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# Design of sub-wavelength acoustic absorbing panels using accumulation of resonances due to slow sound

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***Abstract*** **– We report sub-wavelength resonant panels for low-frequency sound absorption by using the accumulation of cavity resonances due to the slow sound phenomenon. The panels are built by bricks of sub-wavelength thickness with periodically distributed horizontal slits, in the walls of which identical Helmholtz resonators are attached (HRs). Due to the presence of the HRs, the propagation in the slit is highly dispersive with low group velocity that goes to zero due to the gap produced by the resonance of the HRs. In the slow-sound regime, the resonant frequency of cavity modes in the slit are shifted down and they are accumulated before the HRs frequency. Near the accumulation point, simultaneously symmetric and antisymmetric modes can be nearly critically coupled and produce nearly perfect absorption. However, due to the distribution of the resonant modes perfect absorption is not possible in this kind of structures. Therefore, we further analyze the case of different HRs in the slit, in which the structure of resonances is modified and symmetric and antisymmetric resonances can be simultaneously critical coupled. In this conditions we report perfect absorption for a transparent panel of 1/26 wavelength thickness.**

## I. Theoretical models

We consider a flat vertical panel of thickness $L$ perforated with open slits of height $h$ separated a distance $d$, as shown in Fig. 1. A periodic array of a rectangular cross-section Helmholtz resonators (HRs) was placed in the top of the slits following a square lattice of step $a$. The wall boundary conditions was assumed to be perfectly rigid. Thermo-viscous losses were accounted for in the slits and the ducts that compose the HRs through a complex and frequency dependent density and bulk modulus [1]. The HRs impedance, $Z_{\mathrm{HR}}$, was calculated accounting for the end corrections at the neck-slit and neck-cavity boundaries [2]. A modal expansion method was developed to represent the field on each $\Omega^{[i]}$ domain, where the effect of the resonators is accounted by a continuous effective impedance at the wall of the slit, given by $Z_i = Z_{\mathrm{HRs}}/\phi_n$ with $\phi_n = S_n/a^2$ the slit porosity where $S_n$ is the cross section of the neck similarly to [3]. Assuming the continuity of pressure between domains, and considering perfectly rigid boundary conditions for the walls, the reflection, $R$, and transmission, $T$, coefficients can obtained

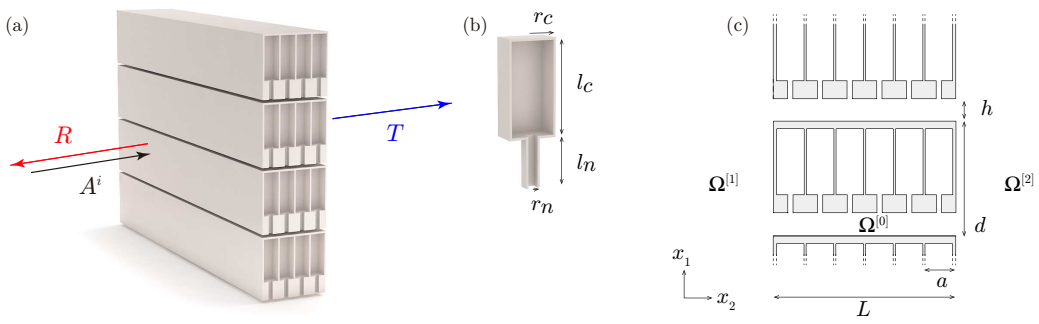



Fig. 1: (a) Conceptual view of the panel and (b) Helmholtz resonator. (c) Scheme of the slitted panel.

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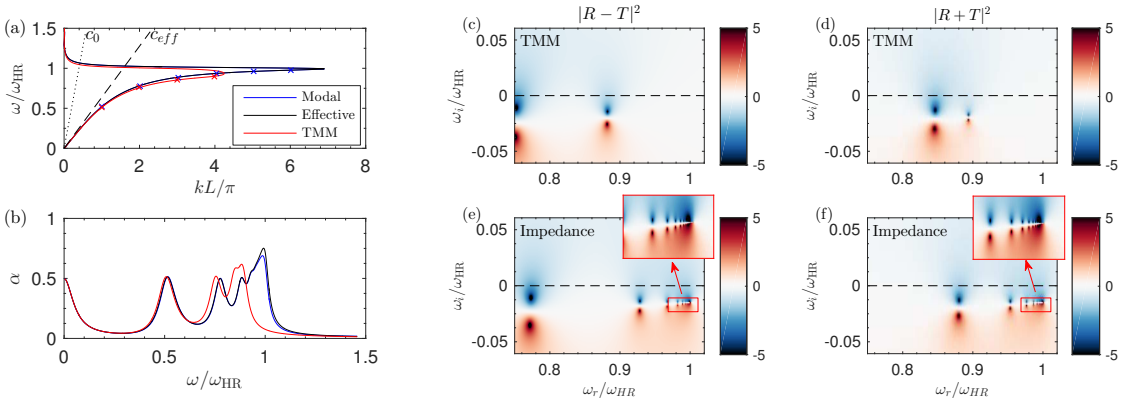



Fig. 2: (a) Dispersion relation in the slit using (red) TMM, (blue) full modal expansion and (black) effective parameters. (b) corresponding attenuation. (c-f) Complex frequency plane of the symmetric and anti-symmetric scattering matrix for the TMM method and impedance model (effective parameters).

by solving the mode-matching system. In the low frequency regime, the effective wavenumber inside the slits of the panel is $k_e = k_2^{[1]} = \sqrt{k_1^2 - k_{10}^2}$ with $k_{10} = 1/h\sqrt{-i\omega\rho_s h/Z_1}$ the transversal component of the wavenumber. Then, using the Helmholtz resonator impedance, $Z_1 = Z_{\rm HR}/\phi_n$, the complex and frequency dependent effective density and bulk modulus of the slit become

$$\rho_e = \frac{\rho_s}{\phi} \quad \text{and} \quad K_e = K_s \left[\phi\left(1 + \frac{K_s(S_c l_c K_n + S_n l_n K_c)\phi_n}{K_n\left(S_n K_c - l_n S_c l_c \rho_n \omega^2\right) h}\right)\right]^{-1}, \tag{1}$$

where $S_i$ and $l_i$ are the surface and length of the $i$ element of the resonator, the subscript $s$, $n$ and $c$ denotes the slit, neck and cavity of the HRs and the total porosity $\phi = h/d$. In the lossless case the low frequency branch of the phase speed can be expressed as $c_{eff} = c_a/\sqrt{1 + V_{tot}\phi_n/hS_n}$. The use of effective parameters allows the calculation of reflection and transmission coefficients, and finally, the absorption of the system as $\alpha = 1 - |T|^2 - |R|^2$. The full modal calculation [3] and its low frequency approximation assumes a wall impedance at the interior faces of the slits, leading to an infinite number of resonances in the slow sound regime. However, in reality the number of resonances is $N-1$, being $N$ the number of resonators in the slit, imposing limitations to the application of slow sound for designing acoustic absorbing systems [2]. Therefore, a discrete model is developed accounting for the finite number of resonators using the Transfer Matrix Method (TMM) [2], in which the end correction is included for accounting for the radiation of the periodic arrangement of slits.

## II. Results

Figure 2 summarizes the results for a panel with $N = 6$ equally sized resonators. The dispersion relation in the interior of the slit is shown in Fig. 2 (a). It can be noticed how when resonators are included an hybridization band-gap is generated at its resonance frequency. Therefore, slow sound conditions are achieved in the first band, where the asymptotic value given by $c_{eff}$ is observed in the low frequency regime. Furthermore, for frequencies just below the band gap extremely slow sound is observed, where sound absorption can be achieved at the induced resonant frequencies $kL/\pi = 1, 2, \ldots$. In addition, important differences are observed between the calculation with impedance models and using TMM. The full modal expansion, red curve in Fig. 2 (a), assumes a continuum wall impedance, $Z_1 = Z_{\rm HRs}/\phi$, and therefore, the effect of the finite number of resonators is not included. In contrast, this effect is accounted by TMM, blue curve in Fig. 2 (a). The result of including $N$ resonators is that the number of possible low-frequency resonances is limited to $N-1$, which are in fact the possible collective modes of the HRs [2]. A more clear view of this phenomena can be observed in the complex frequency plane representation of the transmission and reflection matrix. Figure 2 (c-f) shows the quantities $|R-T|^2$ and $|R+T|^2$ corresponding to the zero-pole structure of the symmetric and anti-symmetric problem respectively. First, it can be observed that the zero-pole structure for the impedance model, Fig. 2 (e-f), presents an infinite collection of zeros and poles that are stretched around $\omega_{\rm HR}$. By contrast, the corresponding structure obtained by TMM, Fig. 2 (c-d), the number of

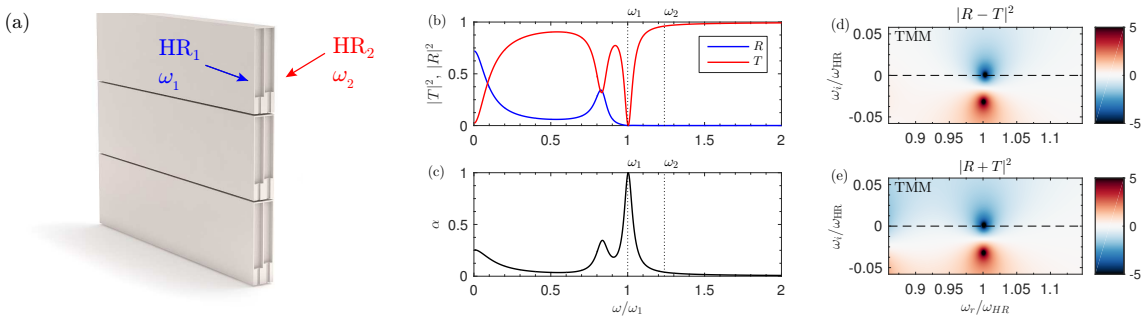



Fig. 3: (a) View of the perfect sub-wavelength absorbing panel. (b) Transmission and reflection using TMM, (c) absorption, (d-e) complex frequency plane of the symmetric and anti-symmetric scattering matrix.

zeros/poles is limited to $N-1$. Secondly, it can be observed that the zeros of the symmetric problem, $\omega^Z_{\text{sym}}$, are located at frequencies staggered to the zeros of the anti-symmetric $\omega^Z_{\text{asym}}$. It is worth noting here that, for a given frequency, perfect absorption can only be achieved if both, the symmetric and antisymmetric zeros are located in the real axis, $\text{Im}(\omega^Z_{\text{sym}}) = \text{Im}(\omega^Z_{\text{asym}}) = 0$, i.e. the zeros of the symmetric and antisymmetric problem must be simultaneously critically coupled. As the zero-pole structure is staggered in frequency, perfect critical coupling is not possible. However quasi-perfect absorption can be achieved. If a single zero is placed in the real axis, then the absorption of the system is $\alpha > 0.5$. However, as zeros present some frequency-width, the system properties can be tuned to place a zero of the anti-symmetric (symmetric) problem very close to the corresponding symmetric (anti-symmetric) one and, simultaneously, both as close as possible to the real axis.

One way to break the symmetry and unstagger the zero-pole structure is to include HRs of different resonant frequencies [4]. This situation is presented in Fig. 3. In Fig. 3 (a-b) the dotted lines indicates the position of the resonances for $N = 2$ resonators. In this conditions, the high frequency resonator acts as a rigid wall and as it can be observed in Fig. 3 (a) the transmission is forbidden at band-gap frequencies. Then, the parameters of the lower frequency resonator can be tuned to include the exact amount of losses to reduce the reflection. In the complex frequency plane, this reduces to localize both zeros of the symmetric $(R-T)$ and anti-symmetric $(R+T)$ on the real axis as shown in Fig. 3 (c-d). The absorption of the sample reaches $\alpha = 1$ for a sample of thickness $L/\lambda_0 = 26$ for the transmission problem.

## III. Conclusions

The limits of acoustic absorption in sub-wavelength transparent panels using slow sound are discussed. The current work shows that as long as the zero-pole structure of the symmetric and anti-symmetric problems is staggered, no perfect absorption is possible using regular arrays of resonant elements in a cavity. We show that using different resonators the symmetry can be broken and therefore perfect absorption by critical coupling is achieved for sub-wavelength thickness transparent panels.

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