# SPATIOTEMPORAL DYNAMICS IN A RING OF COUPLED PENDULA: ANALOGY WITH BUBBLES

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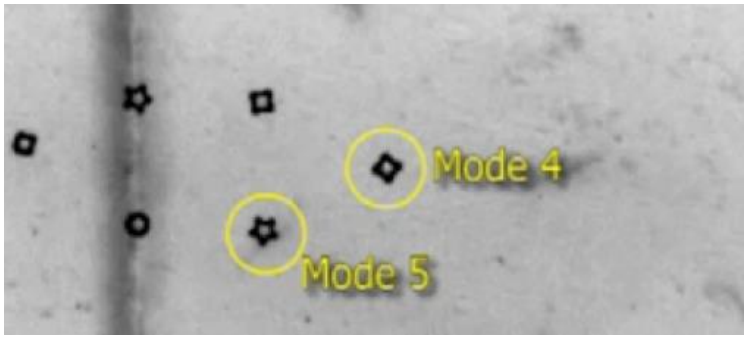
*Unité Mixte de Recherche «Imagerie et Cerveau» INSERM U930 - CNRS ERL, Blois (France)*

# Outline

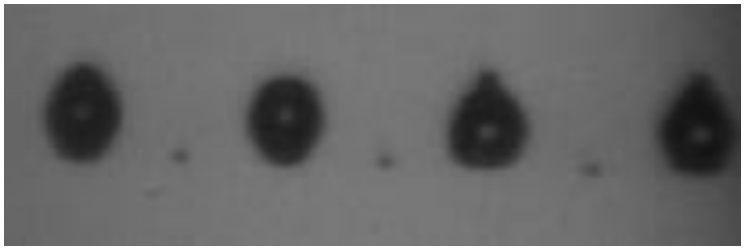
- Motivation: the acoustic bubble
- Ring chain of coupled pendula: modeling and experiment
- Experimental results: patterns, breathers and kinks
- Numerical results: inhomogeneous chains

# Motivation: bubbles in a ultrasound field

- Gas bubbles driven by an ultrasound field develope surface modes.
- Mechanism: parametric instability
- There are also evidences of localized modes in bubbles

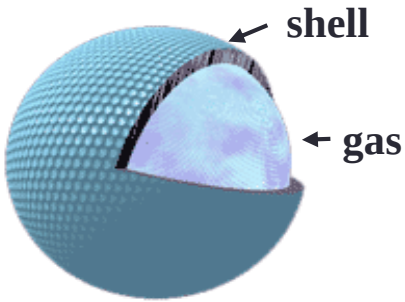


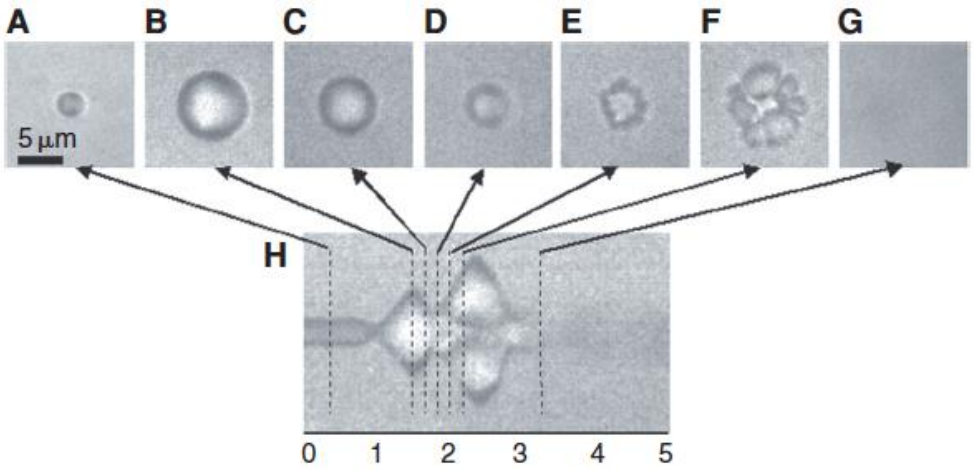


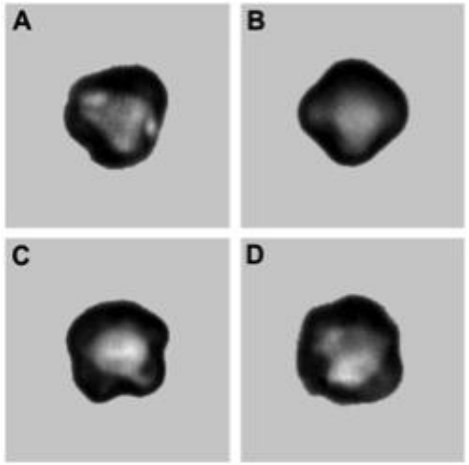



# Motivation: bubbles in a ultrasound field (2)

- Recent interest: coated microbubbles for medical applications (ultrasonic contrast agents)




Oscillation and destruction of lipid-shelled microbubble

# Basics of bubble dynamics (1)

Radial mode dynamics: Rayleigh-Plesset equation

$$\rho\left(R\ddot{R}+\frac{3}{2}\dot{R}^2\right)=P_g-P_0-P_A(t)-\frac{2\sigma}{R}-4\mu\frac{\dot{R}}{R}$$

$$P_A(t)=p_A\cos(\omega t)$$

Approach 1: spherical modes projection

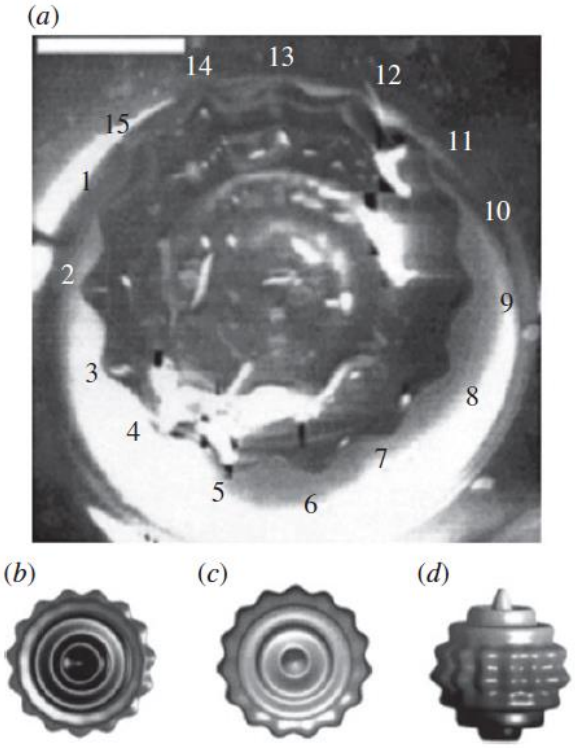
$$R(\theta,t)=R(t)+\sum_n a_n(t)Y_n(\theta)$$

Linearizing:

$$\ddot{a}_n+2\gamma\dot{a}_n+\omega_n^2 a_n=0$$

$$\gamma=\frac{3\dot{R}}{R}+\frac{2(n+2)(2n+1)\mu}{\rho R^2}$$

$$\omega_n^2=\frac{(n-1)(n+1)(n+2)\sigma}{\rho R^3}+\frac{2(n-1)(n+2)\mu\dot{R}}{\rho R^3}$$



# Basics of bubble dynamics (2)

$$r\left(R\ddot{R}+\frac{3}{2}\dot{R}^2\right)=P_g-P_0-P_A(t)-\frac{2s}{R}-4m\frac{\dot{R}}{R}$$

Approach 2: slow envelopes

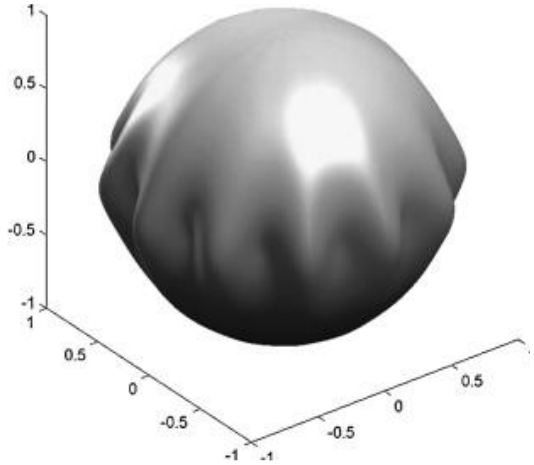
$$R(q,j,t)=R_0+x(q,j,t)$$

$$x(q,j,t)=\left(\frac{1}{2}\tilde{x}(q,t)e^{-iwt}+c.c.\right)\cos(nj)$$

$$2iw\frac{\partial\tilde{x}}{\partial t}=D\tilde{x}+d\frac{\partial^2\tilde{x}}{\partial x^2}+a\tilde{x}\left|\tilde{x}\right|^2+2iwd\tilde{x}+b\tilde{x}^*$$

$$D=\left(w_1^2-w^2\right),\ d=\frac{3w_1^2}{2n^2},\ a=\frac{17}{32}w_1^2\left(\frac{n}{R_0}\right)^2$$

$$w_1^2=s(n+1)(n+2)(n-1)\left(r_0R_0^3\right)^{-1}$$



# Analogies bubble-pendula

- Parametrically driven-damped chain

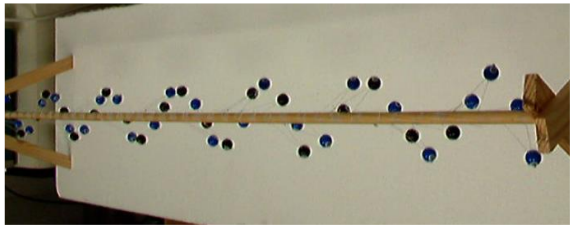
$$\frac{d^2 q_n}{dt^2} + w_0^2 \sin q_n - (q_{n+1} + q_{n-1} - 2q_n) = -\, b\frac{dq_n}{dt} - h\cos(2w_e t)\sin q_n$$

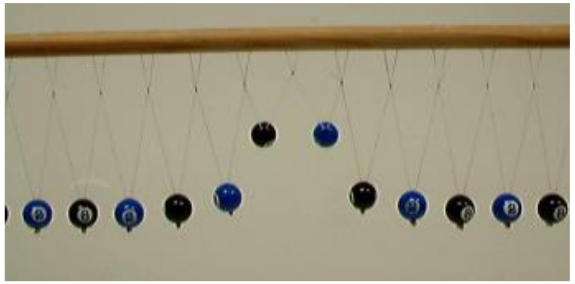
- Slow envelope approach (Denardo, PRL92)

$$2iw\frac{\partial \tilde{x}}{\partial t} = \mathrm{D}\tilde{x} + d\frac{\partial^2 \tilde{x}}{\partial x^2} + a\tilde{x}\left|\tilde{x}\right|^2 + 2iwd\tilde{x} + b\tilde{x}^*$$

$$\mathrm{D} = \left(w_0^2 - w^2\right),\ d = \frac{ga^2 d}{4L(L-d)},\ a = \frac{1}{2}w_0^2$$

$$w_0^2 = g/L$$





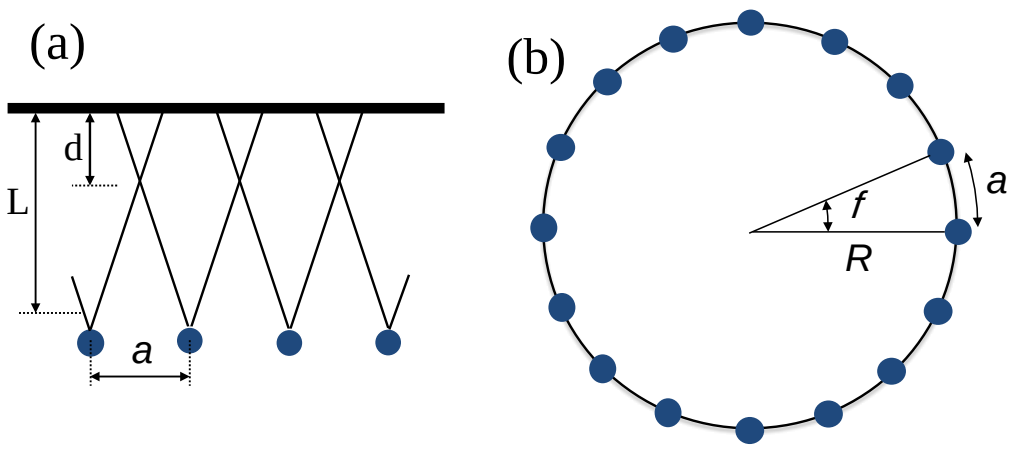
It predicts breathers and kinks

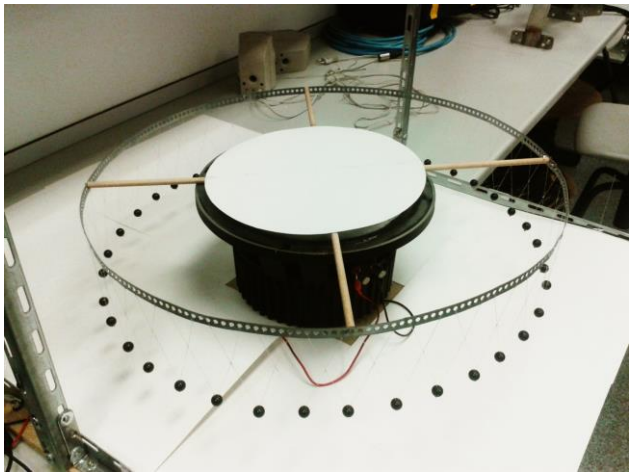
# Chains: straight vs circular

$$\frac{d^2 q_n}{dt^2} + w_0^2 \sin q_n - (q_{n+1} + q_{n-1} - 2q_n) = -b\frac{dq_n}{dt} - h\cos(2w_e t)\sin q_n$$

$$q_{n+1} = q_1$$

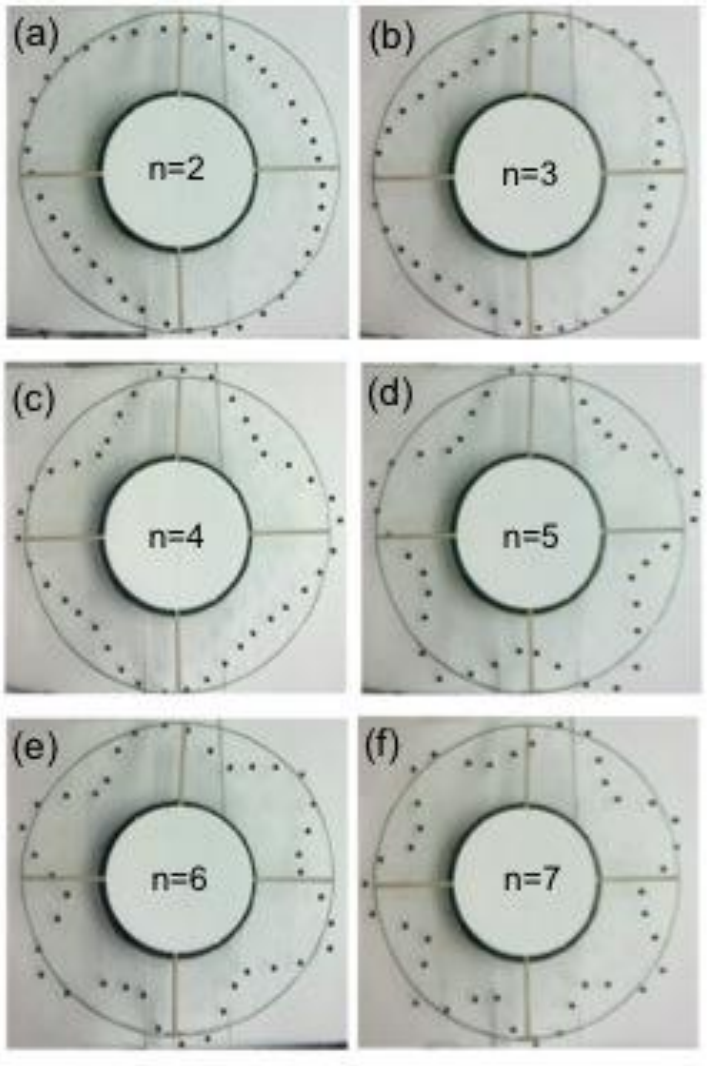
Curvature effects (asymmetry) for large amplitudes



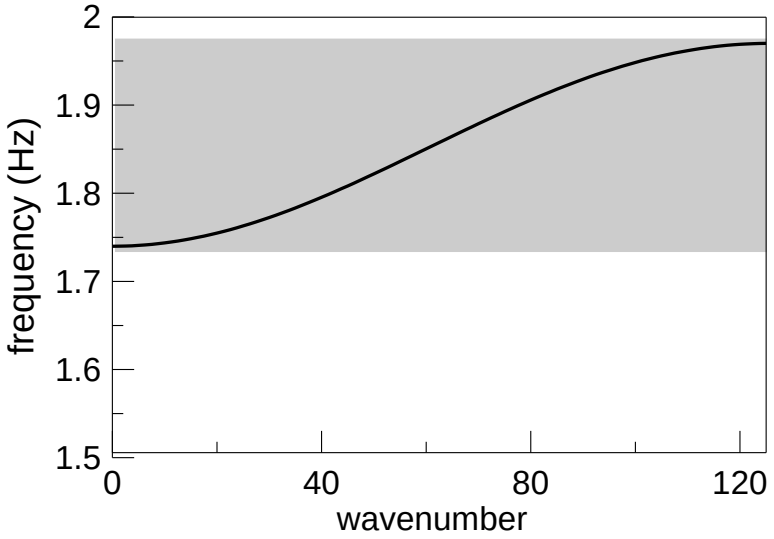
Our chain: N = 50, L = 5 cm, d= 1.6 cm, R = 60 cm

# Eigenmodes



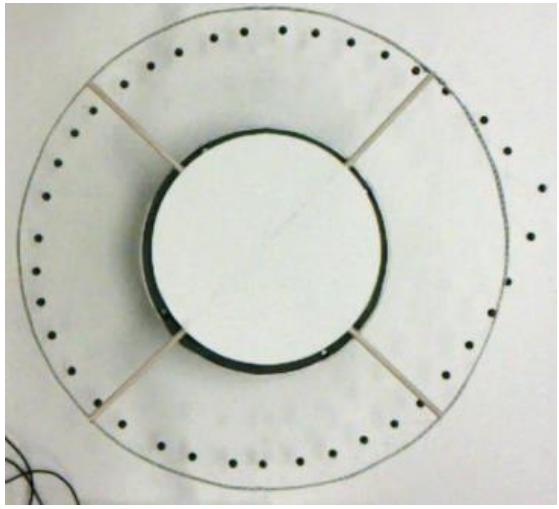
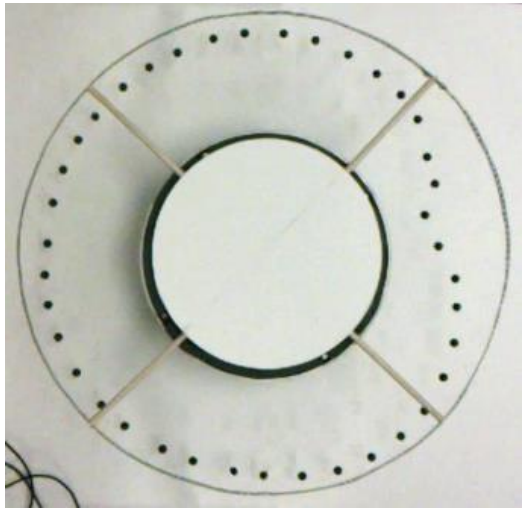
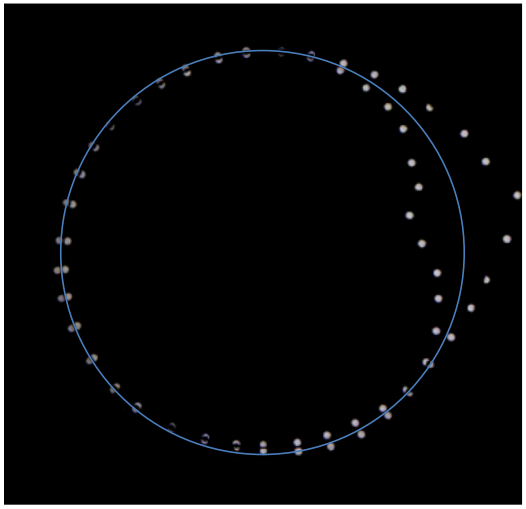

Dispersion relation

$$\omega_n^2 = \frac{g}{L-d}\left(1-\frac{d}{L}\cos\frac{n\pi}{4R}\right)$$



# Single breathers

- Exist for driving frequencies below lower cutoff frequency
- The shape of the breather in expansion-compression phases is not symmetric: bubble effect

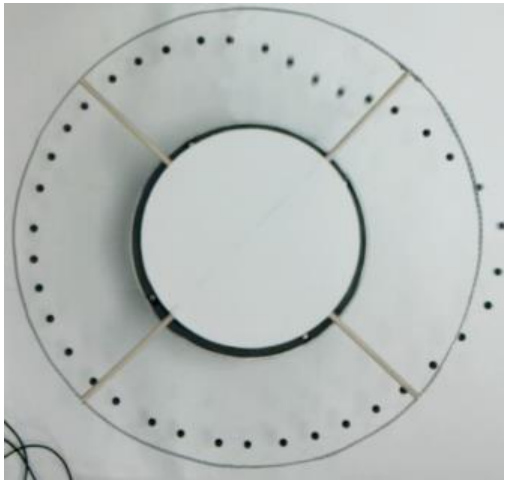




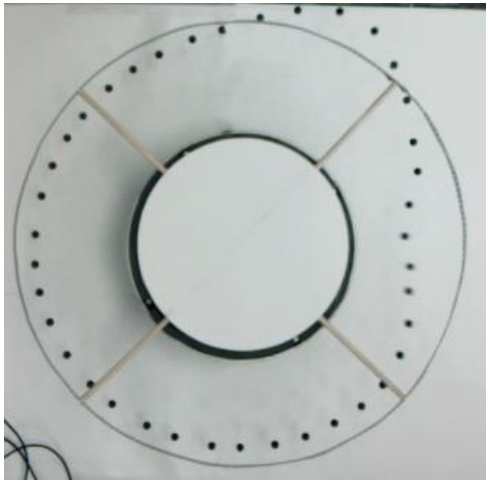
# Single breathers

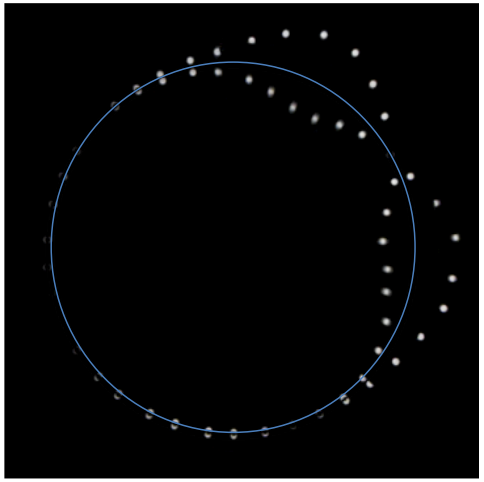


# Double breather

- Coexist with single breather
- Out of phase motion
- Breather profile is more rounded





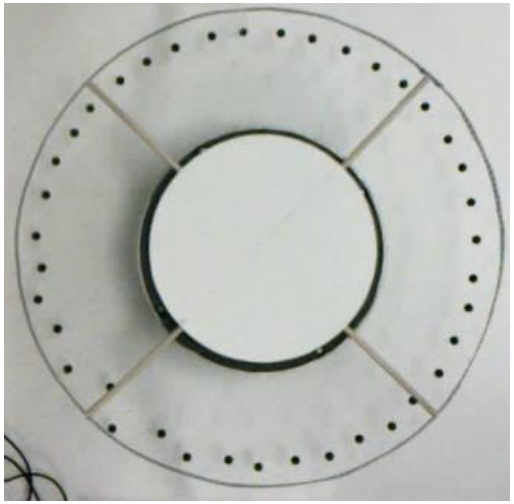


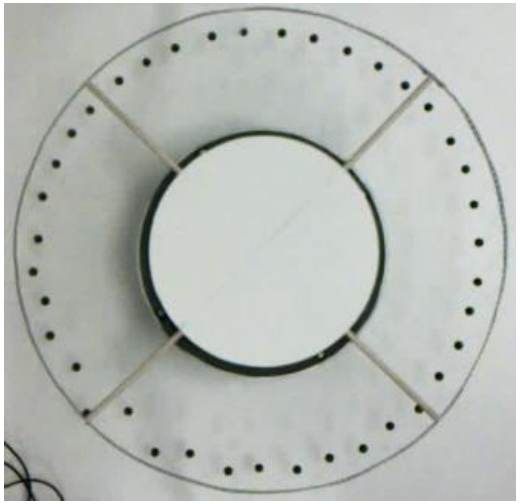
# Double breathers

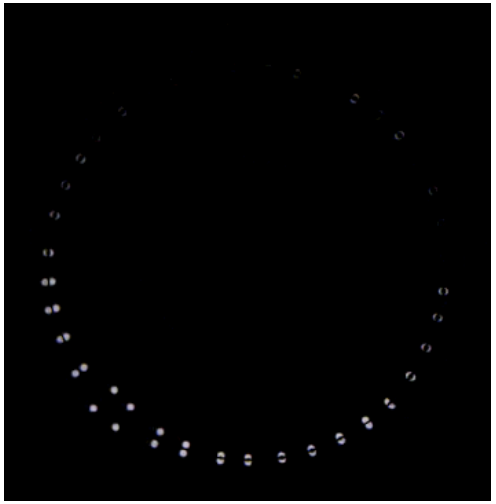


# Upper cutoff structures: breathers and kinks

- Neighbour particles moving out of phase
- Lower amplitude than low frequency breathers







# Upper cutoff breather in motion



# Upper cutoff kink in motion

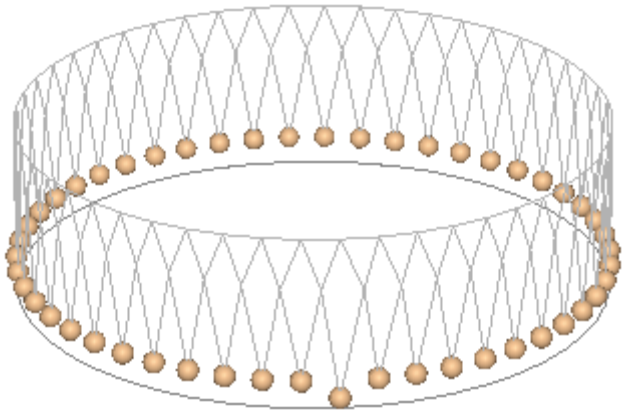


# Numerical simulation

The homogeneous chain does not show the growth of breathers

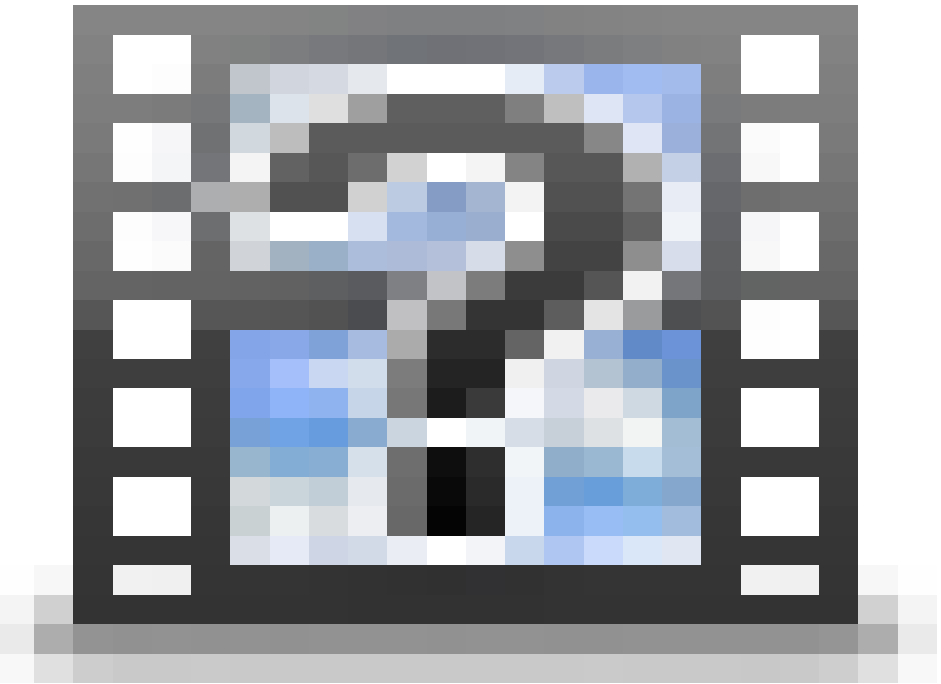
$$\frac{d^2 q_n}{dt^2} + w_0^2 \sin q_n - (q_{n+1} + q_{n-1} - 2q_n) = -b\frac{dq_n}{dt} - h\cos(2w_e t)\sin q_n$$

We consider instead the chain with a local defect (as proposed in Alexeeva et al, PRL2000)

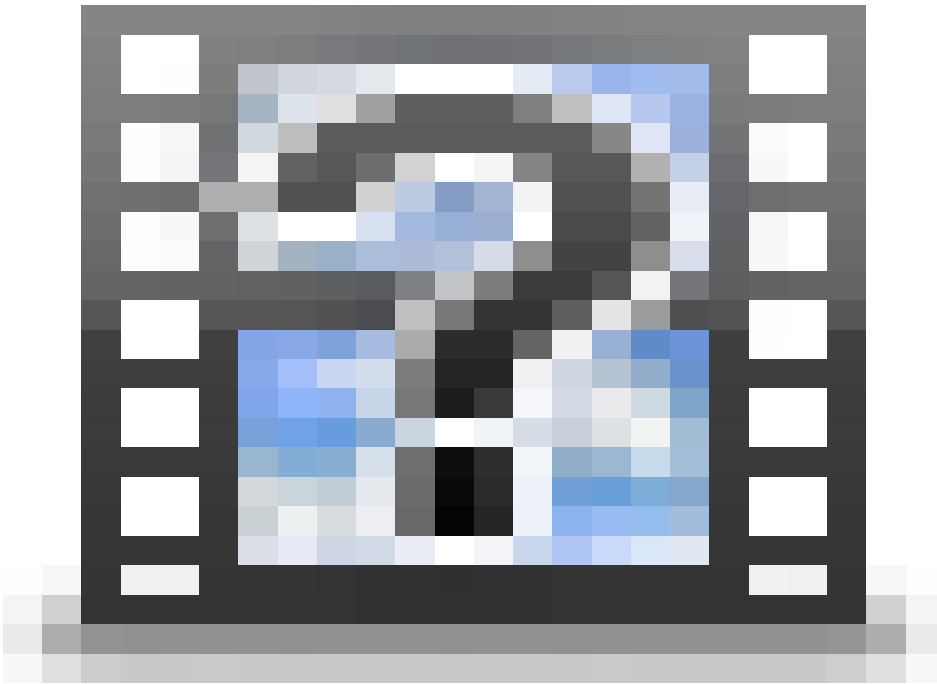


# Numerical simulations

Weak coupling

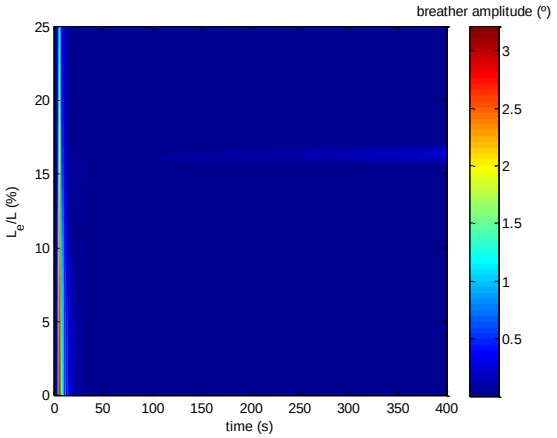


Strong coupling

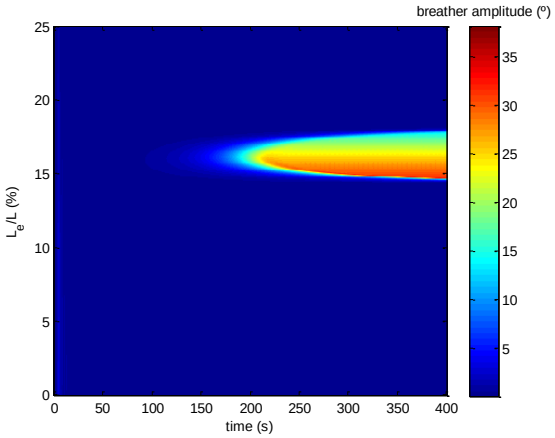


# Inhomogenities in the chain: thresholds

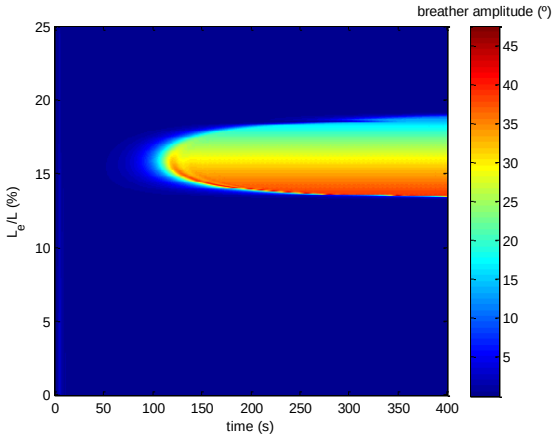
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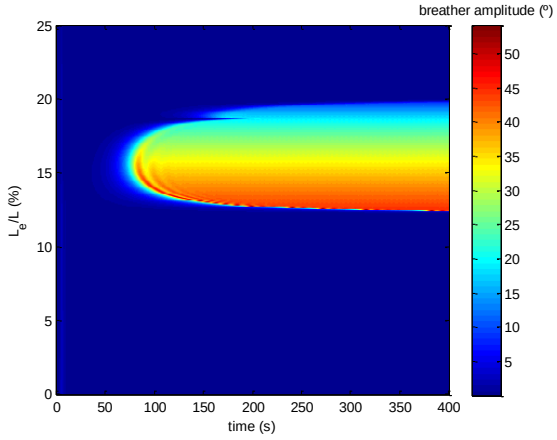

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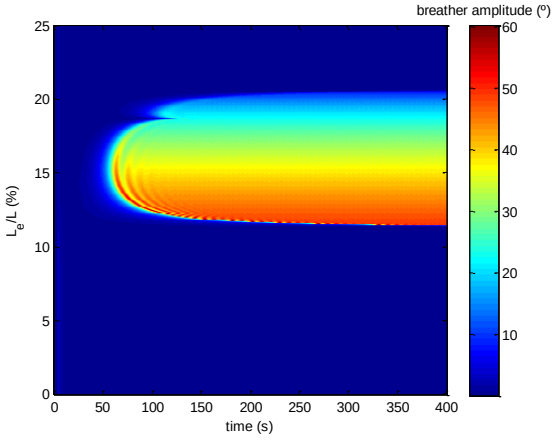

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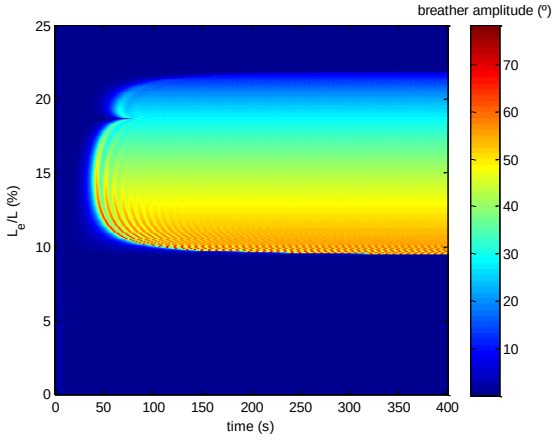

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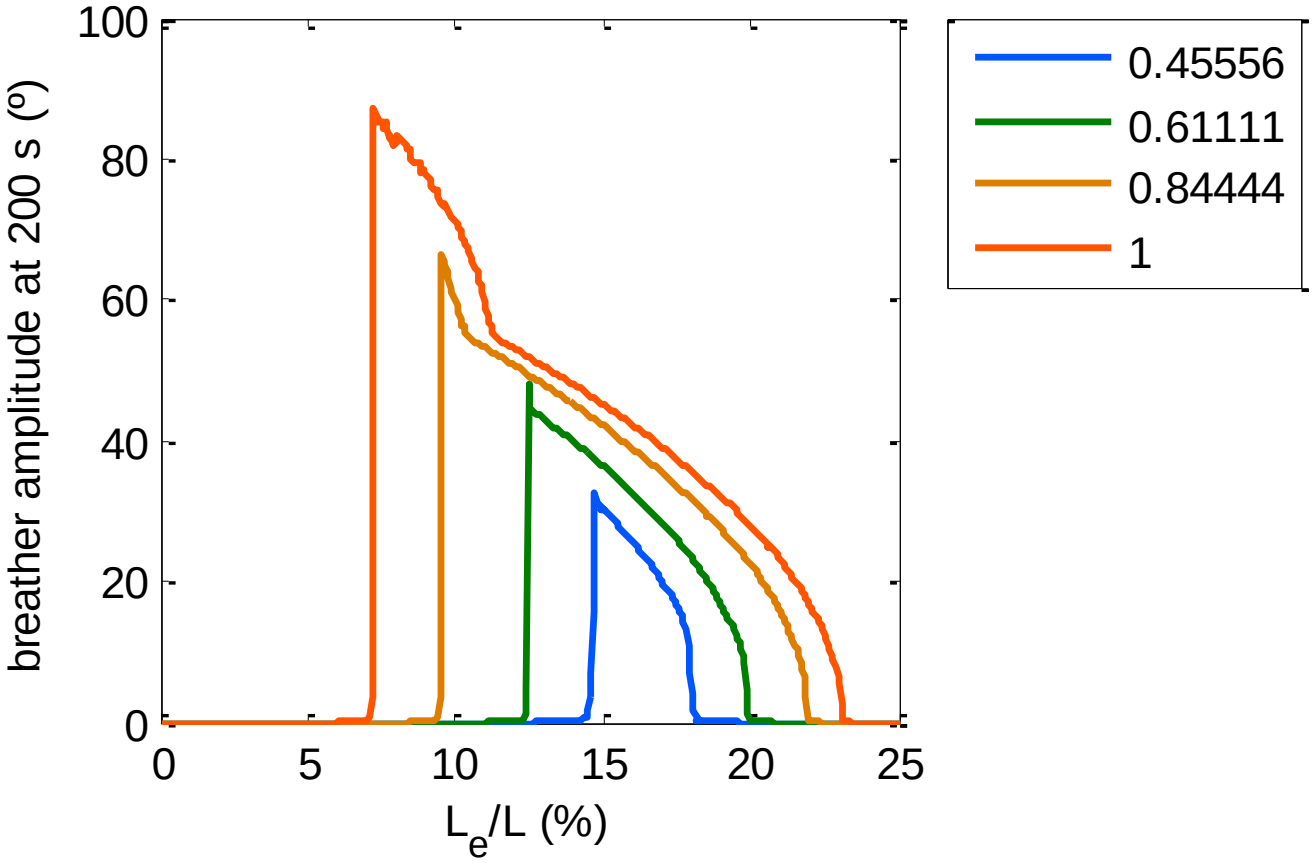

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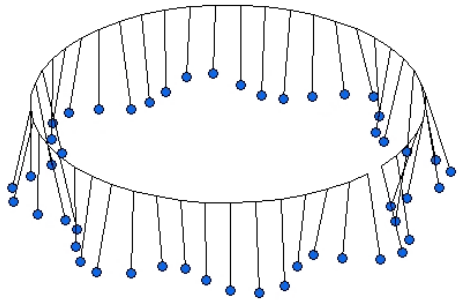

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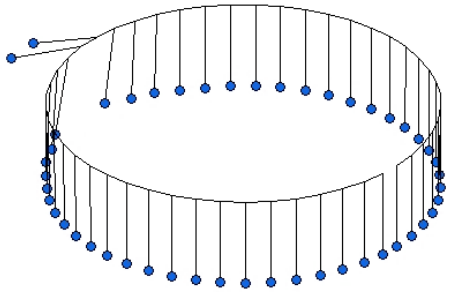


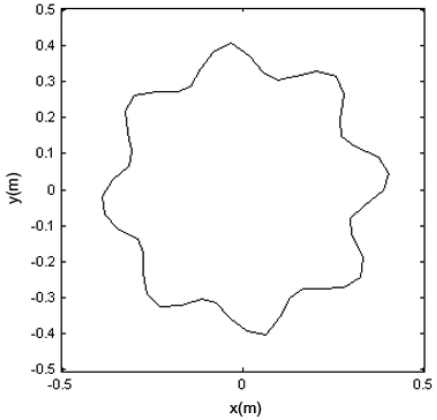
# Inhomogenities in the chain: thresholds

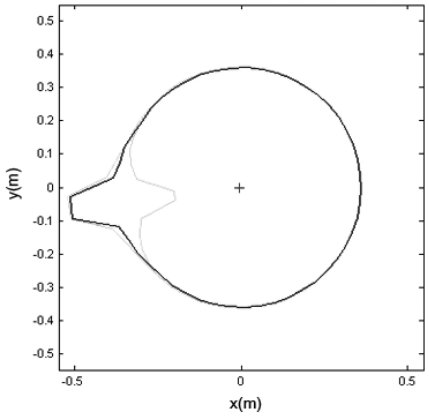


# Summary






0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
y(m)
-0.5
0
0.5
x(m)


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
y(m)
-0.5
0
0.5
x(m)



