Vibration modes in a pendulums ring: Analogy with gas microbubbles surface modes

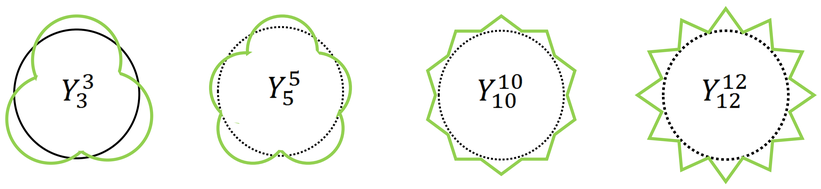
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Introduction

Ultrasound Contrast Agents are made of gas microbubble (1- 8 µm) that present surface patterns when subjected to specific ultrasound excitation (see scheme). The radius can thus be described on the basis of spherical harmonics:

$$R(t,\theta,\phi) = R(t) + \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_l(t) Y_l^m(\theta,\phi)$$

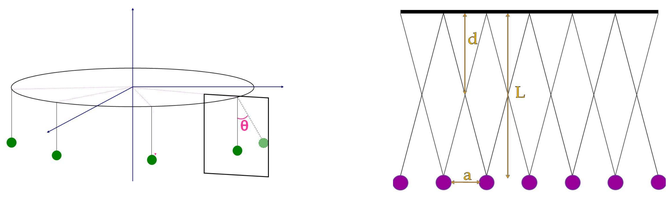


The surface oscillatory behavior of a microbubble is investigated through an acousto-mechanical analogy. It consists of a macroscopic experiment of coupled pendulums parametrically excited by a vertical force.

The aim of such study is to determine optimal excitation conditions of the microbubble for imaging and therapeutic applications.

Theory

The system is a chain of 54 pendulums of mass *m* fixed on a ring with nylon string forming a «V» shape with the vertical axis. The coupling is obtained by overlapping the strings and fixing them by a nod.



Motion equation

$$\ddot{\theta_n} + \beta\dot{\theta_n} + \left(\omega_0^2 + A\cos(2\omega_e t)\right)\sin(\theta_n) + k(\theta_{n-1} - 2\theta_n + \theta_{n+1}) = 0$$

with
β : damping coefficient
ω_e, A: excitation pulsation, amplitude
k: coupling constant between pendulums

$$\omega_0 = \sqrt{\frac{g}{L}}$$

Material & Methods

The chain lies on a loudspeaker linked to a waveform generator (Agilent 33220A) and an amplifier generating a sinusoidale excitation. Two excitation parameters (*i.e.* frequency and amplitude) have been studied and the generated surface modes have been acquired.

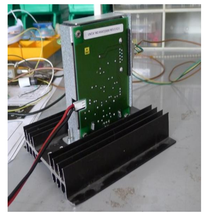
Chain characteristics:



- N=54 pendulums
- m=6 g
- Aluminum support
- Diameter=61 cm
- d= 6 cm
- a= 3,5 cm
- L=10 cm

Excitation system:

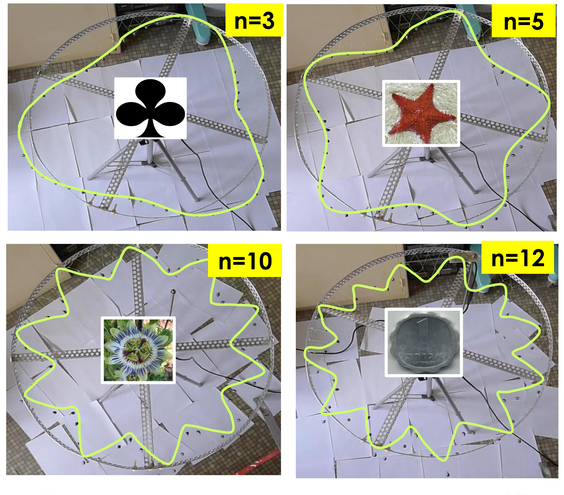






- *Frequency range:* f_e=0.5 Hz – 5 Hz
- *Amplitude range:* A= 0.5 Vpp - 3.5 Vpp

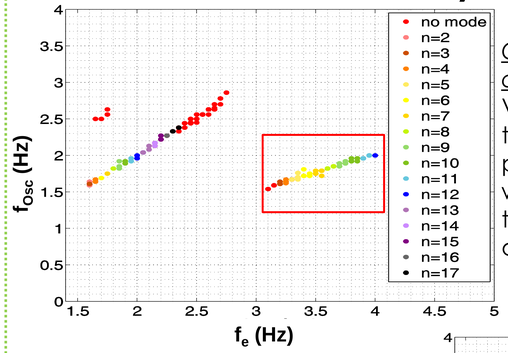
Results – Qualitative analysis



Observation of vibration modes up to n=17

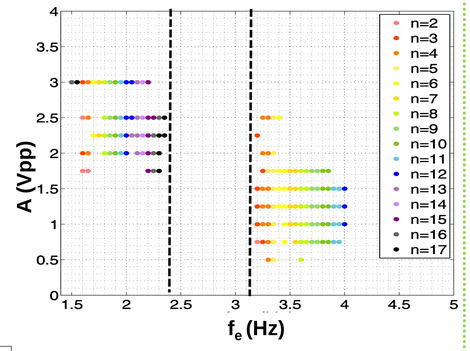
Videos here

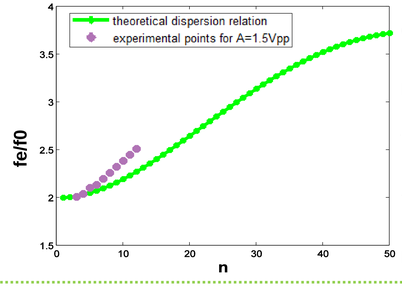
Results – Quantitative analysis



Oscillations at subharmonics of the emitted excitation:
Vibration modes observed in the red rectangle are the parametric modes expected when the ring is excited at twice the natural frequency of the pendulums.

Observation of a bandgap:
Between f_e=2.4Hz and f_e=3.15 Hz no vibration mode is observed.
On the left side, vibration modes were not expected. However, simulations show few vibration modes in this part for high excitation amplitudes. We are in a region where the parametric resonance is smaller but where modes can appear.





On the right side of the bandgap, parametric vibration modes up to n=12 are observed.

The first experimental points that have been plotted are closed to the theoretical dispersion relation. The following points deviate and required more accurate measurements

Conclusions

The same mechanisms seem to be at the origin of surface vibration modes of the pendulums ring and microbubbles. The results are promising and consistent with theory. In addition, simulation results seem to point in the same direction. Further studies are expected.. However, such acousto-mechanical analogy of macroscopic behavior and microscopic one provides important insights for the comprehension of coated microbubble's dynamics for both imaging and therapeutic applications.

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