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Nonlinear focal shift in medium Fresnel-number focused acoustic beams

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***Abstract*— The study of the acoustic field characteristics generated by focusing sources, both in linear and nonlinear regime, is an active field of research as they are relevant in most of the ultrasonic applications in medicine and industry. Particularly, the linear focal shift phenomenon (i.e. the distance between the geometrical focus of the focused source and the on-axis maximum pressure position in linear regime, real focus) was explained in 1982 by Lucas and Muir. Also, the nonlinear focal shift phenomenon (the displacement of the pressure and intensity maxima position of focused acoustic beams under increasing driving voltages) has been related and interpreted in previous works. Nevertheless, till the moment it has not been published a specific experiment with the objective to study, experimentally and numerically, the focal region of medium Fresnel-number transducers, and the magnitude of the this shift. This is the purpose of this work. The nonlinear focal shift phenomenon is presented for the case of a medium Fresnel-number beam (N_F=6). The experimental results show the existence of a short nonlinear shift in the position of the on-axis maximum pressure (6 mm). These results are coherent with numerical KZK model predictions. It is shown that in the nonlinear regime and for this focusing degree the position of the on-axis maximum pressure can surpassing the geometrical focal length. Contrary, the on-axis minimum pressure approaches the transducer under increasing driving voltages, and at high excitation the distance between on-axis maximum and minimum pressure is closer to 1 cm.**

I. Introduction

The study of the acoustic field characteristics generated by focusing sources, both in linear [1-3] and nonlinear [4-8] regime, is an active field of research as they are relevant in most of the ultrasonic applications in medicine and industry.

It is known that the position of the on-axis maximum pressure differs from the location of the geometrical focus in a focused transducer. There are two sides of this problem. First, in linear regime, the action of focussing and diffraction effects causes a shift (towards the transducer) in the position of the real focus from the geometrical focus. This phenomenon is known as *linear focal shift* [3,9]. Previous works [12] have shown that the *linear focal shift* decreases when focusing increases, showing values close to 2.5 cm for Fresnel number beams of 1.28 and values close to some millimeters when the focusing reach values like 6, still far from the values associated to HIFU devices (it is N_F>20). On the other hand, under nonlinear propagation conditions, the appearance of higher harmonics causes the movement of the maximum pressure position along the axis [9] away the transducer. This phenomenon is known as *nonlinear focal shift* and has been observed both in unfocussed beams and in focused sources [10, 11]. It decreases when focusing increases. For Fresnel number 1.28 it was evaluated in 2.4 cm [12].

Although the nonlinear focal shift has been observed and explained [4-8], till the moment it has not been published a specific experiment with the objective to study, experimentally and numerically, the focal region of medium focusing transducers, and the magnitude of this shift. The aim of this work is to evaluate the nonlinear shift of an ultrasonic beam with medium Fresnel number (N_F = 6), as well as to measure if the nonlinear shift is able to move the on axis maximum pressure location beyond the geometrical focus.

II. Materials and Methods

A. Experimental set-up

The experimental setup follows the classical scheme of confronted emitting transducer and receiving calibrated membrane hydrophone in a water tank filled with degassed and distilled water. The experimental design is shown in Fig. 1.

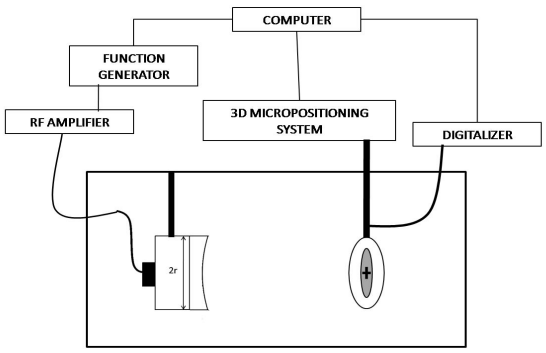


Fig. 1 Experimental design for pressure measuring in water.

The ultrasonic source was formed by a plane single element piezoceramic crystal (PZ 26, Ferroperm Piezoceramics) mounted in a custom designed steel housing and a methacrylate focused lens with diameter 50 mm and radius of curvature 70 mm (R).

The resonant frequency of the system was 2.227 MHz, the aperture 50 mm ($2a$) and the geometrical focal length 157 mm (F), evaluated from Eq. (1).

$$F = \frac{R}{1 - \frac{c_m}{c_l}} \quad , \tag{1}$$

where c_m and c_l are the sound velocity of the water tank and the methacrylate used to build the lens respectively.

The transducer was driven with pulse bursts (30 cycles-sine wave bursts) using a function generator (14 bits, 100 MS/s, model PXI5412, National Instruments) and a linear RF amplifier (ENI 1040L, 400W, +55dB, ENI, Rochester, NY). To measure the acoustic waveforms a NTR PVDF membrane hydrophone (0.2229 V/MPa sensitivity, model MH2000B with 200 μm active diameter, NTR/Onda Corp.) and a digitizer (64 MS/s, model PXI5620, National Instruments) were used. A three-axis micropositioning system was used to move the hydrophone in three orthogonal directions with an accuracy of 10 μm (OWIS GmbH,). All the signal generation and acquisition process is based on a National Instruments PXI-Technology controller NI8176, which also control de micropositioning system.

B. Numerical Model

Numerical modelling of the experiment was performed using the KZK equation for axisymetric beams:

$$\frac{\partial^2 p}{\partial z \partial t'} = \frac{c_0}{2}\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right) + \frac{\delta}{2c_0^3}\frac{\partial^3 p}{\partial t'^3} + \frac{\beta}{2\rho_0 c_0^3}\frac{\partial^2 p}{\partial t'^2}, \tag{2}$$

where $t'=t-z/c_0$ is a retarded time, c_0 the propagation speed, δ the sound diffusivity, β the coefficient of nonlinearity, and ρ_0 the ambient density of the medium. Eq. 2 is valid in the paraxial approximation ($ka \gg 1$) and takes into account nonlinearity, diffraction and thermoviscous absorption. The focusing effect is considered through initial conditions and the numerical scheme used to solve the equation is based on a time domain algorithm described in [14, 15].

C. Measurement Procedure

The acoustic waveforms were evaluated in 25 planes along the z axis of the micropositioning system. These planes were transversal to the z axis, 6×6 mm (x-y planes) and waveforms were measured with 0.25 mm spatial resolution. From the measurement of the pressure distribution in each x-y plane (144 measurement points) we were able to evaluate the maximum pressure amplitude and its coordinates (x_{max},y_{max}), allowing us to recover the axis of radiator symmetry.

The most of the planes were located close to the on-axis maximum pressure location (see Fig. 2) with minimal separation of 1 mm between them. This spatial resolution in z was especially necessary in order to evaluate the position of the on axis maximum pressure with an accuracy better than 3 millimetres, as the nonlinear focal shift phenomenon has been estimated in less than 1 cm from numeric simulations [12]).

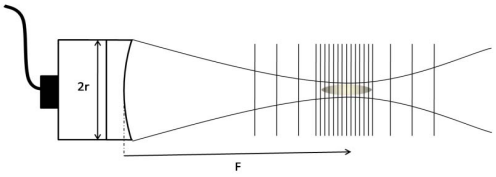


Fig. 2 Measuring plan. Waveforms evaluated in 25 planes along the z axis of the micropositioning system.

As the pressure measurement presented random error estimated in 2%, the uncertainty in the determination of the location is a little higher than 1 mm.

In order to study the evolution of the acoustic field characteristics from linear to nonlinear regimen, the measurement procedure was repeated 8 times with increasing voltages at the transducer terminals, in the range from 2.5 V_{pp} (linear regimen, p_0 = 2 kPa) to 125 V_{pp}.

III. RESULTS

A. Linear Characterization of the Beam

The characterization of the beam in linear regimen allows us to know the characteristics of our acoustic source (aperture and geometrical focus) and the position of the on-axis maximum pressure, i.e. the linear focal shift.

The linear characterization has been executed in three steps: first, the nominal values given by the lens manufacturer is used to evaluate the nominal geometrical focal length. Second, the analytic O'Neil solution [1] for the linear focused field is adjusted to experimental data. This adjustment provides a new value for the geometrical focal length and the aperture. And third, the numerical simulation of the beam based on the KZK equation is adjusted to match the experimental data in the linear regime, and the results in aperture and geometrical focal length obtained will be used to simulate the beam in nonlinear regimen.

The geometrical focal length and the aperture of the transducer were nominally stated by the manufacturer as 157 mm and 50 mm respectively. This implies a Fresnel Number of 5.9, and a gain G=19. The adjustment of the analytic *O'Neil* solution to the experimental data (see Fig. 3) provides an effective aperture of the transducer $2r_0$ = 51.6 mm and an effective geometrical focal length F = 158.2 mm. The on axis maximum pressure obtained from the analytic expression is located at 153 mm from the transducer, i.e. the 96.7 % of the geometrical focal length, what is in good agreement with the value of the linear focal shift predicted by Makov et.al [12].

KZK simulations were performed for different values of aperture and geometrical focal length in order to obtain the best fit to experimental data (see Fig. 3). The effective

aperture of the transducer was found to be 50.2 mm, and the geometrical focal length 157 mm.

The results of both models, calculated with the best fit aperture and geometrical focal length, are in good agreement with the experimental data.

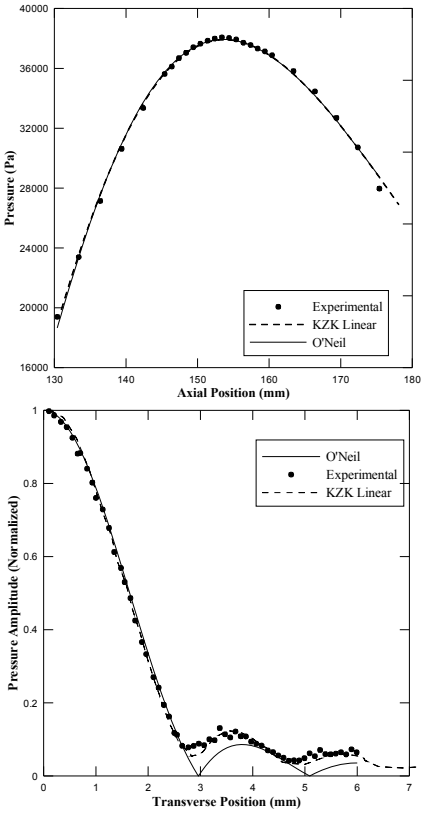


Fig. 3 (a) On axis pressure distribution in linear regimen. (b) Transversal normalized pressure. Experimental values, analytical O'Neil expression and KZK simulation.

B. The Nonlinear Behavior

Fig. 4 shows the variation of the on axis maximum pressure position with the input voltage measured experimentally (dots). Also, starting from the values obtained for the aperture and geometrical focal length that matched KZK simulations to the experimental data in the linear case, we have studied the behavior of the acoustic field in the nonlinear regime with the simulation (line). Both, experiment and simulation show the same two relevant conclusions: a) the on axis maximum pressure position moves away from the transducer when exciting power increases, and b) these positions can surpass the position of the geometrical focus.

The behavior of the maximum pressure position presented in Fig. 4 can be understood considering the appearance of higher harmonics during the nonlinear wave propagation. At higher frequencies the diffraction effect decreases and the real focus moves toward the geometrical focus.

Experimental and simulated values match perfectly in the near linear zone (lower input voltages) but they differ slightly when power increases and propagation gets the nonlinear regimen, so that the nonlinear shift is higher in the simulation. There are several possible reasons that explain it: First, the frequency response of the hydrophone imposes an upper limit of 20 MHz which affects the higher harmonics registration of the signal. Second, the sound field does not present a flat and uniform distribution over the active area of the receptor (200 μm active diameter), thus the measure will be underestimated because registration is the spatial averaging of the measure zone, on the contrary, the simulation maximum are the KZK solution for an infinitesimal field point. Another possible source of error is due to the non-uniform vibration of the transmitter. The numerical model assumes that the vibration of the transducer surface is uniform; however, the actual transducer does not operate as a piston with a perfect uniform vibration.

Fig. 5 shows the variation of the on axis minimum pressure position with the input voltage. Experimental values are represented by dots and the solid line denotes the displacement of the minimum pressure obtained with the KZK simulation. The rarefaction displacement is approximately 6.2 mm while in compression a shift of 7.5 mm was obtained.

Figure 4 shows saturation in the nonlinear focal shift. This is due to the appearance of the shock waves. Instead, behavior of minima is quite lineal, as the rarefactions of the waves do not saturate even when shock appears.

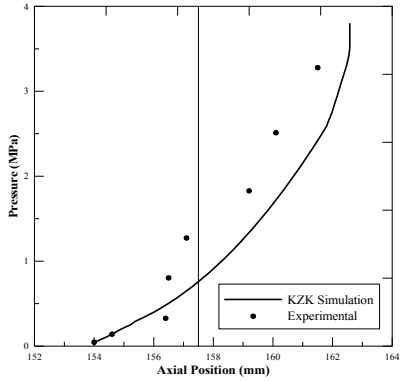


Fig. 4 On axis maximum positive pressure. Experimental values (points) and KZK simulation (solid line). Vertical line denotes the geometrical focus site in 157.5 mm. Input values are 2, 9, 21, 45, 65, 85, 100 and 125 Vpp from bottom to top.

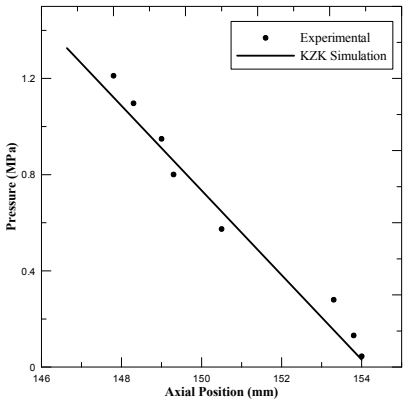


Fig. 5 On axis minimum pressure. Experimental values (points) and KZK simulation (solid line). Input values are 2, 9, 21, 45, 65, 85, 100 and 125 Vpp from bottom to top.

C. Beamwidth

Fig. 6 shows experimental results for the two beam pressure profiles (compression and rarefaction). It was found convenient to define the beam width at -6 dB of the maximum pressure in the plane. R + and R- denote the movement range of maximum and minimum pressure respectively with increasing voltage. In the extreme case (with larger excitation power) the position of maximum pressure is 4 mm beyond the geometrical focus, in this case the distance between the position of maximum compression and rarefaction is 13.7 mm.

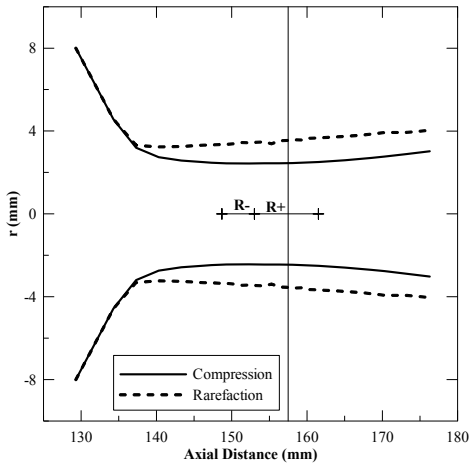


Fig. 6 Half amplitude beam shapes, for positive (solid lines) and negative pressures (dashed lines). Experimental values.

IV. Conclusion

The acoustic field of a medium focused transducer ($N_F = 6$) has been studied in order to fix the characteristics of the linear and nonlinear focal shift. In linear regime it has been observed that the maximum pressure is located at 154 mm from the transducer, which indicates a linear shift of 3.5 mm. This agrees with Makov et al. [12] results in their study about the dependence of the linear shift with the Fresnel number.

In nonlinear conditions it has been observed a maximum pressure position displacement (both in experiment and simulation) due to the increasing input voltage, even exceeding the geometrical focus.

When maximum power is applied to the transducer the on axis maximum pressure position exceed the geometrical focus in 4 millimeters, and the separation between the on axis maximum and minimum positions is as far as 13.7 millimeters. Future plans in this line include the study of the behavior of the on axis intensity, radiation force and phase of the beam.

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