Lattice of nonlinear coupled oscillators: An acousto-mechanical analogy of gas microbubble

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***Abstract*—Expensive and complex experiments and/or simulations are required to study microbubble's dynamics considering its size and the complexity of the associated physical phenomenon of bubble-ultrasound interactions. To overcome this, the use of a macroscopic acousto-mechanical analogy is proposed. In this study, microbubble's surface modes are investigated through an experimental analogy based on coupled pendula. The data are further compared to numerical simulations. Vibration modes up to $n = 22$ have been observed for excitation frequency ranging from $1Hz$ to $5Hz$. Simulations shows mode mixing phenomena and an influence of the amplitude on the dispersion relation. The relevance of the analogy is discussed.**

I. Introduction

The oscillatory behavior of a bubble in an excitation field is a widely studied phenomenon by researchers since the beginning of the twentieth century [1], [2], [3], [4]. Indeed, bubbles present a rich dynamics under a specific acoustic excitation. Nowadays, microbubble dynamics present a great interest in particular in ultrasound imaging and find some applications in targeted therapy [5]. Ultrasound contrast agents (UCAs) currently used in medical imaging [6], are made of gas bubbles of few micrometers ($1 - 8\ \mu m$) coated with a biocompatible shell. When subjected to ultrasonic waves, microbubbles vibrate. Specific bubble feature are exploited to enhance the contrast between the blood and surrounding tissues.

A full understanding of microbubbles dynamics is necessary to, subsequently, study its behavior for both experimental and clinical situations. Nonetheless, considering the size of the microbubble and the complexity of its dynamics, expensive and complex simulations and/or experiments are required. To overcome this limitation,the use of an analogy is proposed to simplify the study of such a complex system since several years from the theoretical point of view[7] and from simulation aspects [8], [9].

The purpose of this important step is to study the interaction modes between microbubbles and acoustic waves through an acousto-mechanical experiment based on couple pendula parametrically excited by a vertical force. The objective of this study is to acquire information about the vibrational behavior of a single microbubble in order to study it in experimental and clinical conditions for both imaging and therapeutic applications.

II. Theoretical model

A. Analogy

The analogy between a microbubble and the pendula ring is based on parameters R, corresponding of the radius of the system under study, and f_e, corresponding to the frequency parameter of the excitation. For classical experiments, microbubble ($R \simeq 10^{-6}$) are insonified with ultrasound in the MHz frequency range. For the acousto-mechanical analogue, the pendula ring ($R \simeq 0.3$) is parametrically excited in the Hz frequency range. According to symmetry analysis, the product $f_e.R$ is an invariant for both experiments and suggest a similarity analysis as classically performed in the context of fluid mechanics [10].

	Microbubble	pendula ring
$R\ (m)$	$\simeq 10^{-6}$	$\simeq 0,3$
$f_e\ (Hz)$	$\simeq 10^{6}$	$\simeq 3$
$f_e.R$	$1\ m.s^{-1}$	

B. Mathematical formulation

1) Motion equation: A circular monoatomic pendulums lattice is considered. Formed by equal masses m coupled to their first neighbors, the whole chain is subjected to a parametric vertical excitation of amplitude A and pulsation ω. The motion of the i^{th} pendulum is described by its angle θ_i with respect to the vertical axis and is given by the equation

$$\begin{aligned} &\ddot{\theta}_i + \gamma\dot{\theta}_i + \left(\omega_0^2 + Acos(2\omega_e t)\right) sin(\theta_i) \\ &-k(\theta_{i-1} - 2\theta_i + \theta_{i+1}) = 0, \end{aligned} \tag{1}$$

where θ_i is the angle of the i^{th} pendulum with the vertical axis, γ is the damping coefficient, $\omega_0 = \sqrt{g/L}$ is the eigen pulsation of the uncoupled pendulum, g is the gravitational acceleration, L is the pendulum length, k is the coupling constant related on the height d of the junction, and A and ω_e are respectively the normalized amplitude and the pulsation of the parametric excitation.

2) Dispersion relation: After linearization, and neglecting the damping and driving terms and assuming a solution in the form of a harmonic wave, the dispersion relation is obtained.

$$\left(\frac{\omega_n}{\omega_0}\right)^2 = \frac{1}{1-\eta}\left(1 - \eta cos^2\left(\frac{\pi n}{2N}\right)\right), \tag{2}$$

with ω_n the pulsation, $\eta = d/L$ the strength of the coupling, d the distance between the ring and the nod, n the mode number and N the number of pendulums.

III. Material and methods

A. Setup

The experimental setup consists of an aluminum ring with a radius of 31 cm, on which 54 pendulums of mass $m = 6g$ are fixed with nylon strings forming a "V"shape with the vertical axis (Fig.1). The coupling is obtained by overlapping the strings and fixing them by a node. The influence of the coupling has been optimized experimentally in order to fit as close as possible to the microbubble. Here $\eta = 0.6$ and $f_0 = \frac{\omega_0}{2\pi} = 1,58Hz$ so our experimental system present a medium coupling. The excitation system consists of a loudspeaker driven by an arbitrary waveform generator (Agilent 33220A) through an audio amplifier which excites mechanically the ring. The sinusoidal excitation varies from $f_e = 0\ Hz$ to $f_e = 5\ Hz$ with amplitudes varying from $A = 0.5\ Vpp$ to $A = 3.5\ Vpp$ corresponding to a vertical displacement of 2 mm to 8 mm. The frequency range is chosen in order to satisfy the similarity property given in the table.

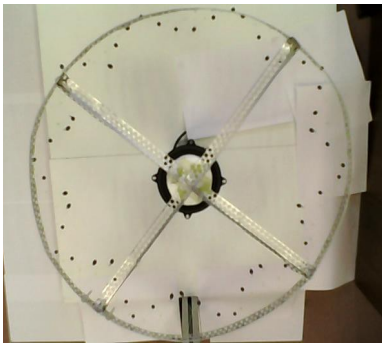


Figure 1. Experimental setup of the acousto-mechanical analogue.

B. Principle

The pendula ring is excited with a sinusoidal sound wave positioned in a vertical position. The amplitude and the frequency of the excitation sound wave can be varied (A and f_e). The pendula are driven near the volumetric resonant frequency as to allow the development of surface modes during the mechanical excitation. Depending on the sound wave parameters, different vibration modes are observed (fig.2).

IV. Experimental data

Beyond a critical threshold of the excitation amplitude A, vibration modes can be generated after a transient regime which is highly dependent on the experimental setup. No vibration modes were observed between $f_e = 0Hz$ to $f_e = 1.5Hz$ because the the amplitude is not high enough to induce a motion of the pendula. Results show surface modes up to $n = 22$ (Fig.2) with an unstable mode for $n = 0$. For $n = 1$, the displacement of the center of mass is observed. Modes oscillating at subharmonics of the parametric excitation are observed for amplitudes varying from $A = 0.5\ Vpp$ to $A = 3\ Vpp$ and an excitation frequency equal to twice the natural frequency f_0 of the simple pendulum. Graphical representations show two groups of vibration mode, the one on the left side correspond to a resonance region where $f_e/f_0 = 1$, on the right side, it is a parametric resonance where $f_e/f_0 = 2$. Results show that for a given excitation frequency, the amplitude of the excitation seems to have no effect on the appearance of specific vibration mode, including a kind of synchronization process.

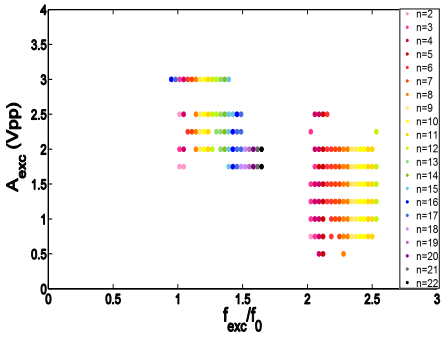


Figure 2. Appearance of vibration mode versus the excitation amplitude and frequency.

The dispersion relation obtained with our experimental data is compared to the theoretical relation given by Eq.2 (Fig.3).

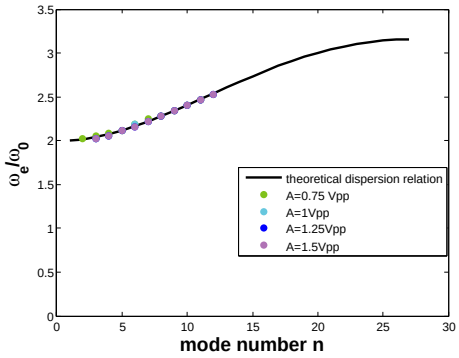


Figure 3. Parametric dispersion relation for different excitation amplitudes.

The curve shows that the first experimental points of the parametric modes are closed to the theoretical predictions. In contrast, the following data quickly deviate from the theoretical curves.This result will be discuss thereafter.

V. Simulation

To validate the experimental data, simulations have been carried out. The phase diagram obtained presents the two groups of vibration modes (Fig.4). On this graphic, each color represents a vibration mode. Additionally, mode mixing phenomena can be observed.

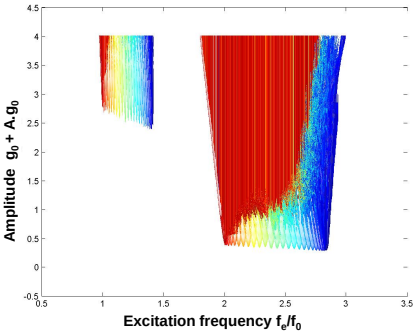


Figure 4. Simulation data for the appearance of vibration mode versus the excitation amplitude and frequency.(one color=one mode)

The representation of the parametric dispersion relation (Fig.5) shows that for increasing amplitudes, the relation distorts due to nonlinearity. Here, each color represents an excitation amplitude.

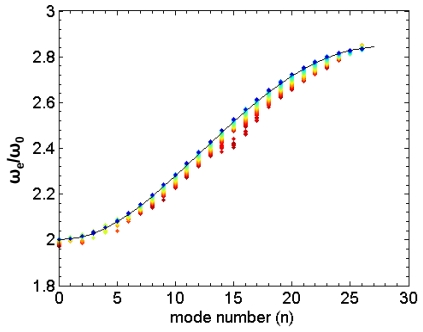


Figure 5. Parametric dispersion relation for increasing excitation amplitudes.(one color=one amplitude)

VI. Discussion

Vibration modes up to $n = 22$ have been observed in the acousto-mechanical analogue of a gas microbubble. . Above $f_e = 5\ Hz$, measurement can't be performed due to the system vulnerability. Indeed, beyond this excitation frequency, the oscillations become too great for the coupling is maintained between the strings.In the experimental phase diagram (Fig.2), that the preferential mode number does not depend on the excitation amplitude whereas in (Fig.4) we can see clearly that increasing amplitudes at fixed excitation frequency will shift dominant modes. This is explained by the fact that, in the experimental setup, the vibrational modes are evaluated by choosing the apparent pattern. While in numerical results, for an apparent mode 5, modes 3, 4 and so on are oscillating, but at lower amplitudes.
Experimental data have been plotted for different excitation amplitudes to see the effect of the latter on the dispersion relation. Experimentally the amplitude seems to have no effect but numerically, a distortion of the dispersion relation due to nonlinearity can be observed for increasing amplitudes. Moreover, the experimental points deviate from the theoretical model. Accurate measurements are required for the following points. This result shows that the current theoretical model was probably derived assuming hypothesis that should be improved.

VII. Conclusion

An analogy between a microbubble and a lattice of coupled oscillators has been proposed to study surface modes. Such acousto-mechanical analogy provides important informations for the understanding of microbubble dynamics. The results obtained are promising and in agreement with those obtained for microbubble [11]. Nonetheless, the current theoretical model needs to be completed, in particular by including effect of curvature in the nonlinear dynamics of the chain. Experiments with different coupling are ongoing and further studies are expected, in particular on the interaction with a wall.

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