Nonlinear waves in phononic rotational lattices

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Abstract: Wave propagation in a highly nonlinear system of coupled rotators is investigated. A discrete nonlinear wave equation is obtained, and the main characteristics of its propagating solutions are discussed. An experiment was designed to test the analytical predictions. Nonlinear signatures as amplitude-dependent dispersion, bistability and self-modulations are observed and discussed.

Highly nonlinear systems, including those where any linear response is absent (fully nonlinear systems) have been much less investigated than weakly nonlinear systems. Some known examples are granular media [1], or connected discs performing torsional motions [2]. In such systems, any arbitrarily small perturbation may lead to significant nonlinear effects. The absence of linear waves defines also the so-called *acoustic vacuum*, a medium where the speed of sound as defined by classical acoustics vanishes. Such systems are also known to support highly localized waves or *compactons*.

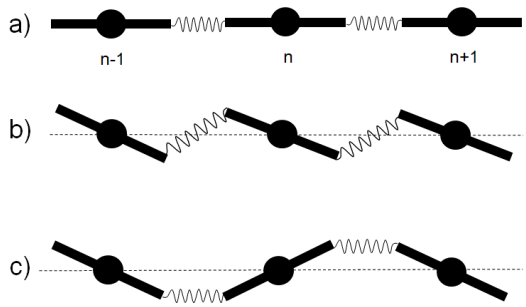


Figure 1 Three neighbour elements of the infinite rotator lattice. a) The lattice at rest, with rotating elements connected by unstressed springs. b) Lattice excited with a symmetric mode k=π and c) Lattice excited with the antisymmetric mode k=2π.

We present a theoretical/experimental study of the dynamics of mechanical waves propagating in a highly nonlinear one-dimensional lattice consisting of rotating elements, with mass *m* and inertia moment *I*, coupled by linear springs (Fig. 1). The nonlinearity, of geometrical origin, is of cubic type. In the case of absence of pre-stress of the springs in the rest state (Fig. 1a), linear terms are absent and the system is purely nonlinear. The equation of motion for this lattice has derived for a general geometry (for arbitrary spring-to- rotator length ratio), and its different limits have been explored.

The equation of motion for rotator *n* is defined by its angle φ_n. Wave propagation in this system is described by the nonlinear discrete equation

$$\ddot{\varphi}_n = -\left(\frac{1-\bar{\mu}^2}{12}\right)\left[\varphi_{n-1}^3 + 2a\varphi_n^3 + \varphi_{n+1}^3 + 3\varphi_n^2\left(\varphi_{n-1}+\varphi_{n+1}\right) + 3b\varphi_n\left(\varphi_{n-1}^2+\varphi_{n+1}^2\right)\right] \quad (1)$$

where *a*, *b* and μ are geometrical parameters dependent on the spring length l_0 and radius of rotator *R*.

As the system described by Eq. (1) is highly nonlinear, it does not show linear dispersion at all, presenting what has been called an acoustic vacuum. Moreover, the obtained nonlinear dispersion is of optical type, i.e. the wavelength increases with the frequency of the wave. Figure 2 shows the dispersion relation of the first harmonic (the wave at the excitation frequency). Note the presence of upper-

lower bandgaps. Note also that, since the frequency in Fig. 2 is normalized to the amplitude of excitation, the system, this means that, for a given excitation frequency, the dispersion relation can be tuned by the excitation amplitude,

Analytical solutions for the amplitude of the fundamental wave and its third harmonic have been obtained in the rotating wave approximation. Other interesting feature of the system is that the amplitude of the third harmonic vanished for a particular k mode, i.e. the system supports purely harmonic waves.

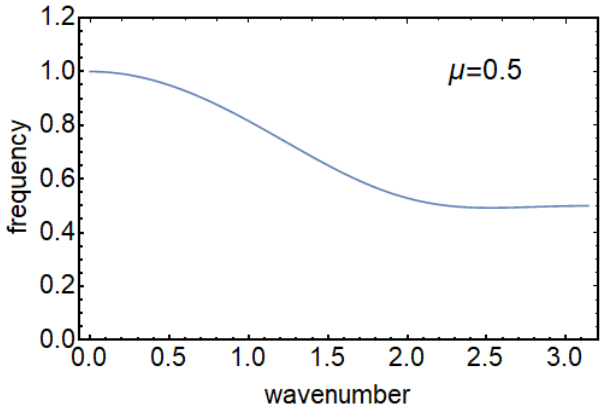


Figure 2 Dispersion relation for the case $l_0=R$ (μ=0.5). Frequency is normalized to the amplitude of the excitation, therefore waves can be propagating or evanescent depending on the initial amplitude.

An experiment has been designed to check the main predictions of the theory. The system is excited at the boundary by driving harmonically the angle of the first rotator, and the motion of each rotator is tracked by video and processed to obtain the time evolution of the angles. Measurements show evidences of the nonlinear characteristics of the system, as the generation of harmonics, bistability and self-modulations, and the predicted amplitude-dependent transition from evanescent to propagating states, in agreement with the nonlinear dispersion characteristics obtained analytically. Other nonlinear features of this system are also reported.

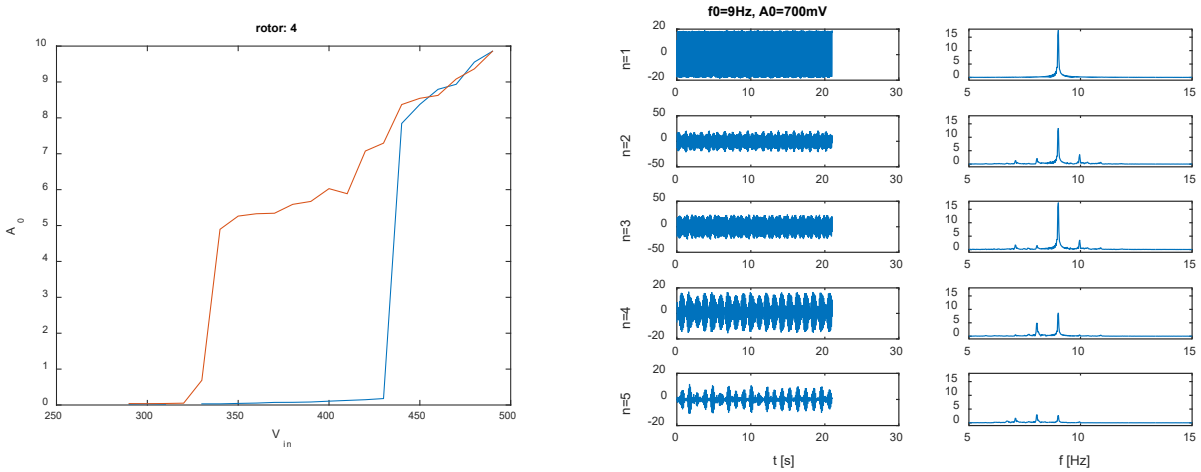


Figure 3 Experimental results obtained with the rotator chain (a) oscillation amplitude of the rotator n=4 for increasing (red) or decreasing (blue) excitation amplitudes. Note the bistable behaviour. In (b) we show the time evolution and spectrum of the first 5 rotators, where self-modulation is also evident.

References

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