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Vibration modes in a pendulums ring: analogy with gas microbubbles surface modes

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***Abstract*—When subjected to ultrasound activation, microbubbles** $(1-8\ \mu m)$ **present a fairly complex dynamics. Their behavior can be well modeled by coupled oscillators system. In this study, microbubble surface modes have been investigated through an acousto-mechanical analogy base on coupled pendulums. Observation of parametric vibration modes up to** $n = 17$ **for a ring of pendulums excited at frequencies between 1**Hz **and 4** Hz **is presented. The relevance of the use of the pendulums ring for studying microbubble dynamics is discussed.**

I. Introduction

The behavior of a gas bubble, subjected to an external force is a widely studied phenomenon since the beginning of the 20^{th} century [1]. Many studies on bubbles oscillations and especially on nonlinear oscillations were carried on in different research fields [2], [3], [4]. Since the last decades, bubbles oscillations present a great interest in targeted ultrasound imaging and find some applications in targeted therapy [5]. Ultrasound contrast agents (UCAs) currently used in medical imaging [6], are one of the major innovations in the field of echography. Contrast agents are made of gas bubbles of several micrometers $(1-8\ \mu m)$ coated with a biocompatible shell (fig.1). When subjected to acoustic waves, microbubbles vibrate thus becoming echogenic to ultrasound. This feature enhances the contrast between blood pool and surrounding tissues. Furthermore, it is also hypothesize that microstreaming and microjet from microbbuble's oscillation and explosion play a role in the mechanisms at the origin of sonoporation [7].

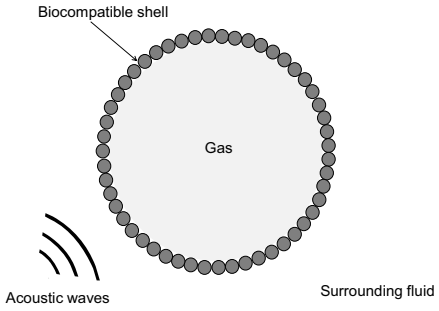


Figure 1. The microbubbble contrast agent is filled with a gas and coated by a biocompatible shell.

The role of microbubbles oscillations in the mechanisms of sonoporation is still a complex problem depending on various excitation parameters. Several authors suggested complex modeling of bubble's dynamics [8], [9], [10] which are experimentally difficult to rank and which are highly dependant on the initial configuration (excitation, conditioning, ...). A full understanding of the microbubbles dynamics is necessary to, subsequently, study it in experimental and clinical conditions. The purpose of this work is to study the interaction modes between these microbubbles and acoustic waves and especially the modulational instability of asymmetrical modes that microbubbles present under ultrasound excitation [11]. Under specific ultrasound activation, microbubbles undergo nonlinear oscillations and present surface patterns called surface modes (fig.2). Surface mode oscillations have been studied for micron-sized and millimeter-sized bubbles by several research groups ([12] and references therein). Such behavior concerns many nonlinear systems when subjected to an external force. In the case of microbubble contrast agent, the complexity and the smallness of the phenomenon studied require complex modeling and/or expensive experiments where high-speed camera or optical trap system are required. To overcome this, our work is based on an acousto-mechanical analogy. Indeed, the dynamics features can be captured with coupled oscillators' models.



Figure 2. Representation of surface modes (3^{rd} mode, 5^{th} mode, 10^{th} mode and 12^{th} mode). The notation Y correspond to spherical harmonics which are eigen functions of radial symmetry systems.

In this study, the surface oscillatory behavior and the surface modes of a microbubble are investigated through a macroscopic experiment of coupled pendulums parametrically excited by a vertical force. This work aims to select optimal excitation conditions of the microbubble for both imaging and therapeutic applications. Based on a discrete nonlinear model of coupled pendulums [11] and a continuous model describing microbubble's dynamics, vibration modes are investigated theoretically and experimentally in complement of numerical

studies performed previously [13].

II. Mathematical formulation

A. Microbubble dynamics

Under the action of an external driving pressure, microbubble can present surface modes due to the emergence of a parametric instability. This has been studied by Francescutto and Nabergoj [14]. Based on a linearized Rayleigh-Plesset equation, surface modes dynamics was given by Mathieu equation. The parametric instability, which is characterized by a time variation of a constant parameters, is due to the growth of small perturbations applied to the spherical interface. Microbubble radius can thus be described as a space dependant function and developed on the basis of spherical harmonics:

$$R(t) \rightarrow R(t,\theta,\Phi) = R(t) + \sum_{n=0}^{\infty}\sum_{m=-n}^{n} a_n(t) Y_n^m(\theta,\Phi), \quad (1)$$

with $Y_n^m(\theta,\Phi)$ the spherical harmonics of order n and a_n surface modes amplitude. The dynamics of a bubble is described by the Rayleigh-Plesset equation

$$\begin{aligned}\rho_l\left(R\ddot{R}+\tfrac{3}{2}\dot{R}^2\right) = & \left(P_0+\tfrac{2\sigma}{R_0}\right)\left(\tfrac{R_0}{R}\right)^{3\kappa} \\ & -P_0 - P_A(t) - \tfrac{2\sigma}{R} - 4\mu\tfrac{\dot{R}}{R},\end{aligned} \quad (2)$$

with R the instantaneous radius of the bubble, ρ_l the density of the surrounding fluid, P_0 the hydrostatic pressure, σ the surface tension, R_0 the equilibrium radius of the bubble, P_A the acoustic pressure, κ the polytropic exponent and μ the viscosity of the surrounding fluid. From this, a dynamical equation for the amplitude of surface modes can be derived [15]

$$\ddot{a}_n(t) + B_n(t)\dot{a}_n(t) - A_n(t)a_n = 0, \quad (3)$$

where the amplitude $A_n(t)$ and $B_n(t)$ are given by

$$A_n(t) = (n-1)\left(\frac{(n+1)(n+2)\sigma}{\rho_l R^3} + \frac{2(n+2)\mu\dot{R}}{\rho_l R^3} - \frac{\ddot{R}}{R}\right), \quad (4)$$

$$B_n(t) = \frac{3\dot{R}}{R} + \frac{2(n+2)(2n+1)\mu}{\rho_l R^2}. \quad (5)$$

Equation 3 gives the distortion amplitude for a given surface mode n.

B. Pendulums ring dynamics

A circular monoatomic pendulums lattice is considered. Formed by equal masses m coupled to their first neighbors, the whole chain is subjected to a parametric vertical excitation of amplitude A and pulsation ω. The motion of the i^{th} pendulum is described by its angle θ_i with respect to the vertical (fig.3), and is given by the equation

$$\begin{aligned}&\ddot{\theta}_i + \gamma\dot{\theta}_i + \left(\omega_0^2 + A cos(2\omega_e t)\right) sin(\theta_i) \\ &-k(\theta_{i-1} - 2\theta_i + \theta_{i+1}) = 0,\end{aligned} \quad (6)$$

where θ_i is the angle of the i^{th} pendulum with the vertical axis, γ is the damping coefficient, $\omega_0 = \sqrt{g/L}$ is the eigen pulsation of the uncoupled pendulum, g is the gravitational acceleration, L is the pendulum length, k is the coupling constant related on the height d of the junction, and A and ω_e are respectively the normalized amplitude and the pulsation of the parametric excitation. After linearization neglecting the damping and driving term and assuming a solution in the form of a harmonic wave, the dispersion relation is obtained.

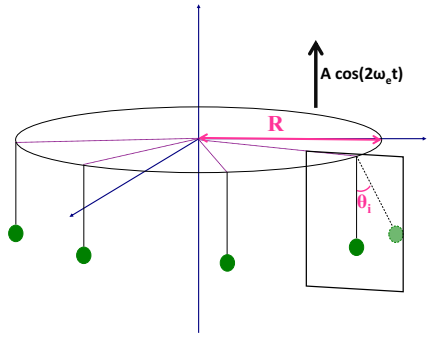


Figure 3. Simplified scheme of the acousto-mechanical system. The motion of the n^{th} pendulum is described by its angle θ_n with respect to the vertical.

$$\left(\frac{\omega_n}{\omega_0}\right)^2 = \frac{1}{1-\eta}\left(1 - \eta cos^2\left(\frac{\pi n}{2N}\right)\right), \quad (7)$$

with ω_n the pulsation, $\eta = d/L$ the strength of the coupling, d the distance between the ring and the nod, n the mode number and N the number of pendulums.

III. Material and methods

Original experiment[11] has been modified and consists in studying vibration modes of coupled pendulums ring subjected to a parametric excitation of which one varies the amplitude A or the frequency f_e. The study of the nonlinear behavior aims to understand the mechanisms at the origin of the modulational instability of these vibration modes. The experimental setup was improved and consists of an aluminum ring with a diameter of 62 cm, on which 54 pendulums of mass $6g$ are fixed with nylon strings forming a "V"shape with the vertical axis. The coupling is obtained by overlapping the strings and fixing them by a node (fig. 4). Here $\eta = 0.6$ so our experimental system present a strong nonlinear coupling.

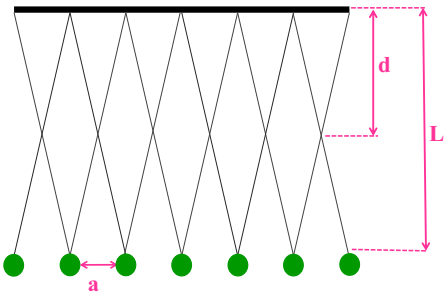


Figure 4. Nonlinear coupled oscillators. Here $a = 3.5\ cm$; $d = 6\ cm$; $L = 10\ cm$.

The excitation system consists of a loudspeaker driven by an arbitrary waveform generator (Agilent 33220A) through an audio amplifier which excites mechanically the pendulum ring (fig.5). The sinusoidal excitation varies from $f_e = 0\ Hz$ to $f_e = 4\ Hz$ with amplitudes varying from $A = 0.5\ Vpp$ to $A = 3.5\ Vpp$ corresponding to $A = 2 - 8\ mm$).



Figure 5. Experimental setup of the acousto-mechanical analogue. The ring lies on the excitation system.

The pendulums are driven near the volumetric resonant frequency as to allow the developpment of surface modes during the mechanical excitation through a parametric instability. The chain is subjected to a parametric vertical excitation of which one varies the amplitude or the frequency. Depending on the sound wave parameters (A and f_e) different vibration modes are observed (fig.6).

IV. Results

Beyond a critical threshold of the excitation amplitude A, vibration modes can be generated after a transcient regime which is highly dependant on the experimental setup. The threshold depend on the diameter of the ring which is associated to the bubble's diameter. From $f_e = 0Hz$ to $f_e = 1.5Hz$ no vibration modes were observed because the excitation is not strong enough to induce a motion of the pendulums. Between $f_e = 1.5Hz$ and $f_e = 4Hz$, surface modes appear with localized oscillations up to $n = 17$ (fig. 6).

Moreover, results show modes oscillating at subharmonics of the parametric excitation ($f_{osc} = f_e/2$). They correspond to the parametric modes expected when the ring is excited at a frequency two times greater than the natural frequency of a pendulum ($f_e \simeq 2f_0$). The dispersion relation with our experimental data is compared with the theoretical relation given by Eq.7 (fig.7). The curve shows that the first experimental points of the parametric modes are similar to the theoretical predictions but the following points deviate from theory. More accurate measurements are hence required in order to derive an enhanced dispersion relation taking into account the real physical properties of the ring. Between $f_e = 2.5Hz$ and $f_e = 3.2Hz$ the system presents a "bandgap"in which no vibration mode is observed (fig.8). Vibration modes on the right side are modes that oscillate at subharmonics of the parametric excitation whereas on the left side, the presence of vibration modes where not expected. This results will be discuss in the next part.

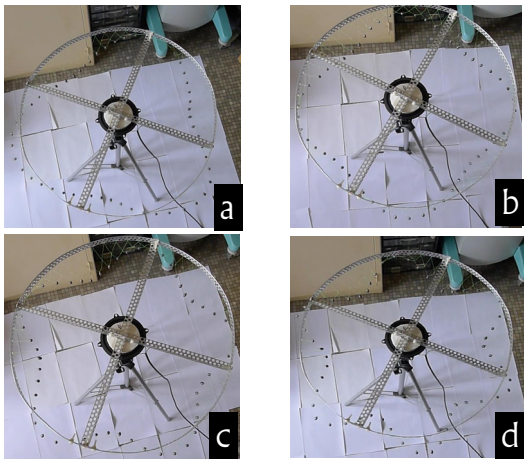


Figure 6. Vibration modes of the pendula ring for $a = 1.20$ Vpp.a: 3^{rd} mode, $f = 3.12Hz$; b:4^{th} mode, $f = 3.21Hz$;c: 5^{th} mode, $f = 3.26Hz$;d: 6_{th} mode, $f = 3.31Hz$

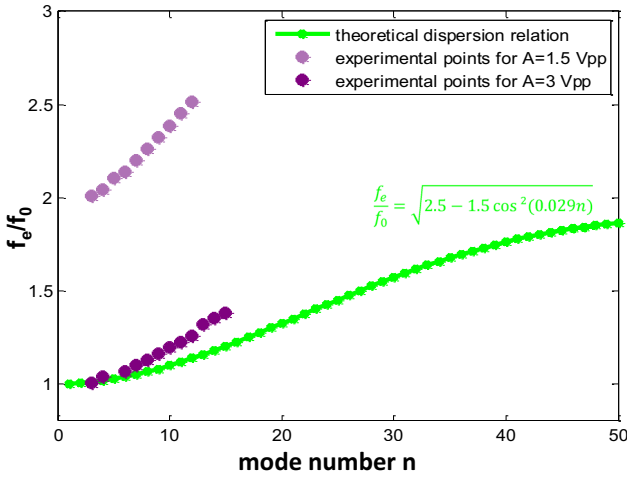


Figure 7. The smooth curve is the theoretical dispersion relation. The dots correspond to experimental points observed for an excitation amplitude $A = 3\ Vpp$ (dark purple) and $A = 1.5\ Vpp$ (light purple).

V. Discussion

Vibration modes up to $n = 17$ have been observed. Unfortunately, due to the system vulnerability, measurements above $f_e = 4\ Hz$ were difficult to perform. Indeed, beyond this excitation frequency, pendulums' oscillations are so important that the coupling break. The appearance of a "bandgap"between $f_e = 2.4\ Hz$ and $f_e = 3.15\ Hz$ is an unexpected result. The presence of this frequency window where no mode is observed is typical of diatomic chains. Except that in our case, the ring is made with equal masses and there is only a single cutoff frequency. On the right side of the bandgap ($f_e \simeq 2f_0$) and on the left side ($f_e \simeq f_0$), parametric vibration modes have been observed. These two sets of modes, that were not expected at the beginning, can be interpreted as branches of the Mathieu equation for a single pendulum. On the left side, we are in a region were the resonance is smaller thant on the right side were ($f_e \simeq 2f_0$). These results have been confirmed by simulations [13]. The first experimental

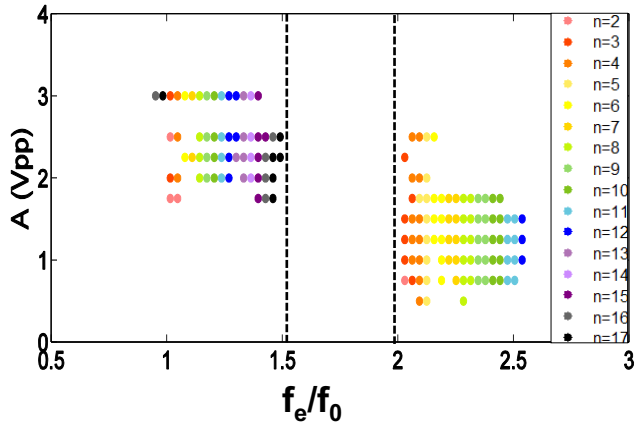


Figure 8. Appearance of modes versus the excitation amplitude and frequency. A bandgap emerge between$f_e = 2.5\ Hz$ and $f_e = 3.2\ Hz$, between the two dotted lines.

points that have been plotted for an excitation amplitude $A = 3\ Vpp$ are in good agreement with theoretical dispersion relation given by Eq. 7, were $f_e = \omega_e/(2\pi)$ and $f_0 = \omega_0/(2\pi) = 1.58\ Hz$. However, additional measurements are required for the following points. This result is also a confirmation that the theoretical dispersion relation was probably derived assuming hypothesis that should be revised.
These results are compatible with those found for real microbubble [12]. Furthermore, it is found on figure 8 that the preferential mode number does not depend on the amplitude excitation A, provided of course it is beyond the critical threshold. These results confirms also the previous experiments [11], [16].

VI. Conclusion

Surface modes of a microbubble have been studied through an acousto-mechanical analogy of coupled pendulums. The results obtained are promising and in agreement with theory. Studies for a ring presenting a low coupling are carried on. The same mechanisms seem to be at the origin of surface vibration modes of the pendulums ring and microbubbles. Such analogy between a macroscopic behavior and a microscopic one provides important insights for the comprehension of coated microbubble's dynamics for both imaging and therapeutic applications. Further studies are expected, particularly for understanding the interaction with a wall for sonoporation applications.

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