terms of the right hand side is zero, but the last 2 terms, that account for the nonlinearity of the material derivative, does not vanish. Thus, for a plane wave propagating in the x direction, the acoustic radiation force per unit volume reads

$$F_x^V = -\left\langle \rho_0 \frac{\partial v_1^2}{\partial x} \right\rangle . \tag{9.6}$$

Then the volume radiation force for a decaying plane harmonic wave with instantaneous intensity $I' = p_1 v_1 = \rho_0 c_0 v_1^2$, traveling in an homogeneous-lossy media can be expressed as

$$F_x^V = -\frac{\partial}{\partial x} \left\langle \frac{I'}{c_0} \right\rangle , \tag{9.7}$$

or in terms of the time-average intensity I in a period $T = 2\pi/\omega$

$$I = \frac{1}{T} \int_t^{t+T} I' \, \mathrm{d}t = \langle I' \rangle , \tag{9.8}$$

the volume radiation force is proportional to the gradient of the intensity divided by the sound speed

$$F_x^V = -\frac{\partial}{\partial x} \left(\frac{I}{c_0} \right) . \tag{9.9}$$

Here we shall recognize that the quantity I/c_0 can be expressed as

$$\frac{I}{c_0} = \frac{1}{2} \rho_0 v^2 + \frac{1}{2} \frac{p^2}{\rho_0 c_0^2} , \tag{9.10}$$

where it is evident that is the total energy density for a plane wave. Thus, is clear that Eq. (9.9) essentially states that an acoustic radiation force must appears when a spatial change in the energy density distribution is produced. The derivation of this equation was developed for homogeneous media, thus, strictly speaking a change of the energy density can only be produced by absorption processes. However, it is well-known that acoustic radiation forces also appear when a change of the energy density is produced by reflection between media with different acoustic properties. Here, we ignore these contributions and assume a homogeneous absorbing soft tissue media, where the contribution of the tissue attenuation to the absorption of the wave is dominant compared to the internal scattering processes.

Small amplitude waves

Thus, if considering an absorbing-media with an attenuation coefficient α, a monochromatic plane wave traveling will present a mean intensity as

$$I = \rho_0 c_0 V_0^2 \exp(2\alpha x)/2, \tag{9.11}$$

with V_0 the excitation particle velocity.

Substituting the intensity of Eq. (9.11) in Eq. (9.9), the time average force per unit volume reads

$$F^V_{x,\mathrm{lin}} = 2\alpha\frac{I}{c_0} = \alpha\frac{{p'}^2}{\rho c_0^2}\,, \tag{9.12}$$

that is the most common approach to calculate the acoustic radiation force in absorbing media. We shall note some peculiarities of this simple expression. It can be observed that the resulting force is proportional to the wave intensity and the attenuation coefficient. For diagnostic ultrasound frequencies the absorption coefficient is typically high, around 1 dB/cm or even higher, and for the commonly used intensities the magnitude of the acoustic radiation force is weak. Therefore, the induced displacements are had to detect ($< 1\mu$m) (Doherty et al., 2013). Thus, for induce detectable displacements (typically from 1 to 10 μm) two basic mechanisms are used. First is focusing. By increasing the gain of the source the intensity can be increased locally inducing higher ARF and displacements. On the other hand, the intensity of the field can be increased directly by increasing the amplitude of the excitation. However, for both approaches the calculation of the radiation force becomes complex.

In the first place, Eq. (9.12) was derived for plane waves and therefore, the calculation of the acoustic radiation force for more complex fields by using this expression is not accurate. For directional beams this expression is valid for low focused fields and only at the focal, where the wavefront can be considered quasi-plane. However, for strongly focused fields the assumption of planes waves no longer holds. For an accurate description of the acoustic radiation force the spatial features of the beam must be included. In the case of paraxial beams analytic approaches can be obtained for intense waves (Rudenko et al., 1996; Ostrovsky et al., 2007; Ostrovsky, 2008), but for strongly focused sources a better theoretical foundation is needed. Approaches include calculating the full radiation stress tensors numerically or by indirect measurement of the acoustic radiation forces, e.g. by estimating streaming in frequency power law absorbing fluids.

Nonlinear regime

Secondly, if the source amplitude is increased, nonlinear effects can be produced. In the case of weakly nonlinear effects, the absorption of each harmonic is different following the tissue frequency power law attenuation. If Eq. (9.12) is used the force can be calculated by the sum of the contributions of each harmonic separately. However, strictly speaking this equation is not exact in nonlinear regime. The main reason is that in nonlinear regime the intensity distribution of the wave is not exactly the given by Eq. (9.11). As long the energy is transferred to the higher harmonics, and the higher spectral components present higher attenuation, the intensity decay is not exponential, leading to the so-called nonlinear absorption. Thus, in the nonlinear regime these processes leads to a increasing rate of momentum transfer from the wave to the media, and therefore the acoustic radiation force is increased.

If the losses are caused by thermo-viscous effects, alternatively the nonlinear acoustic radiation force can be calculated directly using the expression (Rudenko et al., 1996)

$$F^V_{x,\text{thermo}} = \frac{b(\beta' + 1)}{c_0^5 \rho_0^3} \left\langle \left(\frac{\partial p'}{\partial t} \right)^2 \right\rangle , \tag{9.13}$$

where β' is the nonlinear compressibility (often neglected) and the dissipation coefficient, b, is given by

$$b = \frac{4}{3}\mu + \mu_B + \kappa \left(\frac{1}{C_V} - \frac{1}{C_p} \right) , \tag{9.14}$$

that accounts for the quadratic frequency attenuation caused sound thermo-viscous diffusion processes, being μ the shear viscosity, μ_B the bulk viscosity, κ the thermal conductivity and C_V and C_p the specific heats at constant volume and pressure respectively.

Equation (9.13) is valid in both, linear and nonlinear regimes. In the first case, substitution of a monochromatic wave in Eq. (9.13) gives Eq. (9.12). On the other hand, Eq. (9.13) states that for complex nonlinear acoustic fields with sharp waveforms, as those containing shock waves, the squared time-derivative of the pressure will have a non negligible period average value. Therefore, the nonlinear acoustic radiation force is increased when sharp waveforms are present in the domain.

Sawtooth regime

If the nonlinear processes are strong enough to produce very sharp wave steepening, the wave can be described by the a sawtooth waveform. This regime occurs at distances $\sigma > 3$ and Gol'dberg ratios $\Gamma >> 1$. Thus, an analytic expression can be obtained for waveforms containing shock waves by substituting a sawtooth waveform in Eq. (9.13). The substitution of a sawtooth profile into Eq. (9.13) gives (Pishchalnikov et al., 2002; Rudenko et al., 2004)

$$F_{x,\mathrm{saw}}^{V} = \frac{2\pi^2}{3}\left(\beta' + 1\right)^2 \omega \frac{\rho_0}{c_0^2} u_0^3\left(x\right), \tag{9.15}$$

where $u_0(x)$ is the amplitude of the particle velocity in one period of the sawtooth. It is important to note that in the sawtooth regime the acoustic radiation force does not depend on the absorption of the media. This effect is due to the nonlinear intensity absorption dominates over the tissue attenuation in the sawtooth regime. On the other hand, the cubic amplitude dependence, instead of the quadratic one for the linear estimation, evidences the great potential of the sharp waveform to enhance acoustic radiation force in the nonlinear regime.

Thus, the acoustic radiation force for plane waves is well understood in the case of thermo-viscous losses, where the magnitude of the force varies between to limits (1) the small amplitude regime, where the force decays exponentially with distance, and (2) the nonlinear absorption regime, where the ARF is proportional to the cube of the sawtooth peak particle vibration.

9.3 Nonlinear radiation force in tissue-like media

In the case of frequency power law absorption media, the acoustic radiation force for small amplitude waves is equivalent by selecting the appropriate attenuation coefficient and the intensity of each spectral component. However, Eq. (9.13) is no longer valid for media with non-quadratic frequency dependent attenuation, and therefore the calculation of the ARF in the nonlinear regime becomes complex. Here, we calculate the ARF using the mean change in momentum by solving numerically the full constitutive relation of nonlinear acoustics by using the method described in Chapter 7.

The aim of our work is to understand the role of the frequency dependent attenuation in the nonlinear generation of acoustic radiation forces.

We start presenting the intensity of a plane wave in two absorbing media, for thermo-viscous absorption, where the exponent of the frequency power law is $\gamma = 2$ and for tissue absorption, $\gamma = 1$. Figure 9.1 shows the intensity distribution for these two attenuation models, where each curve represents a different Gol'dberg ratio: for $\Gamma >> 1$ the nonlinear effects dominate over attenuation and the propagation can be considered lossless, and for $\Gamma \to 0$ the nonlinear effects are negligible and the propagation can be assumed linear. The analytical intensity for a lossless media is also presented (Hamilton et al., 1998a), were it can be appreciated the

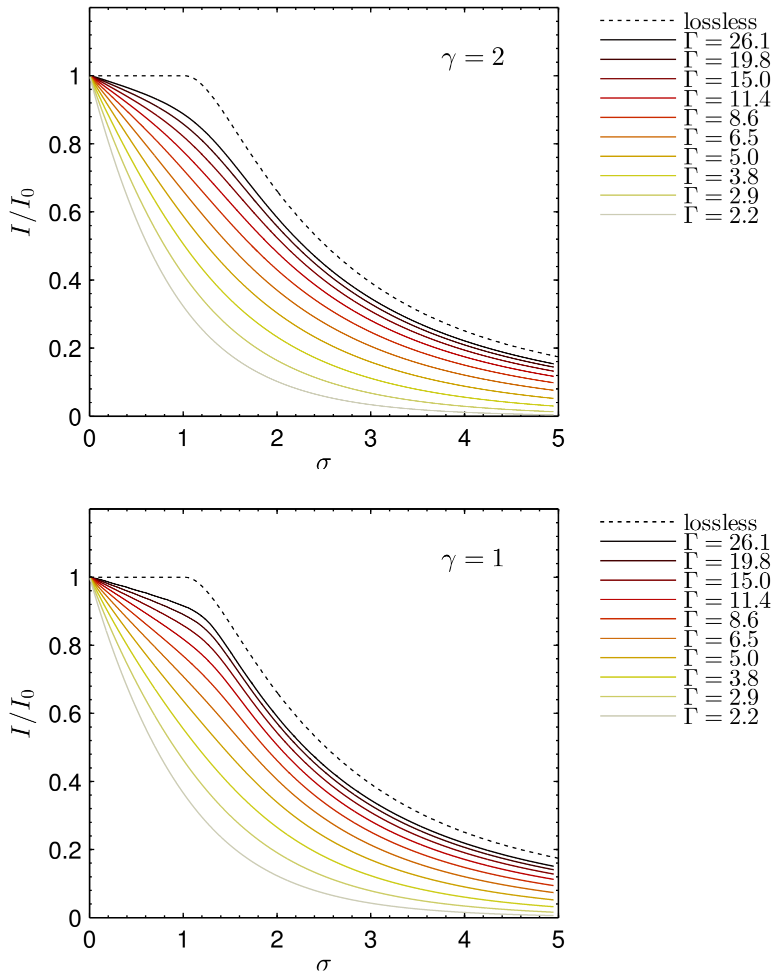

Figure 9.1: Spatial distribution of the nonlinear intensity in (top) viscous media and (bottom) tissue for increasing ratio between nonlinear and attenuation effects (Γ is the Gol'dberg ratio).

well-known distribution: in the shock-free region the intensity is constant and beyond $\sigma > 1$ the nonlinear absorption activates and the intensity drops, even for lossless media.

For lower Gold'berg ratios the intensity decays exponentially in both lossy media. However, for higher wave amplitudes, when $\Gamma >> 1$ the intensity does not decays exponentially and appreciable differences can be observed between thermoviscous and tissue model. Only for the very high nonlinear regimes the intensity distribution in both models converges to the lossless case. In these cases the nonlinear effects develop fast and the attenuation have no appreciable effect. Also,

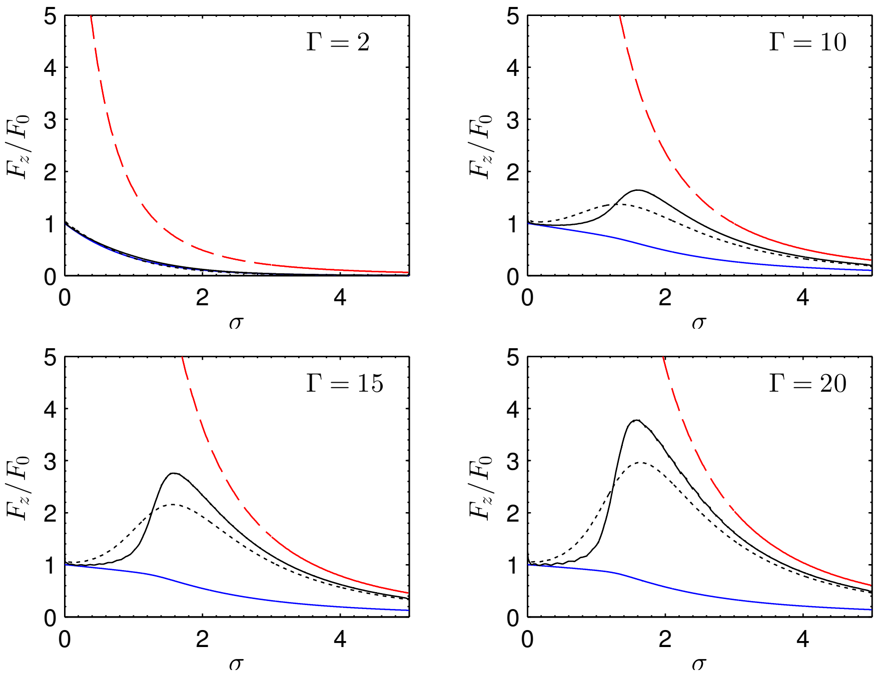

Figure 9.2: Spatial distribution of the normalized acoustic radiation force (ARF) for different Gold'berg ratios (Γ). The distribution of the ARF is calculated for (black continuous line) soft-tissue and for (black dotted line) thermo-viscous media. The limiting case of linear ARF calculation using Eq. (9.12) is shown in blue line, while the sawtooth regime using Eq. (9.15 (valid for $\sigma > 3$) is shown in red lines.

in all cases it can be seen that the intensity distribution have different shape in the shock-free region and beyond the shock formation distance.

Thus, as long the media is homogeneous and sound speed is constant, is evident that there exist differences in the gradient of the energy density. Therefore the resulting acoustic radiation force is amplitude dependent, and also different between attenuation models.

A close view can be observed in Fig. 9.2. There, the acoustic radiation force is shown for four selected amplitudes, being $\Gamma = (2, 10, 15, 20)$ respectively. The limiting cases of linearized calculation using Eq. (9.12), and the sawtooth regime using Eq. (9.15) are shown in blue and red lines respectively. First, for $\Gamma = 2$, the propagation is nonlinear, but the attenuation processes still comparable to the nonlinear processes. In this case, the intensity decay is almost exponentially, and therefore the ARF is proportional to the attenuation coefficient and wave intensity. The ARF in this case decays also almost exponentially and the expression (9.12), although not exact, is accurate to describe the ARF generation for both, thermo-viscous and absorbing media.

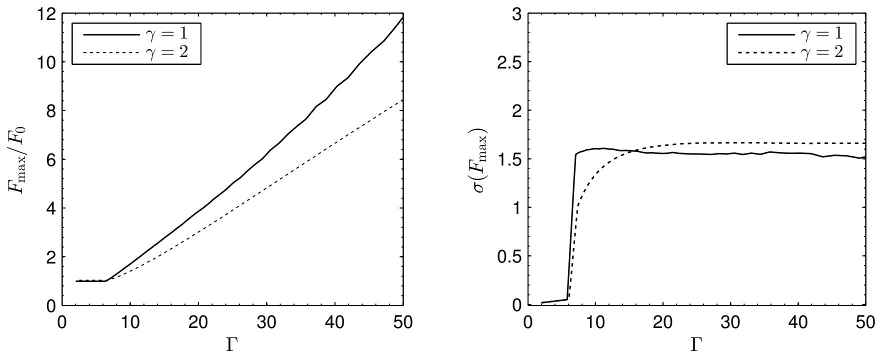

Figure 9.3: Peak acoustic radiation force value and its location in σ coordinate as a function of the Gol'dberg ratio for (continuous line) soft-tissue model and (dotted line) thermo-viscous lossy medium.

However, if wave amplitude is increased, i.e. for $\Gamma = (10, 15, 20)$ in Fig. 9.2, the ARF distribution is no longer accurately described by Eq. (9.12). Moreover, there exist remarkable differences between tissue and thermo-viscous models. First, we set the same attenuation coefficient at the fundamental frequency for both models. Therefore, due to the attenuation is given by $\alpha = \alpha_0 \omega^\gamma$, in the case of thermo-viscous model the high spectral components present high overall attenuation than in the tissue model. Thus, in the nonlinear regime high spectral components are generated in both models, but in the thermo-viscous model they are attenuated faster and close to the source. The momentum transfer is increased near the source. By contrast, in the tissue model the attenuation of higher spectral components is lower, and therefore the momentum transfer is weaker near the source.

On the other hand, at distances near the shock formation distance this situation is inverted. At these distances, the amplitude of the harmonics in the tissue model is higher than in the thermo-viscous model due to the weaker high-frequency absorption. Thus, when nonlinear absorption activates and strongly dominates over tissue absorption, the momentum transfer of the tissue model is remarkable higher than in thermo-viscous lossy media.

Finally, for higher distances, and very strong nonlinearity, the ARF observed in both models converge to the sawtooth profile given by Eq. (9.15). This fact evidences that also in tissue model, the momentum transfer for waves including shocks is governed by the nonlinear absorption processes and almost independent of the tissue properties: the attenuation value and the exponent of the frequency power law attenuation. However, we remark that for intermediate distances at which there exists shocks, but the sawtooth profile is not yet developed, let say $1 < \sigma < 3$, the specific tissue power law strongly affects the acoustic radiation force generation.

The maximum value of the acoustic radiation force and its location is presented in Fig. 9.3. First, it can be seen that for low Gol'dberg ratios the acoustic radiation force maximum in located at the source and the value is proportional to the source intensity $F_0 = 2\alpha I_0/c_0$. In these situations the media attenuation is stronger than the nonlinear absorption, and the ARF decays from its initial value. However, for higher Gol'dberg ratios the strong nonlinear effects dominates over absorption. As explained above the momentum transfer is increased due to nonlinear absorption processes and the acoustic radiation force is strongly enhanced. For these plane wave simulations, even without focusing, the nonlinearity increases the acoustic radiation force a order of magnitude relative to the acoustic radiation force at the source. On the other hand, an interesting feature is observed: due to the maximum of the acoustic radiation force is produced before the sawtooth regime, the maximum enhancing depends on the power law exponent.

Thus, this results evidences the importance of including soft tissue losses for estimate acoustic radiation forces, even in situations with strong nonlinearities. On the other hand, the maximum momentum transfer is located around $\sigma = \pi/2$, where shocks are fully developed and the shock amplitude (the pressure jump at the discontinuity) is maximum. It can be observed that in Eq. (9.13), the maximum possible value of the time-derivative will be produced by a sharp time waveform. Thus, the maximum value is obtained at the location where the discontinuity jump reaches its maximum, for lossless propagation occurs exactly at $\sigma = \pi/2$.

9.4 Power law dependence

We have shown the nonlinear generation of the acoustic radiation force for the case of linear frequency power law and for thermo-viscous losses. Here we extend the analysis for intermediate values of the exponent of the frequency power law. Thus, Fig. (9.4) presents an overview, where it is shown the acoustic radiation force generated as a function of the distance and as a function of the exponent of the power law. In first place, Fig. (9.4) (top) presents a case where nonlinear effects are of the order of the attenuation processes. Here, the ARF decays almost exponentially and there does not exist remarkable differences between the frequency power law attenuation models. The ARF is slightly greater for the higher exponents of the power law due to the absorption of the generated high spectral components is greater. Note in the linear regime the effects of the exponent of the power law are negligible for monochromatic plane waves.

However, for high nonlinear regimes, there can be observed a high (an non-trivial) dependence in the ARF and the exponent of the power law. There can be seen that the maximum ARF are generated in the case of linear power law. For thermo-viscous losses ($\gamma = 2$) the ARF is generated at higher rate near the source and the peak value is lower than for the linear power law case. But for intermediate values, there exist a trade-off between these effects: in one hand, the specific power law increases the attenuation in the high frequency limit, the absorption of the spectral components is increased rising the ARF near the source. On the other hand, when the power law exponent is increased, the nonlinear intensity absorption is increased near the source and therefore the intensity of the wave when shocks are produced is reduced. Thus, around $\sigma = \pi/2$ the wave amplitude strongly depends on the attenuation produced before, near the source. Therefore the enhancement of the ARF generated by the sharp waveforms beyond the shock formation distance strongly depends on the trade-off between media absorption and the specific wave amplitude due to cumulative nonlinear absorption processes.

Figure 9.5 shows the ARF at four distances as a function of the exponent, where the above described effects can be observed. Near the source ($\sigma = 0.5$), the momentum transfer is governed by the media attenuation. Higher the exponent higher the momentum transfer for the superior harmonics and therefore, higher the nonlinear acoustic radiation force. This behavior holds until the existence of shocks in the solution, at $\sigma = 1$. For longer distances, the situation is inverted. At the location where the discontinuity jump is maximum ($\sigma = \pi/2$) the maximum of the ARF is achieved for linear frequency power laws. For higher exponents, the value of the ARF is reduced. However the minimum is not located for the thermo-viscous case, but slightly before, around $\gamma = 1.8$.

Here, we need to add another effect that we have omitted, that is the weak dispersion of the frequency power law attenuation media. As seen in Chapter 8, the efficiency in the harmonic generation is also dependent of the exponent of the power law. Essentially, the energy transfer from fundamental to higher harmonics is maximized for nondispersive case ($\gamma = 2$), and being reduced for nearly linear power laws due to weak dispersion. Thus, the harmonic energy transfer is increased for thermo-viscous losses, and due to its higher attenuation of the higher spectral components the ARF is slightly increased. The competition between nonlinear generation, attenuation and dispersion effects, and the nonlinear intensity absorption leads to the specific curves obtained numerically, where it is evident the critical effect of the power law absorption to accurately describe the acoustic radiation force. Differences between absorption models can lead to differences of the ARF values (about 25%) and also strong differences in the spatial distribution.

9.5 Relation of nonlinear absorption and tissue heating rate

Nonlinear absorption in tissue

One of the factors that are closely related to the nonlinear behavior of the acoustic radiation force is the nonlinear intensity absorption. It is clear from Eq. (9.9), that the acoustic radiation force is closely dependent on the intensity distribution. On

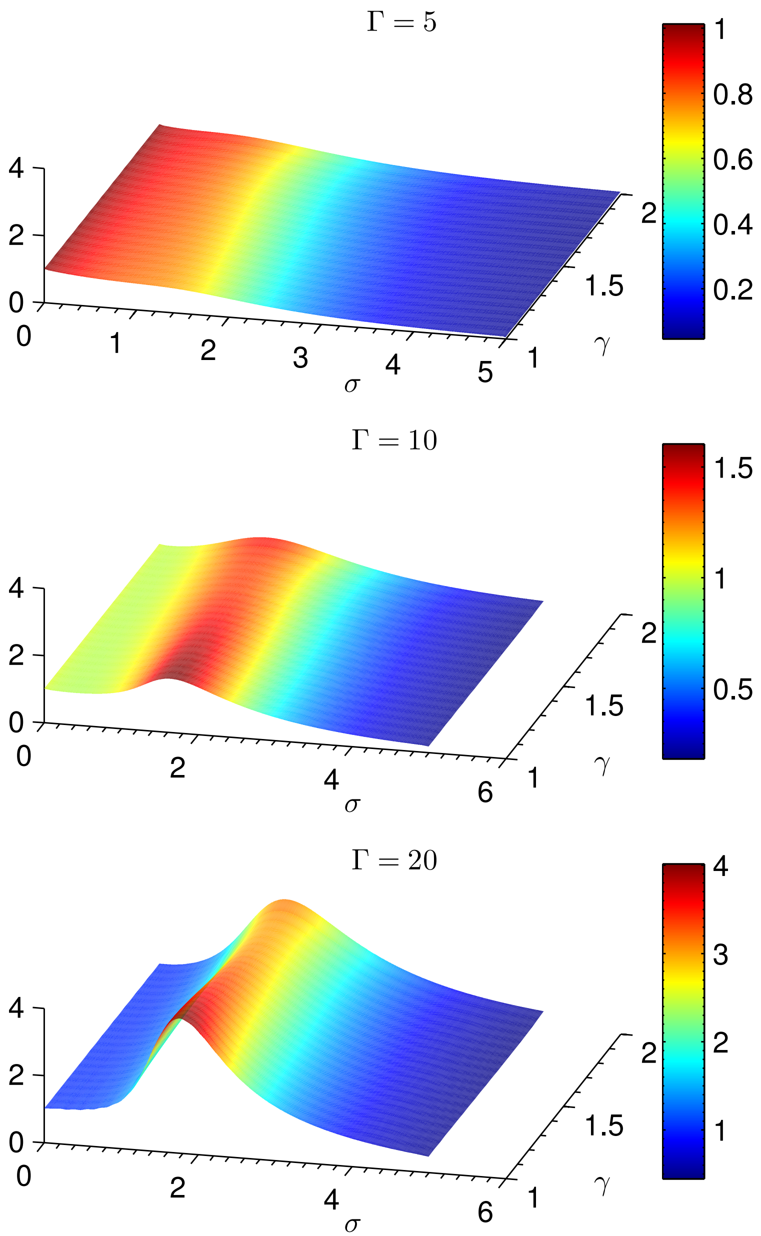

Figure 9.4: Spatial distribution of the acoustic radiation force for $\Gamma = (5, 10, 20)$ as a function of the exponent of the frequency power law attenuation, γ and the propagation distance. Colorbars in F/F_0 units where F_0 is the ARF at the source.

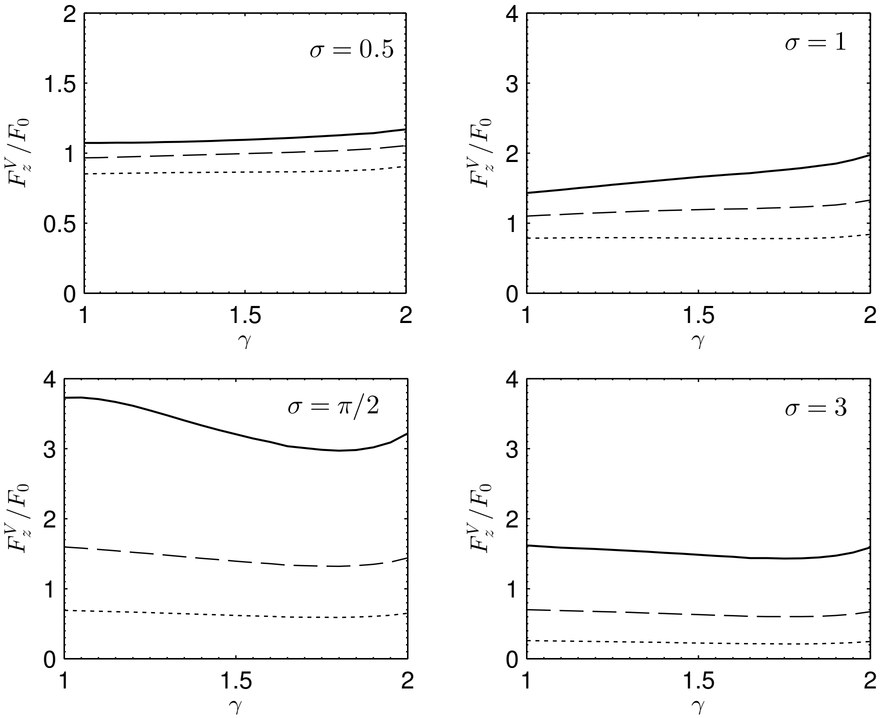

Figure 9.5: Acoustic radiation force (ARF) as a function of the power law exponent measured at distances $\sigma = (0.5, 1, \pi/2, 3)$. Dotted lines are ARF for a Gol'dberg ratio $\Gamma = 5$, dashed lines for $\Gamma = 10$ and continuous lines for $\Gamma = 20$.

the other hand, by the definition of the absorption we get

$$\alpha_f = -\frac{\nabla \cdot I}{2I}. \tag{9.16}$$

For small amplitude plane wave with $I = I_0 \exp(-2\alpha x)$, it reduces to

$$\alpha_f = -\frac{1}{2I}\frac{\partial I}{\partial x} = \alpha, \tag{9.17}$$

that is the linear absorption coefficient. However, for finite amplitude waves the effective intensity absorption is increased. For the lossless case, (equivalent to $\Gamma \to \infty$), the nonlinear absorption is a function of the shock amplitude, P_{sh}, (Hamilton et al., 1998a) as:

$$\alpha_f = -\frac{\frac{2}{3}P_{\text{sh}}^3}{\pi - \sigma P_{\text{sh}} + P_{\text{sh}}\cos\sigma P_{\text{sh}} + \frac{2}{3}\sigma P_{\text{sh}}^3}\,. \tag{9.18}$$

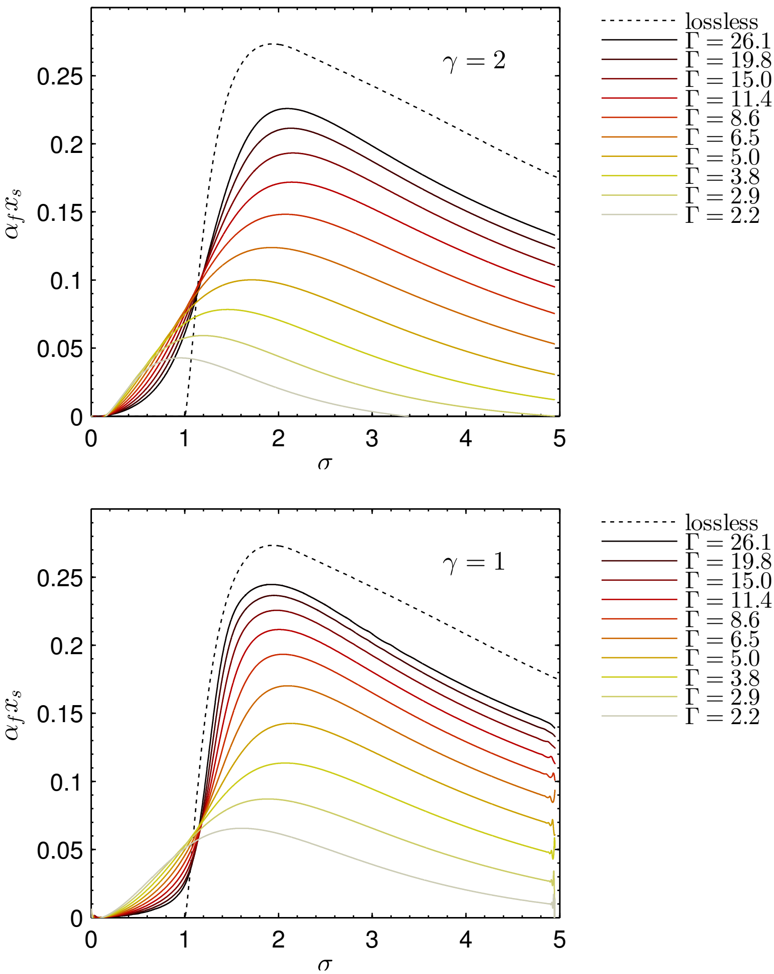

Figure 9.6: Nonlinear absorption for different Gol'dberg ratios. (Top) thermo-viscous absorption model, $\gamma = 2$, and (bottom) tissue absorption model, $\gamma = 1$.

In the case of thermo-viscous absorption the intensity can be obtained analytically from th Mendousse solution (Hamilton et al., 1998a), and the nonlinear absorption calculated using Eq. (9.16). In the case of other tissue model, the nonlinear absorption is calculated by obtaining numerically the intensity and then apply Eq. (9.16).

Thus, Fig. 9.6 shows the nonlinear absorption obtained for thermo-viscous and tissue models. These results have been partially shown in the literature previously, see e.g. the work of Kashcheeva et al. (2000), we review here for convenience. We can see that the nonlinear absorption is present in all simulations. Near the source,

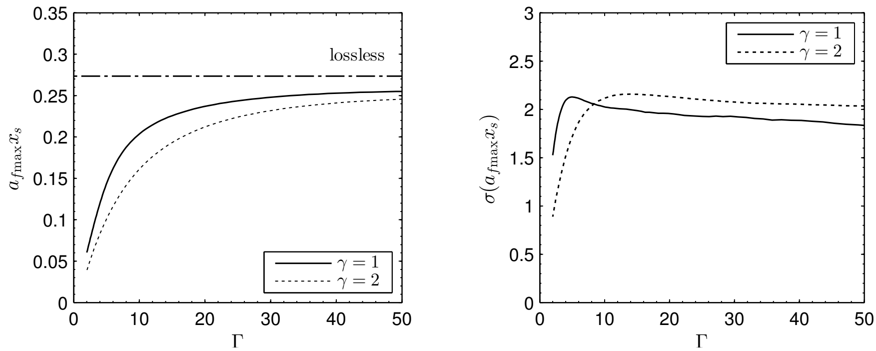

Figure 9.7: (Left) Maximum value of the nonlinear absorption as a function of the Gol'dberg ratio for (continuous line) tissue and (dotted line) thermo-viscous media. (Right) Location in the σ coordinate as a function of the Gol'dberg ratio.

in the case of thermo-viscous losses the effective intensity absorption is higher than in the linear power law due to strong attenuation at the higher spectral components and when shocks are present, as explained above, the nonlinear absorption is higher for linear power law. The nonlinear intensity converges to the lossless estimation for very strong nonlinearities. In the case of linear power law the convergence is faster due to the low effective attenuation near the source.

These features are consistent to the acoustic radiation force, higher the effective attenuation, higher the ARF value. However, the maximum of the acoustic radiation force does not necessary coincide with the maximum of the nonlinear absorption, even in plane waves. By comparing Eq. (9.9) and Eq. (9.16), it can be appreciated the difference: the acoustic radiation force is proportional to the change of the energy density, while the nonlinear absorption is proportional to the change of the energy density *normalized* to the energy density. Note that in the linear regime, the acoustic radiation force decreases exponentially while the effective absorption is constant in space. For weak nonlinear effects, as shown by the lower curves in Fig. 9.6, the acoustic radiation force decays almost exponentially and the maximum is located at the source, while the maximum of the nonlinear absorption is always not at the source, but around $\pi/2$. See also Figs. 9.7 and compare to Figs. 9.3. Both magnitudes are closely related though the momentum transfer, energy deposition, and intensity reduction, but are not the same.

Nonlinear tissue heating rate

As studied above, the result of the momentum transfer from the wave to the medium due to absorption processes is the appearance of a radiation force. On the other hand, the result of the dissipation of the acoustic energy of the wave is that the medium is heated. The heating rate per unit volume, Q_v, at which the temperature T is increased is:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{Q_v}{\rho_0 C_p}\,, \tag{9.19}$$

where C_p is the specific heat capacity at constant pressure (per unit mass) of the medium. On the other hand, the heating rate can be related to the wave intensity as (Hamilton et al., 1998a):

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{1}{\rho_0 C_p}\nabla \cdot I\,. \tag{9.20}$$

As can be seen by simple comparison with Eq. (9.9), for a plane wave the heating rate of the medium is proportional to the acoustic radiation force as

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{c_0}{\rho_0 C_p} F_x^V\,. \tag{9.21}$$

Therefore, for a plane wave the heating rate coefficient, Q_v, is proportional to the acoustic radiation force:

$$Q_v = c_0 F_x^V\,. \tag{9.22}$$

For linear plane waves this relation reduces to the widely expression $Q_v = 2\alpha I$. However, as seen from the results of the preceding sections, for nonlinear waves both the amplitude and the spatial distribution of the heating rate calculated from Eq. (9.21) will not match the predictions using the linearized expression $Q_v = 2\alpha I$. The prediction of heating rate using the linearized expression will underestimate the temperature increasing in the nonlinear regime. Huge literature is devoted to the heat deposition by nonlinear fields, most of them for HIFU therapy applications and is well-known that the heat rate is increased by nonlinearity. However, most of the literature works still modeling the heating rate using the linearized expression.

On the other hand, once the medium is locally heated over the ambient temperature processes as diffusion, convection, conduction and radiation transfer heat from warmer to cooler regions. For tissues, the blood flowing trough capillaries and blood vessels redistributes heat. This process, known as tissue perfusion, and the heat diffusion due to thermal conductivity can be modeled by the Pennes bioheat transfer equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \kappa \nabla^2 T - \frac{(T - T_0)}{\tau} + \frac{Q_v}{\rho_0 C_p}, \tag{9.23}$$

where κ is thermal diffusivity, τ is the constant for perfusion and T_0 is the ambient temperature. It is worth noting here that due to the Laplacian term, temperature will spread strongly for sharp temperature distributions. Thus, the heat distribution will depend initially to the specific acoustic radiation force field (acting as heat source) and in second term (but coupled in time) to the diffusion/perfusion processes. As seen before, there exist remarkable differences on the (realistic) nonlinear estimation of the acoustic radiation force and its linearized approach (blue curves in Fig.9.2). These differences, not only in the value but in the distribution shape of the ARF, will lead to different heating patterns and must be included for realistic heating models if nonlinear effects are strong.

In the case of finite amplitude acoustic beams, it was shown previously that the peak intensity location does not necessary match the location of the ARF maximum, see e.g. (Camarena et al., 2013b). Thus, the heating pattern will be different using directly $Q_v = 2\alpha I$, or by estimating the energy deposition rate using the momentum transfer, i.e. the nonlinear acoustic radiation force distribution in the tissue.

9.6 Conclusions

In this Chapter we have calculated numerically the acoustic radiation force for intense ultrasound plane waves in biological media. By neglecting diffraction, the particular features of the acoustic radiation force in absorbing biological media can be studied and clearly understood. Thus, the relevance of the exponent of the frequency power law absorption is studied, showing that the correct tissue attenuation model is critical, even in the case of very strong nonlinearity. On the other hand, we have shown that in the nonlinear regime, accounting for the proper acoustic radiation force calculation is critical in order to do not underestimate the force value.

Furthermore, the relation of the acoustic radiation force to the effective or nonlinear absorption in tissue model is presented. Also, considerations on the effective heat deposition rate are also underlined, where it was shown that the nonlinear acoustic radiation force accurately describe the heating rate in the tissue.

However, more complete study is needed for realistically describe the acoustic radiation force in biomedical applications. In first place, in the case of acoustic beams, including diffraction not only implies the calculation of the acoustic radiation force in the direction of the beam, but also in the other directions. In other words, if diffraction is included, the nonlinear stress tensor must be calculated (Rudenko et al., 1996; Ostrovsky et al., 2007; Ostrovsky, 2008). Of special interest is the calculation of the nonlinear stress tensor for non-paraxial beams. Moreover, other interest related problems are the accurate modeling of the shear waves generated by nonlinear acoustic radiation force in tissue, and the acoustic radiation force generated due to tissue internal reflections, heating patterns with realistic blood streaming due to high intensity fields, or acoustic radiation force interaction with microbubbles inside capillaries.

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