Modulational Instability of Microbubbles Surface Modes

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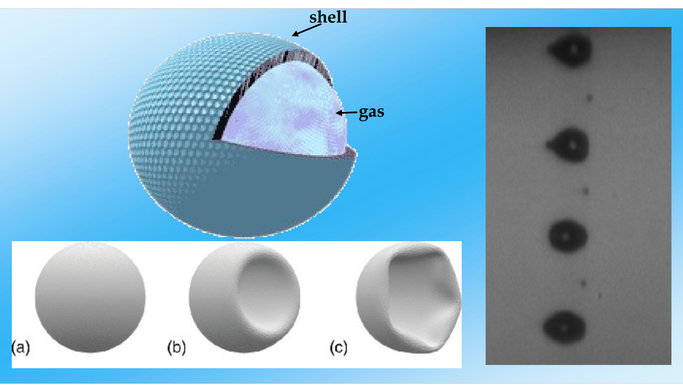
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Abstract

Microbubble surface modes appear when ultrasound contrast agents (UCA) are insonified with specific ultrasound pulses. In this case, the radius is a space-dependent function which can be expanded on the basis of spherical harmonics describing the spatial vibrational modes of the bubble. Modulational Instability (MI) is an instability of the vibration modes in nonlinear systems induced by an external driving. Thanks to similarity analysis[1], the bubble can be studied with a macroscopic mechanical 1D ring chain (diameter =73 cm) using N=52 nonlinearly coupled pendulum of mass m=6g and length =3.2cm with periodic conditions. Simulations consist in studying numerically the MI criterion versus the amplitude of external field applied to the bubble. The observation of ILM in 1D chain of coupled oscillators, presenting many similarities with the discrete bubble model, showed to play a role in creating spatio-temporal localized excitations is observed. In our periodic pendulum lattice apparatus describing the macroscopic equivalence of the bubble, the system parametrically driven at 5.4 Hz presents subharmonic behavior and localized modes oscillating at 2.7 Hz. Versus the amplitude and the frequency of the driving, the mechanical ring presents parametric instability surface modes and localized modes oscillating at subharmonics of the parametric excitation.

Ultrasound Contrast Agents (UCA)



The bubbles are coated with biocompatible materials. UCA buckling is observed when solid shell bubbles are insonified with ultrasound [2]

Mechanical Model

Experimental set-up



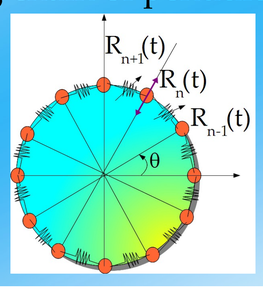
external driving:

parametric driving of the gravity

Simulations

macroscopic mechanical 1D ring chain of pendulum

$$R(t,\theta,\phi) \rightsquigarrow R(t,\theta) \rightsquigarrow R_n(t)$$



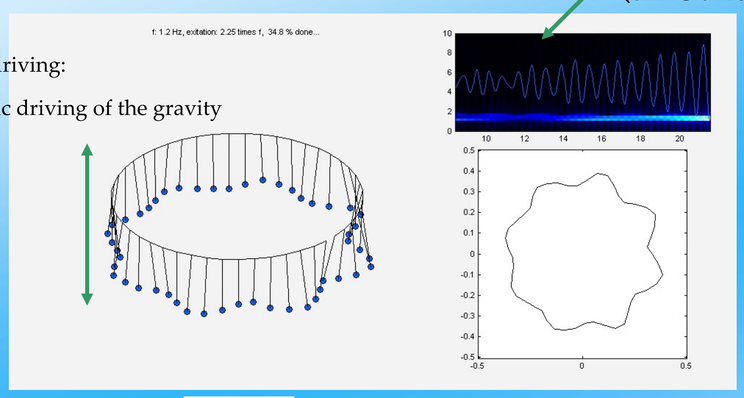
external driving:
parametric driving of the gravity

$$\ddot{R}_n(t) + \sin\left(R_n(t)\right)\left[\omega_0^2 + \eta\cos(2\omega_{ext}t)\right] + \gamma R_n'(t) = f\left(R_{n-1}(t) - 2R_n(t) + R_{n+1}(t)\right)$$

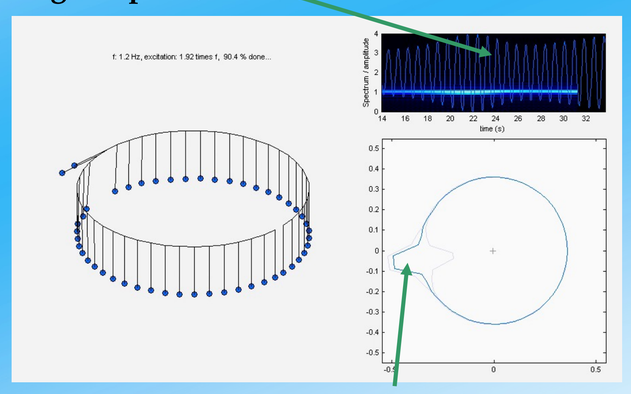
Modulational Instability (MI) (time and spectrogram plots)

external driving:

parametric driving of the gravity



$$\simeq Y_{l=8}^{(8)}(\theta)$$



Intrinsic Localized Mode (ILM) [4]

The spherical mode is unstable, *i.e.* $R(t) \rightsquigarrow R(t,\theta)$,
MI appears and localized behavior is observed when $\omega_{ext} = 1.92 f_0$

Theoretical Modelling

$$\rho_l\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) - \left(P_0 + \frac{2\sigma(R_0)}{R_0}\right)\left(\frac{R}{R_0}\right)^{-3\kappa}\left(1 - \frac{3\kappa}{c}\dot{R}\right) + P_0 + 2\frac{\sigma(R)}{R} + \frac{4\mu\dot{R}}{R} + \frac{4\kappa_s\dot{R}}{R^2} = -P_{ac}(t)$$

Linearization consist in assuming

$$R(t) = R_0 + \varepsilon r(t)$$

and

$$\sigma(R) = \sigma(R_0) + 2\varepsilon\kappa\left(\frac{R}{R_0} - 1\right)$$

In this case, it can be shown that

$$\ddot{r}(t) + 2\gamma\dot{r}(t) + \omega_0^2 r(t) = -\frac{P_{ac}(t)}{\rho_l R_0^2}$$

where ω_0 is given by

$$\omega_0^2 = \frac{3}{\rho_l R_0^2}K_V = \frac{3}{\rho_l R_0^2}\left(\kappa P_{atm} + \frac{2(3\kappa-1)\sigma}{R_0}\right)$$

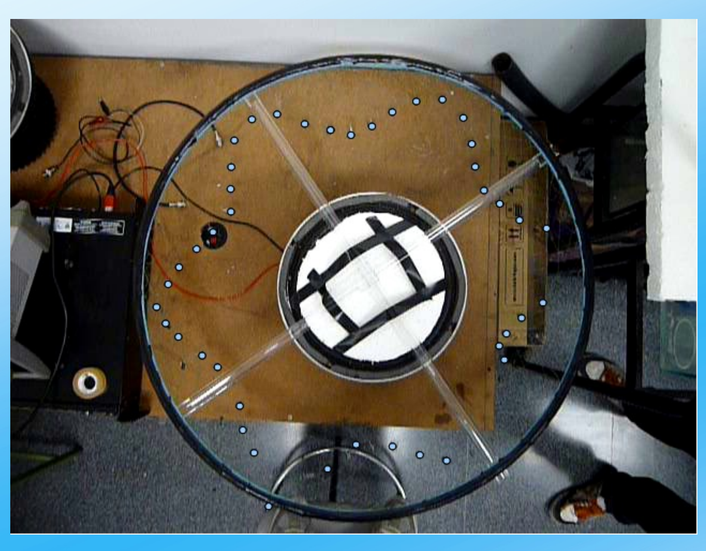
Bubble parameters [2]

$$\rho_l, \sigma, \kappa, c, \mu, \kappa_s$$

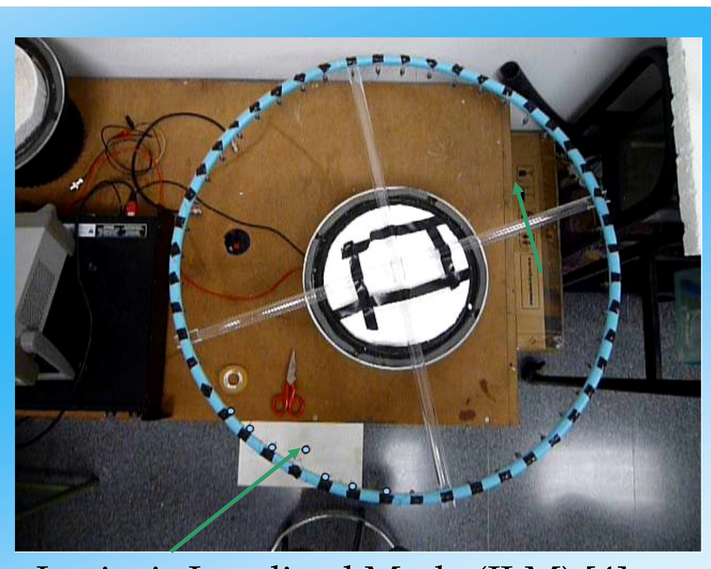
External driving (acoustic pressure)

$$P = P_0 + P_{ac}(t)$$

Observation of Localized Modes



Surface mode (N=6)



Intrinsic Localized Mode (ILM) [4]

Similarity Analysis and Analogies

Assuming classical values for physical parameters of the bubble, one have

$$f_0[MHz].R_0[\mu m] \simeq 3$$

The similarity property can be written

$$f_0[Hz].R_0[m] \simeq 3$$

Spherical Harmonics [3]

$$R(t) \rightsquigarrow R(t,\theta,\phi) = R(t) + \varepsilon\sum_{(l,m)} a_l(t)Y_l^{(m)}(\theta,\phi)$$

Bubble oscillation is then given by

$$\ddot{a}_l(t) + 2\gamma\dot{a}_l(t) + \omega_l^2 a_l(t) = 0$$

where ω_l is given by

$$\omega_l^2 = (l-1)(l+1)(l+2)\frac{\sigma}{\rho R_0^3}$$

Conclusion and Pespectives

- The localization of energy leading to coated UCA can be studied in analogy with ILM in nonlinear coupled pendulum with periodic properties
- The perspective of this analysis is to define practical optimized ultrasound pulses exciting the bubble and leading to sonodrugs applications

References :

[1] B. J. Cantwell, *Introduction to Symmetry Analysis*, Cambridge University Press, (2002)

[2] P. Marmottant et al, *A model for large amplitude oscillations of coated bubbles accounting for buckling and rupture*, J. Acoust. Soc. Am. 118, 3499–3505 (2005) ; Philippe Marmottant, Ayache Bouakaz, Nico de Jong, and Catherine Quilliet, *Buckling resistance of solid shell bubbles under ultrasound*, J. Acoust. Soc. Am. 129, 1231 (2011)

[3] R. Nabergoj, and A. Francescutto, Journal de Physique 40, 306–309 (1979)

[4] B. Denardo *et al*, *Observations of Localized Structures in Nonlinear Lattices: Domain Walls and Kinks*, Phys. Rev. Lett. 68, 1730 (1992)