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On the nonlinear effects in focused ultrasound beams with frequency power law attenuation

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Abstract

When finite amplitude ultrasound propagation is considered, changes in spatial features of focused ultrasound beams can be observed. These nonlinear effects typically appear in thermoviscous fluids as focal displacements, beam-width variations or gain changes. However, in soft-tissue media, the frequency dependence of the attenuation doesn't obey a squared law. In this way, these complex media response leads to weak dispersion that prevents the cumulative processes of energy transfer to higher harmonics. In this work we explore the influence of different frequency power law attenuation responses and its influence on the self-defocusing effects in focused ultrasound beams. Thus, we numerically explore the spatial field distributions produced by low-Fresnel number devices and High Intensity Focused Ultrasound (HIFU) radiating trough different soft-tissue media.

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1. Introduction

When accounting for finite amplitude acoustic perturbations in biological media the inclusion of the correct frequency dependent attenuation and dispersion is critical to accurately model nonlinear effects in the propagation. Thus, the dynamics of the microscopic-heterogeneous structure of the biological media for wavelengths bigger than the microstructure can be modeled by a macroscopic-homogeneous attenuation and dispersion. In this way, the attenuation presents a complex frequency dependence that in most cases can be approximated by a frequency power-law dependence as $\alpha(f)=\alpha_0 f^{\gamma}$ (Duck, 1990; Hill et al., 2004) where the exponent, γ, is close to the unity. In order to include including the observed losses in the acoustic equations there exist numerous phenomenological approaches (Hill et al,. 2004; Wismer et al., 1995). On the other hand, is common in literature to describe the losses

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in soft tissues as multiple-relaxation processes (Pierce, 1989; Nachman et al., 1990; Hill et al., 2004), where the relaxation frequencies can be associated to either a tissue specific physical mechanism or empirically optimized to fit the observed tissue attenuation (Cleveland et al., 1995; Pinton et al., 2009). Moreover, fractional partial differential operators has been demonstrated the ability to describe frequency power law attenuation (Szabo, 1994; Prieur et al., 2011). These operators can be included in the modeling by means of time (Szabo, 1994), space (Chen et al., 2004) or combined time-space fractional derivatives (Caputo, 1967; Wismer, 2006). In order to solve these models, many time-domain numerical methods have been developed. Attenuation modeled by relaxation processes can be solved by means of finite-differences in time-domain (FDTD) solvers in linear regime (Yuan et al. 1999), and in nonlinear regime applied to augmented Burger's equation (Cleveland et al., 1995), Khokhlov-Zabolotskaya-Kuznetsov (Yang et al., 2005) and Westervelt (Pinton et al., 2009) nonlinear wave equations. On the other hand, time-dependent fractional derivatives can be solved in nonlinear regime (Liebler et al., 2004) by convolutional operators. This approach requires the memory storage of certain time history, and although the memory can be strongly reduced compared to direct convolutions, this algorithm employs up to ten auxiliary fields and a memory buffer of three time steps. In order to overcome this limitation, time-space fractional derivatives or fractional Laplacian (Treeby et al., 2010; Chen et al., 2004) can be used to model frequency power laws without time-domain convolutional operators. Recently *k*-space and pseudo-spectral methods have been applied in order to solve fractional Laplacian operators efficiently in nonlinear regime.

In the present work a FDTD numerical scheme is used in order to solve the constitutive equations of nonlinear acoustics in multiple relaxation media. The aim of this work is to summarize the role of the power law exponent, γ, in the propagation of finitude amplitude plane waves and high intensity focused ultrasound devices.

2. Attenuation and dispersion in soft-tissue

When modelling longitudinal acoustic wave propagation in biological soft-tissue media it is critical to account for the correct attenuation and phase speed. When dealing with linear monochromatic propagation, the values of media attenuation and phase speed for a given frequency can be explicitly included from those obtained from experimental tests. However, if finite amplitude propagation is taking into account the inclusion of a causal-time domain operator that includes frequency power law attenuation is necessary to accurately model the losses of each spectral components. Moreover, the inclusion of a non-squared frequency power law leads to the inevitable inclusion of dispersion. Thus, applying the Kramers-Krönig dispersion relations, the attenuation and change on phase speed for a given power law can be obtained by the relations (Waters et al., 2003):

$$\alpha(\omega) = \alpha_0 \omega^{\gamma} \ , \tag{1}$$

$$c_p(\omega) = \frac{1}{\frac{1}{c_0} + \alpha_0 \tan\left(\frac{\pi\gamma}{2}\right)\left(|\omega|^{\gamma-1} - |\omega_0|^{\gamma-1}\right)} \ . \tag{2}$$

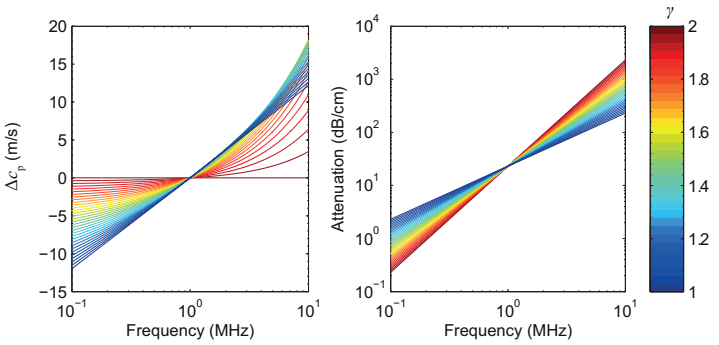


Figure 1. (Left) Frequency dependent phase speed and (Right) attenuation for $\alpha = 2$ dB/cm at 1 MHz

Thus, for a soft-tissue media with typical attenuation of 2 dB/cm at 1MHz, variations of phase speed from Eq. (2) are shown in Fig. 1, where variations of about ± 15 m/s are obtained for a frequency range from 0.1 to 10 MHz.

3. Physical model

The main constitutive relations for nonlinear acoustic waves for a viscous fluid can be expressed as (Waters et al., 2003):

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \ , \tag{3}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla p + \eta \nabla^2 \mathbf{v} + \left(\zeta + \frac{\eta}{3}\right) \nabla (\nabla \cdot \mathbf{v}), \tag{4}$$

where ρ is the total density field, $\mathbf{v}$ is the particle velocity vector, p is the pressure, η and ζ are the coefficients of shear and the bulk viscosity respectively. The acoustic waves described by this model exhibits viscous losses with squared power law dependence on frequency. In order to include a power law frequency dependence on the attenuation, a multiple relaxation model will be added into the time domain equations. The basic mechanism for energy loss in a relaxing media is the appearance of a phase shift between the pressure and density fields. This behavior is commonly modeled as a time dependent connection at the fluid state equation, that for a fluid retaining the nonlinear effects up to second order an be expressed as (Naugolnykh and Ostrovsky, 1998; Rudenko et al., 1977)

$$p = c_0^2 \rho' + \frac{c_0^2}{\rho_0} \frac{B}{2A} \rho'^2 + \int_{-\infty}^{t} G(t - t') \frac{\partial \rho'}{\partial t} dt, \tag{5}$$

where $\rho' = \rho - \rho_0$ is the density perturbation over the stationary density ρ_0, B/A is the nonlinear parameter, c_0 is the small amplitude sound speed, and $G(t)$ is the kernel associated with the relaxation mechanism. The first two terms describe the instantaneous response of the medium and the convolutional third term accounts for the "memory time" of the relaxing media. Thus, by choosing an adequate time function for the kernel $G(t)$ the model can present an attenuation and dispersion response that fits the experimental data of the heterogeneous media. If an sum of N exponential forms of the kernel $G(t)$ is taken into account, the integral form of the state eq. (3) leads to

$$p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0} \frac{B}{2A} \rho'^2 - \sum_{n=1}^{N} S_n. \tag{6}$$

Here the “frozen” sound speed for N mechanisms is defined as

$$c_\infty^2 = c_0^2\left(1+\sum_{n=1}^{N}\eta_n\right), \tag{7}$$

and for each relaxation process, variable S_n obeys

$$\frac{\partial S_n}{\partial t} = -\frac{1}{\tau_n}S_n + \frac{\eta_n c_0^2}{\tau_n}\rho' \tag{8}$$

for each relaxation process. Model equations (3-4, 6, 8) are solved by finite differences in time domain method (Liebler et al., 2004) (FDTD). Thus, a space-time staggered discretization is employed and central finite differences operators are applied for solving both spatial and time differential operators.

4. Results

4.1. Plane wave nonlinear propagation in frequency power-law attenuation media

Numerical tests have been done for plane wave propagation in frequency power-law attenuation media. Selected attenuation was 2 dB/cm at 1 MHz, and the power law attenuation exponent varies from 0.5 to 2. Central frequency of the excitation was 1 MHz, and spatial distribution for the spectral components was carried out up to the shock formation distance. Figure 2 shows the obtained results for the fundamental component, and for the second and third harmonics. Compared with the propagation in a non-dispersive viscous fluid, two main effects can be observed in the numerical solutions obtained. First, while the attenuation value for the fundamental frequency remains constant, as shown in Fig. 1 (right), the attenuation of each spectral component differs from different power law. Thus, higher power laws leads to higher attenuation values for second and third harmonic, and, as a consequence, the amplitude decreases progressively more accentuated for the highest power law values.

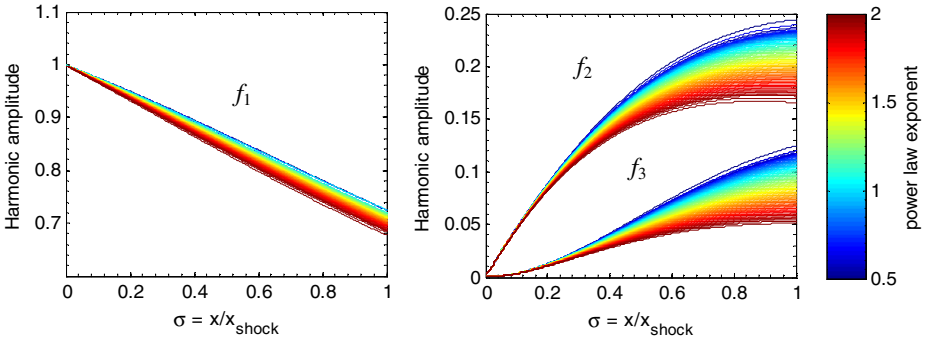


Figure 2. Spatial distribution of fundamental, second and third harmonic for power laws ranging from 0.5 to 2.

On the other hand, the second effect is relative to the dispersion included. While the attenuation for the fundamental component remains constant for all power laws, the amplitude decreases with distance with varying decay rate. Thus, the energy decrease of amplitude of the fundamental frequency is explained as the harmonic cascade is modified: the nonlinear efficiency is modified due dispersion. The most dispersive regime is obtained for a power law of $\gamma = 1$, where the observed amplitude of the fundamental frequency is maximum. Figure 3 shows the spectral components measured at the shock distance, where both effects described can be identified clearly.

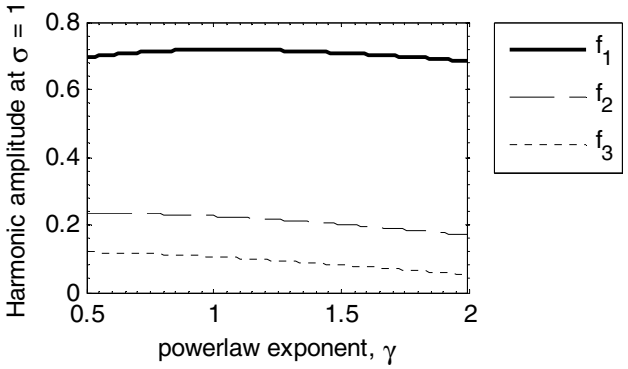


Figure 3. Amplitude for the fundamental, second and third harmonic measured at the shock distance for a progressive plane wave in frequency power law media

In conclusion, the amplitude of the fundamental frequency depends on the selected attenuation, and if dispersion is not negligible, the nonlinear efficiency can be modified, weakly preventing the cascade processes and the energy transfer to higher harmonics. The dispersion is maximized for nearly linear power laws and high attenuation values, so is for this kind of biological media where the nonlinear efficiency is expected to be minimized. On the other hand, the final amplitude of higher harmonics depends on the power law because different attenuation value is used (see Fig. 1 at 2, 3, 4 MHz). However, this effect can be balanced out with the lack of pumping from the first harmonic, where the nonlinear efficiency can be decreased due to dispersion.

4.2. Nonlinear effects for focused beam in soft-tissue media

Inclusion of diffraction in the model for focused ultrasound simulations was taken into account by modelling a moderately focused bowl in an axisymmetric domain. The operating frequency of the device was $f_0 = 1\text{MHz}$, with an aperture of $A = 50$ mm, curvature radius of $F = 50$ mm, linear gain of $G = 26.2$ and $f_{number} = 1$. Figure 4 shows the simulation spatial distribution of the peak pressure for linear and frequency squared power law attenuation media, where differences in the spatial distributions can be appreciated. Those differences can be measured as displacements of the peak pressure maximum, variations in the sonicated area covered by the full width at half maximum (FWHM) or changes in the peak pressure value (not shown in Fig. 4).

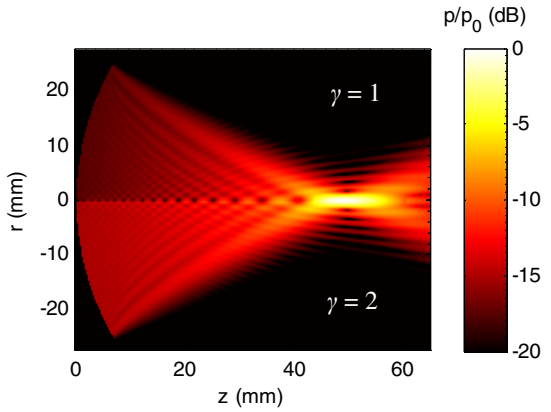


Figure 4. Normalized spatial distribution of the peak pressure for focused device for non-dispersive propagation (lower half) and linear power law (upper half)

In this way, simulations with variations on the power law exponent from 0.5 to 2 have been done. Results for the complete waveform and harmonic peak amplitude are shown in Fig. 5. (left) for nonlinear propagation, where the well-known nonlinear increasing value from the linear gain is observed for all the power laws considered, even considering that linear gain value ($G = 26.2$) is lossless calculated. However higher γ leads to higher attenuation for second and third harmonic than for the fundamental, but the results show an increasing gain value for all the spectral components and the absolute peak pressure value. This effect seems contradictory: for a media ($\gamma = 2$) with higher attenuation at most of the spectral components the final peak value is increased. However, this effect is well understood by the introduction of weak dispersion, where the weak phase mismatch between the nonlinear generated wavevector and the propagating wavevector can lead to a decrease in the nonlinear efficiency of the cascade processes and thus, diminishing the nonlinear peak pressure gain (p / p_0) amplification in a focused ultrasound device, even when the attenuation in the high frequency range is higher.

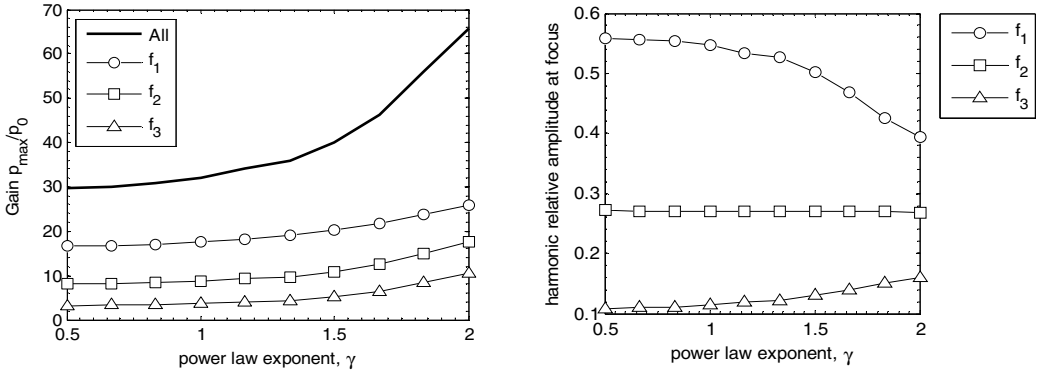


Figure 5. (Left) Effective gain variation and (Right) harmonic relative amplitude at the focus, versus frequency power exponent for fundamental, second and third harmonic

In order to eliminate at first order the effect of the focusing in the discussion, harmonic normalized amplitude is plotted in Fig. 5 (right). Thus, when normalizing the present beam pressure distribution, is clearly observed how the amplitude of the first harmonic decreases with power law exponent. In this way, less energy remains in the fundamental component for non-dispersive media ($\gamma = 2$), and the energy is transferred to higher components that conform the sharp characteristic nonlinear profile. Moreover, here diffraction effects are added to the attenuation and weak phase mismatching effects discussed for plane waves discussed above, so harmonic amplitude depends not only on the balance of nonlinear pumping and attenuation, but also on beam focalization and also self-(de)focusing.

5. Conclusions

A generalized formulation of finite amplitude waves in multiple relaxation media has been developed for the constitutive relations of nonlinear acoustics and solved by finite differences in time domain for axisymmetric focused sources in frequency power law media. Results for varying power law exponent media shows that two main effects describe the final wave amplitude. First, the power law varies attenuation magnitude for spectral components so the amplitude of second and higher harmonics is reduced. Secondly, weak dispersion modifies the nonlinear efficiency and more energy remains in the fundamental component where $\gamma \to 2$. When diffraction is included by a focused beam, the effects remain present: nonlinear effects are more significant for non-dispersive media, even where the attenuation of the higher harmonics is remarkably higher. However, as the diffraction can be modified by choosing different focusing devices, remains open to explore the harmonic amplitude dependence not only on the balance of nonlinear pumping and attenuation, but also on beam focalization and self-(de)focusing.

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