PHONONICS 2015: 3nd International Conference on Phononic Crystals/Metamaterials, Phonon Transport and Phonon Coupling

Paris, France, May 31-June 5, 2015

PHONONICS-2015-0479

Nonlinear Waves in Periodic Media: from Lattices to Superlattices and Phononic Crystals

V.J. Sánchez-Morcillo[1], N. Jiménez[1], A. Mehrem[1], R. Picó, L.M. García-Raffi[2], V. Romero-García[3], V. Tournat[3]

[1] *IGIC, Universitat Politécnica de Valencia, Paranimf 1, 46730 Gandia Spain,*
[2] *IUMPA, Universitat Politécnica de Valencia, Camino de Vera s/n 46022 Valencia, Spain*
[3] *LUNAM Université, Université du Maine, CNRS, LAUM UMR 6613*
Av. O. Messiaen, F-72085 Le Mans, France.
victorsm@upv.es

Abstract: Some effects related to high amplitude acoustic waves in periodic media are investigated. The combined action of nonlinearity (harmonic generation) and periodicity (different propagation velocities and attenuation for the different harmonics) results in novel and unexpected phenomena, with respect to the linear problem, and opens the door to new mechanisms of acoustic wave control and manipulation.

Sound wave propagation in periodic media has become increasingly popular in the last years, after the introduction of the concept of sonic and phononic crystals. Exploiting the analogies with other type of waves (mainly electrons and light) in the corresponding media, many interesting effects as forbidden propagation bands (band-gaps), focalization, self-collimation, negative refraction, and many others have been proposed. The huge majority of the studies have been done assuming low-amplitude waves, or linear regime, where the mentioned analogies apply. In linear systems, frequency is conserved. Intense wave propagation in nonlinear periodic media has been much less explored, particularly in acoustics. Here we present some of new phenomena related to sound wave propagation in a nonlinear medium with periodically modulated properties. Space dependent linear properties (as density, or sound velocity) introduce dispersion, which may be very strong at some frequency ranges. On the other hand, propagation in a nonlinear medium leads to harmonic generation, which in the case of nondispersive media ends up with the formation of a shock wave. This work explore several effects of strong dispersion in acoustic waves propagating in nonlinear media.

The simplest case of nonlinear periodic medium corresponds to a lattice or chain of oscillating masses, coupled by some anharmonic potential and excited at one boundary. Such a lattice, depending on the form of the interation potential, may describe a granular chain (spheres in contact interacting by Hertz potentials), an ionic chain (charges interacting by repulsive Coulomb potentials), and, more generally, a system of masses coupled by nonlinear springs. Most of such complex potentials may be approximated, for small displacements, by a quadratic or cubic dependence on the amplitude, leading to the celebrated FPU lattice. We investigate the process of second harmonic generation in such lattices, under conditions of weak and strong dispersion. Assuming that the wavelength is much larger than the interparticle distance, the lattice behaves as a homogeneous nondispersive medium, where harmonics grow as they propagate propagation; otherwise, dispersion has an impact in the harmonic generation processes. Experiments performed in a chain of repeling magnets are also presented, in good agreement with the main conclusions.

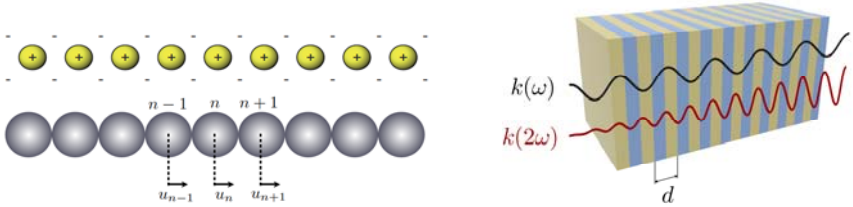


Figure 1. Different types of 1D phononic crystals: Left - ionic (top) and granular (bottom) lattices. Right - superlattice.

PHONONICS 2015: 3nd International Conference on Phononic Crystals/Metamaterials, Phonon Transport and Phonon Coupling

Paris, France, May 31-June 5, 2015

PHONONICS-2015-0479

Once the basic nonlinear process is discussed in the discrete system, we consider a sequence of finite lattices each containing many elements. This is equivalent to a 1D nonlinear multilayered structure, consisting of periodically arranged (along the propagation axis) layers of elastic isotropic materials of two types, with thickness d_α (α = 1,2), being c_α the velocity of the sound wave in the α layer. The structure is periodic with period $d = d_1 + d_2$. Such a periodic structure is often called an elastic superlattice (SL) [1], since it has a macroscopic period *d* which, by definition, greatly exceeds the interatomic distance. Nonlinear acoustical effects in periodic structures have been studied only in few works. Harmonic generation [2] and soliton propagation [3] have been discussed in fluid/fluid and solid/solid multilayers, respectively. More recently, the combined action of nonlinearity and periodicity has been used in [4], to demonstrate an symmetric propagation device (acoustic diode). In [4], the nonlinear and periodicity effects act at separated locations.

The linear dispersion relation, or band structure, for 1D systems can be expressed analytically as [1],

$$\cos(kd) = \cos(k_1 d_1)\cos(k_2 d_2) - \frac{1}{2}\left(\frac{k_1}{k_2} - \frac{k_2}{k_1}\right)\sin(k_1 d_1)\sin(k_2 d_2) \qquad (1)$$

where *k* is the Bloch wave-number, and $k_j = \omega / c_j$. For a wave of frequency ω incident in a medium with known acoustical c_j and geometrical d_j parameters, the above equation results in a band structure of propagating and nonpropagating (bandgap) regions, as shown in Fig. (2). With Eq. (1) we can estimate the effect of periodicity on the different harmonics of the incident wave as they propagate in the multilayer.

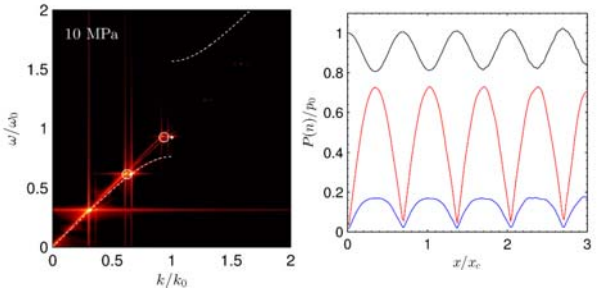


Figure 2. (a) The linear dispersion relation (dotted line) for first and second propagation bands, and the wavenumbers involved in the second and third harmonic generation process. Note that the third harmonic is in the bandgap. The parameters are the thickness of the layers d_1=d_2=0.1 mm and sound velocities c_1=1500 m/s and c_2=500 m/s. (b) First, second and third harmonic amplitudes along the propagation distance, normalized to the coherence length.

The nonlinear propagation of sound in the fluid multilayer can be described by several methods. We solve numerically the Navier-Stokes equations, with space dependent parameters. Nonlinearity appears in the continuity and Euler equations (advection terms) but also in the equation of state relating pressure and density acoustic perturbations in the form $p = c_0^2\rho' + (B/2A)(c_0^2/\rho_0)\rho'^2$ where c_0, ρ_0 are the values of sound velocity and density (in general different for each layer), and B□□□ is the parameter of nonlinearity. We have investigated the process of harmonic generation under conditions weak and of strong dispersion and nonlinearity. We find that the efficiency of harmonic generation is weakened by dispersion, where instead of an unbounded growth, beatings in the amplitude of harmonics are observed. We show that the beating period is dependent on the amplitude (defining a coherence length). In the case of a periodic system, we show that it is also possible to get phase matching of particular sets of harmonics (for example, by fulfilling the condition $k(n\omega) = nk(\omega)$ for only a particular value of *n*), by allowing them to propagate in different dispersion bands. Using this fact we are able to engineer the structure in order to propagate efficiently only particular set of harmonics. The wave propagates as if the multilayered medium had an effective nonlinearity (e.g. cubic) different from that of the constituent layers (quadratic). We have developed a couple mode theory, that allows to treat the problem in a simpler way, and obtain analytical solutions for the different harmonics. Another interesting features of this problem, as KdV soliton propagation, are discussed.

References

[1] A.M. Kosevich, *The Crystal Lattice: Phonons, Solitons, Dislocations, Superlattices*. WILEY-VCH Verlag (2005)
[2] Y. Yun, G.Q. Miao, P. Zhang, K. Huang, R.J. Wei, *Phys. Lett. A*. **343**, 351-358 (2005)
[3] R.J. Leveque and D.H. Yong, SIAM *J. Appl. Math.* **63**, 1539-1560 (2003)
[4] B. Liang, B. Yuan, and J.-ch. Cheng, *Phys. Rev. Lett.* **103**, 104301(2009)