UNIVERSITAT POLITÈCNICA DE VALÈNCIA

IGIC Instituto de Investigación para la Gestión Integrada de Zonas Costeras

UIA ultrasonic industry association powering sound ideas

Nonlinear Ultrasound Simulations Including Complex Frequency Dependent Attenuation

Application to Biological Media

Noé Jiménez, Javier Redondo,
Víctor Sánchez-Morcillo, Francisco Camarena

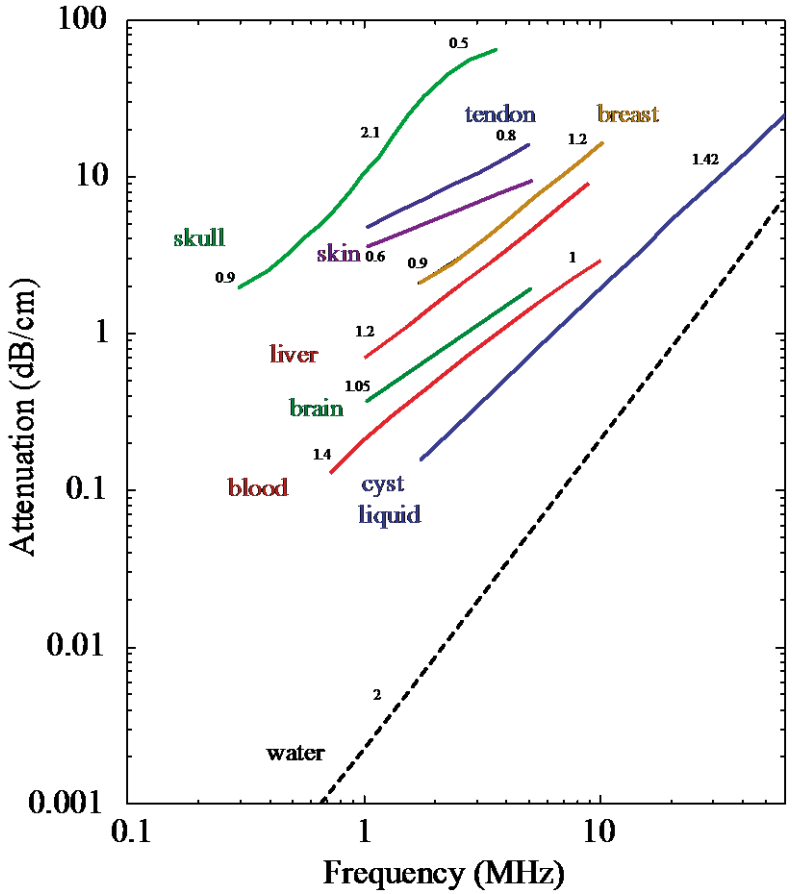
Goal

- Prediction
  - Acoustic waves and related phenomena for Medical Applications
- Biological Media
  - heterogeneous/ complex: dispersion, attenuation ≈ power law
- High Intensity
  - Nonlinear
- Complex Geometry
  - No paraxial
  - No one-way propagation
- Formulation
  - Avoid convolutions
  - All wave phenomena
  - Multi-physics (streams, temperature, optoacoustics, …)

Introduction

- Microscopic structure
  - Heterogeneous (fluid-solid)
  - Complex dynamical response
  - Quasi-periodic…
- Model
  - Micro-modeling
    - λ << structure
  - Homogenization
    - λ >> structure
- Soft-Tissue
  - Attenuation / dispersion

$$\alpha(f) = \alpha_0 f^{\gamma}$$

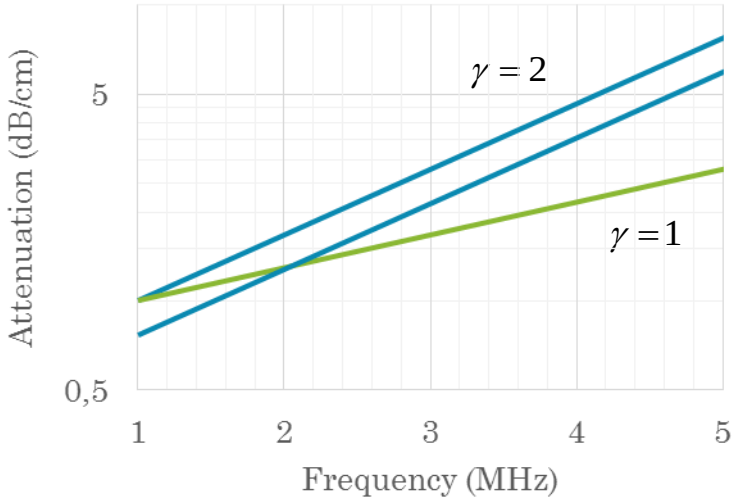


C.R. Hill et. all. Physical Principles of Medical Ultrasonics. John Wiley & Sons, 2004

Why so important?

- Nonlinear regime: always wide spectra
  - Harmonic generation
  - Sub-harmonics, self-demodulation
  - D.C. effects: ARF, streaming
- Soft-tissue frequency power law attenuation $\alpha(f) = \alpha_0 f^{\gamma}$
  - 1. (High) Attenuation
    - Progressive harmonic attenuation
  - 2. (Weak) Dispersion
    - Nonlinear efficiency
    - Phase matching

$$\mathbf{k}(2\omega) \neq 2\mathbf{k}(\omega)$$



Model TD power law

$$\alpha(f) = \alpha_0 f^{\gamma}$$

- Classical operators
  - Damped (f^0)

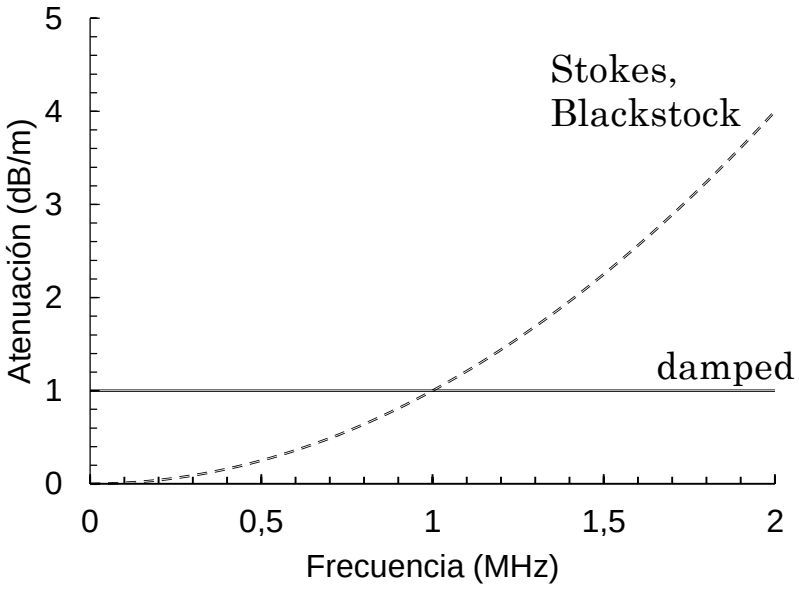
$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \alpha\frac{\partial p}{\partial t} = 0$$

  - Stokes (f^2)

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \eta\frac{\partial}{\partial t}\nabla^2 p = 0$$

  - Blackstock (f^2)

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \eta\frac{\partial^3 p}{\partial t^3} = 0$$

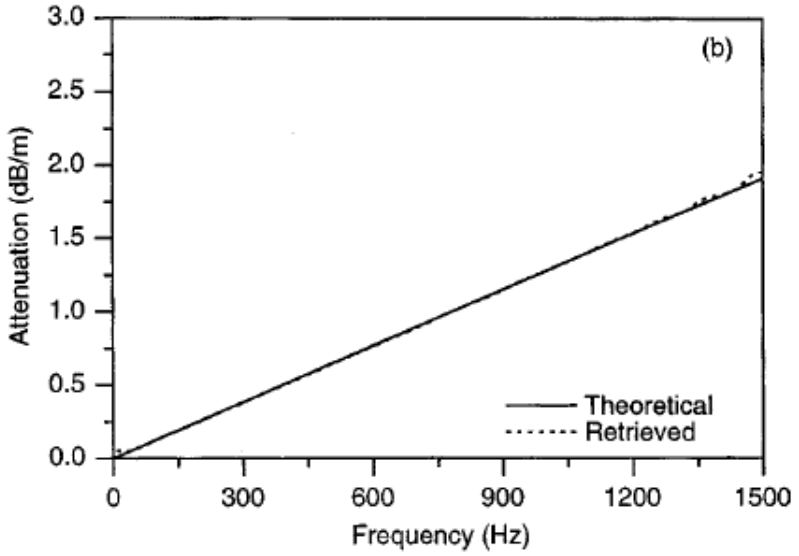


Model TD power law

$$\alpha(f) = \alpha_0 f^{\gamma}$$

- Losses operators
  - Convolutive

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - L_\gamma * p = 0$$



G. V. Norton, *Including dispersion and attenuation directly in the time domain for wave propagation in isotropic media*. J. Acoust. Soc. Am. **113** (6) (2003)

Model TD power law

$$\alpha(f) = \alpha_0 f^{\gamma}$$

- Fractional derivatives

	Damped	Blackstock	Szabo	Stokes	Chen and Holm	Caputo/ Wismer
Π	$\frac{\partial}{\partial t}$	$\frac{\partial^3}{\partial t^3}$	$\frac{\partial^{y+1}}{\partial t^{y+1}}$	$\frac{\partial}{\partial t}\nabla^2$	$\frac{\partial}{\partial t}(-\nabla^2)^{y/2}$	$\frac{\partial^{y-1}}{\partial t^{y-1}}\nabla^2$
$\alpha \propto$	ω^0	ω^2	ω^y	ω^2	ω^y	ω^y

Bradley E. Treeby and B. T. Cox, *Modeling power law absorption and dispersion for acoustic propagation using the fractional Laplacian,* J. Acoust. Soc. Am. **127** 5, May 2010

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} + \tau^{\gamma-1}\frac{\partial^{\gamma-1}}{\partial t^{\gamma-1}}\nabla^2 p = 0$$

J. F. Kelly and R. J. McGough. *Fractal ladder models and power law wave equations* J. Acoust. Soc. Am., Vol. 126, No. 4, October 2009

Model TD power law

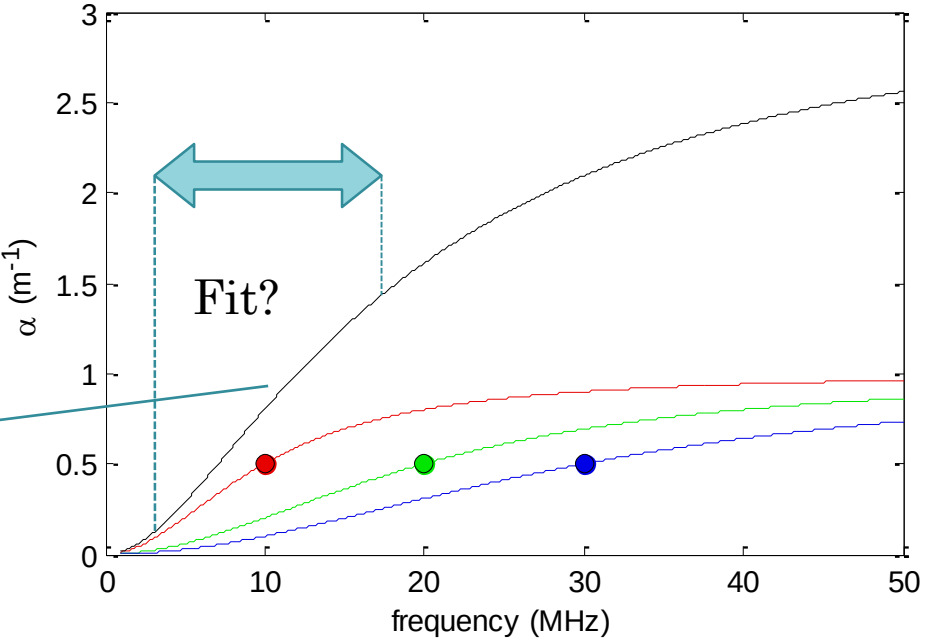
$$\alpha(f) = \alpha_0 f^{\gamma}$$

- Relaxation

$$\alpha(\omega) = \frac{\omega^2}{\rho_0 c_0^3}\left(\zeta + \frac{4}{3}\eta\right) + \sum_{n=1}^{N} \frac{\eta_n}{2c_0\tau_n} \frac{\omega^2\tau_n^2}{1+\omega^2\tau_n^2}.$$

$$\alpha = \alpha_0 f^{\gamma}$$

$$1 < \gamma < 2$$



f_1=10 MHz, f_2=20 MHz, f_3=30 MHz

Model: Nonlinear and relaxation

- Conservative laws of *mass*, *momentum* and *state*

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \eta\nabla^2\mathbf{v} + \left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\mathbf{v})$$

Viscosity $\alpha(f) = \alpha_0 f^2$

$$p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0}\frac{B}{2A}\rho'^2 - \int_{-\infty}^{t} G(t-t')\rho' dt$$

Relaxation $\alpha(f) = L(f)$

Model: Nonlinear and relaxation

- Non-convolutive formulation
  - Multiple exponential relaxation

$$\int_{-\infty}^{t} G(t-t')\rho' dt = \sum_{n=1}^{N} G_n * \rho' \qquad G_n(t) = \frac{\eta_n c_0^2}{\tau_n} e^{\frac{-t}{\tau_n}} H(t)$$

  - State variable $S_n = G_n * \rho'$
  - Evolution

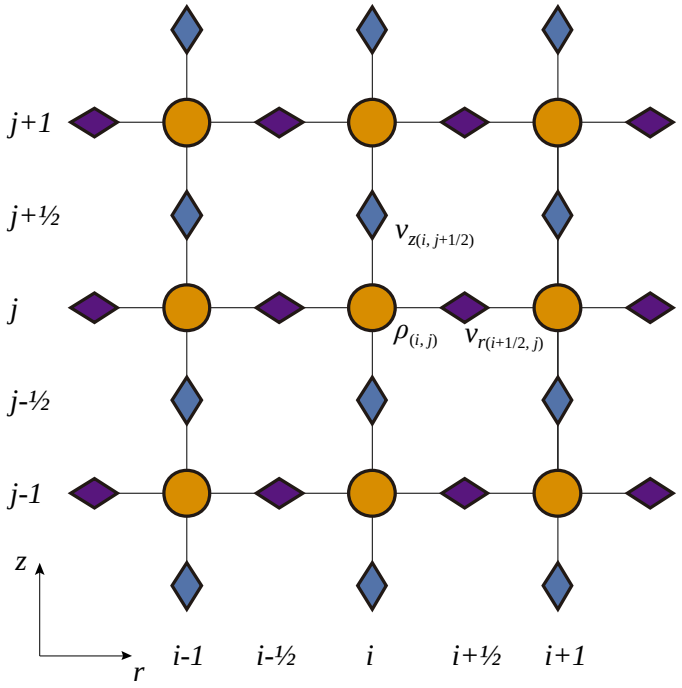
$$\frac{\partial S_n}{\partial t} = \left(-\frac{1}{\tau_n} \frac{\eta_n c_0^2}{\tau_n} e^{-\frac{t}{\tau_n}} H(t) + \frac{\eta_n c_0^2}{\tau_n} \right) * \rho'$$

  - Finally…

$$\boxed{\frac{\partial S_n}{\partial t} = -\frac{1}{\tau_n} S_n + \frac{\eta_n c_0^2}{\tau_n} \rho'} \qquad \boxed{p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0} \frac{B}{2A} \rho'^2 - \sum_{n=1}^{N} S_n}$$

Computational Method FDTD

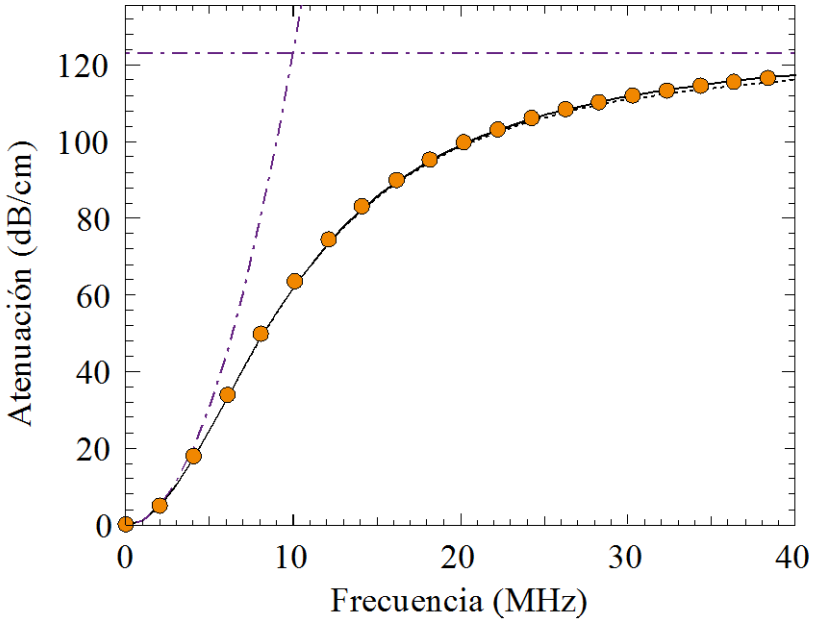
- Features
  - Time domain
  - Nonlinear
  - Attenuation / dispersion
  - Arbitrary Boundary Conditions
  - Arbitrary Initial Conditions
  - No One-way
  - No paraxial
  - Only one auxiliary per relax
  - PML (attenuation 54dB)
  - Nonlinear stability: fourth order artificial viscosity (f^4)

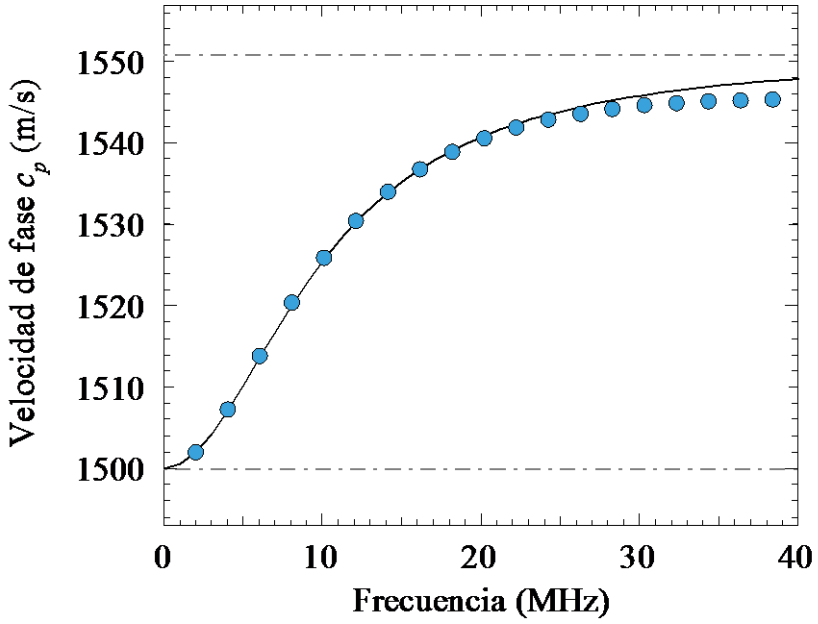


Nonlinear Acoustics time domain method including Frequency Power Law Attenuation for Soft Tissue Modeling
N. Jiménez, J. Redondo, F. Camarena, V. Sánchez-Morcillo, Y. Hou and E. E. Konofagou
Submitted to JCA (2014)

Results: simple process

- Attenuation and dispersion





$$\alpha = \frac{\eta\omega^2\tau^2}{2c_0\tau\left(1+\omega^2\tau^2\right)}$$

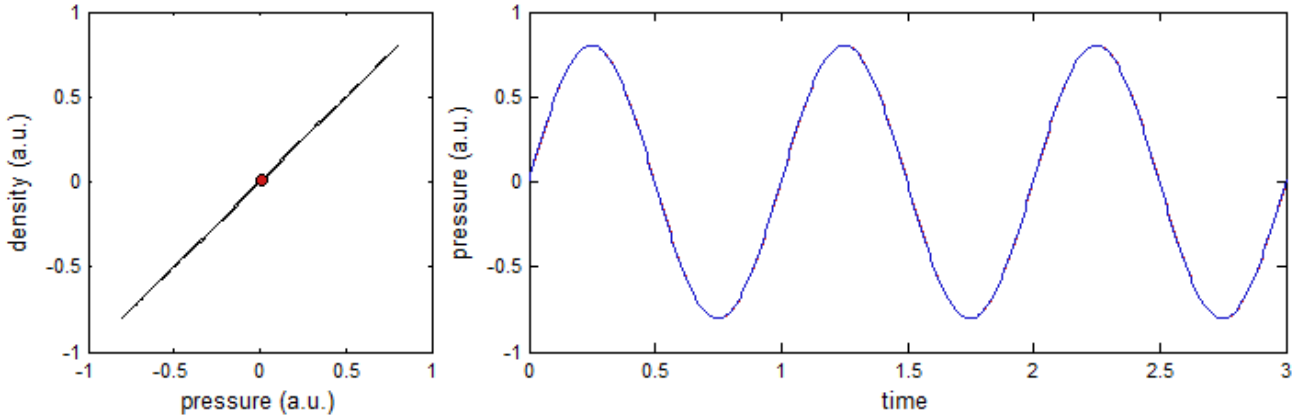
$$c = c_0\left(1+\frac{\eta\omega^2\tau^2}{2\left(1+\omega^2\tau^2\right)}\right)$$

Phase spaces

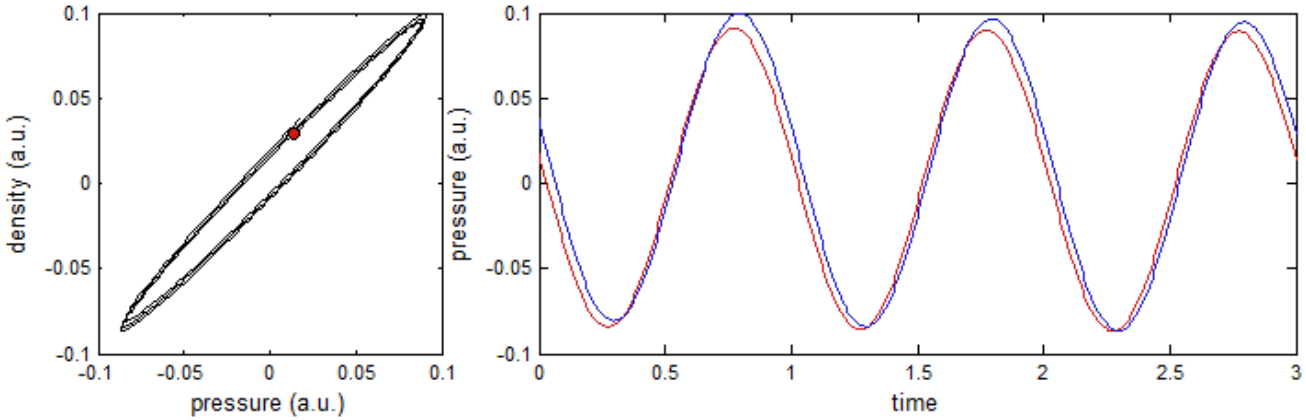
- Linear

$$p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0}\frac{B}{2A}\rho'^2 - \int_{-\infty}^{t} G(t-t')\rho' dt$$

Lossless



Relaxation

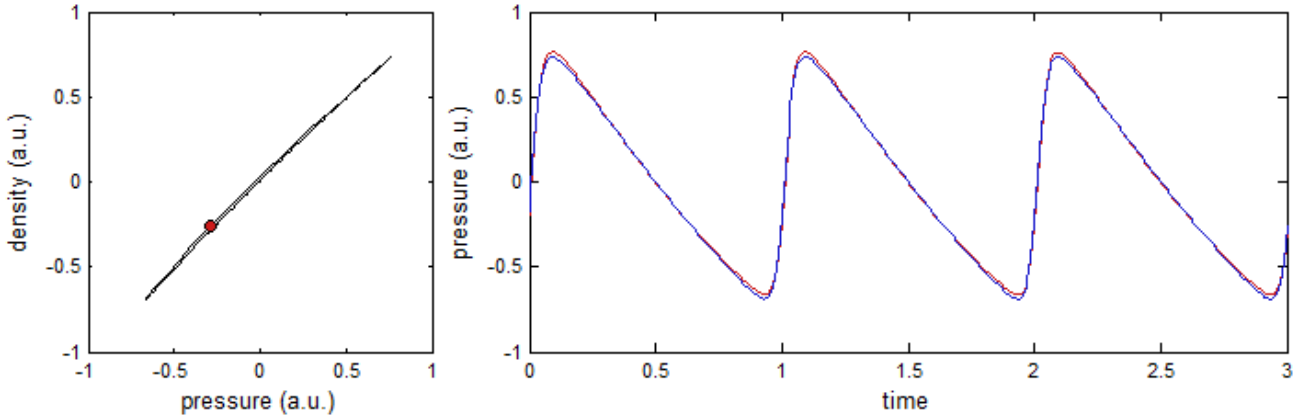


Phase spaces

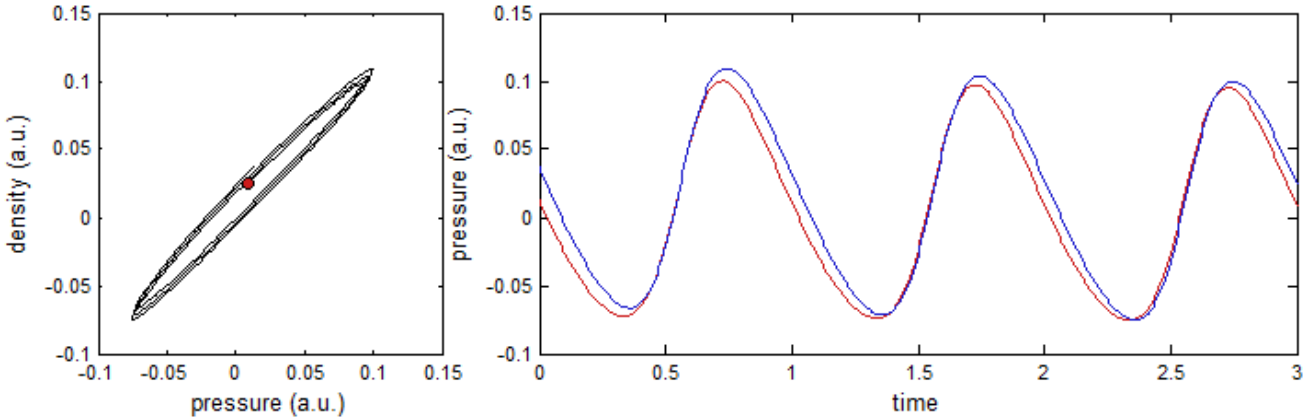
- Nonlinear

$$p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0}\frac{B}{2A}\rho'^2 - \int_{-\infty}^{t} G(t-t')\rho' dt$$

Thermoviscous



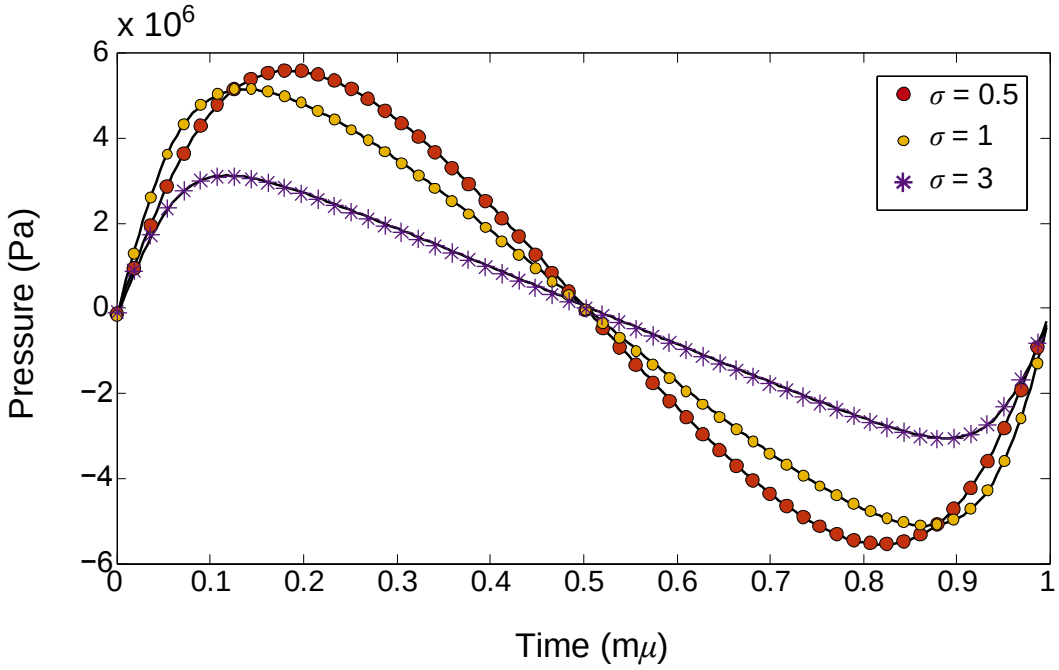
Relaxation



Nonlinear propagation

- Mendousse solution

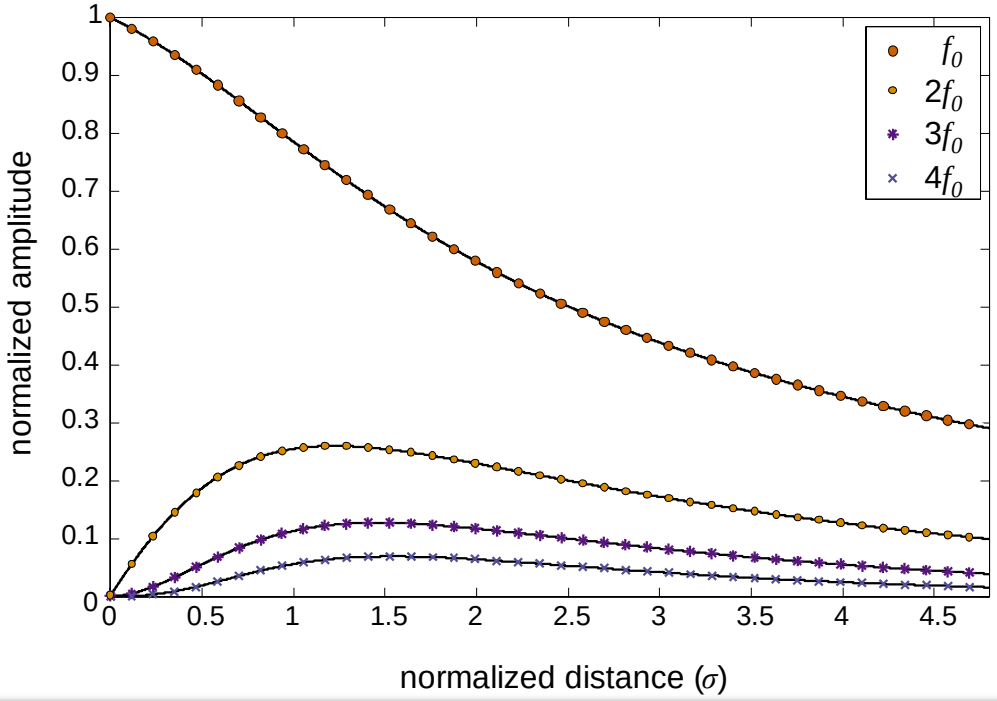
$$\frac{p}{p_0} = \frac{\frac{4}{\Gamma}\sum_{n=1}^{\infty}(-1)^{n+1} I_n\left(\frac{\Gamma}{2}\right) n e^{-n^2\sigma/\Gamma} \sin(n\omega t')}{I_0\left(\frac{\Gamma}{2}\right) + 2\sum_{n=1}^{\infty}(-1)^{n} I_n\left(\frac{\Gamma}{2}\right) n e^{-n^2\sigma/\Gamma} \cos(n\omega t')}$$



Nonlinear propagation

- Mendousse solution

$$\frac{p}{p_0} = \frac{\frac{4}{\Gamma}\sum_{n=1}^{\infty}(-1)^{n+1} I_n\left(\frac{\Gamma}{2}\right) n e^{-n^2\sigma/\Gamma} \sin(n\omega t')}{I_0\left(\frac{\Gamma}{2}\right) + 2\sum_{n=1}^{\infty}(-1)^{n} I_n\left(\frac{\Gamma}{2}\right) n e^{-n^2\sigma/\Gamma} \cos(n\omega t')}$$



Biological tissue modeling

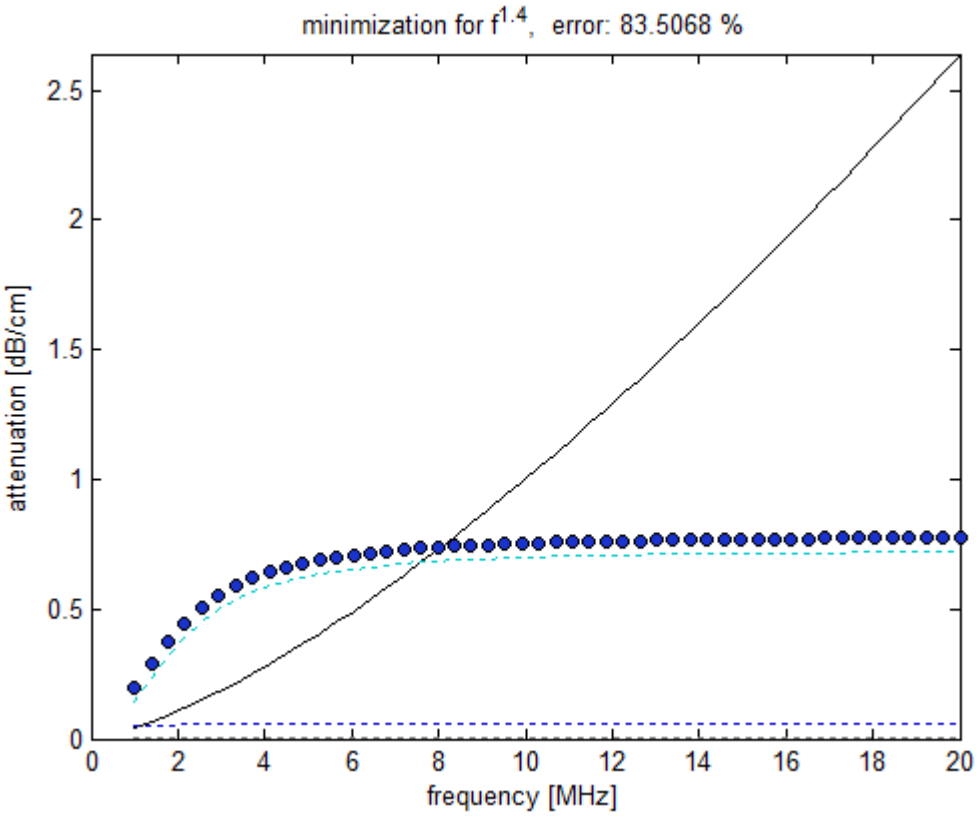
- Optimization

$$\alpha_{goal} = \alpha_0 f^{\gamma}$$

$$\alpha = \frac{\omega^2}{\rho_0 c_0^3}\left(\zeta + \frac{4}{3}\eta\right) + \sum_{n=1}^{N} \frac{\eta_n}{2c_0\tau_n} \frac{\omega^2\tau_n^2}{1+\omega^2\tau_n^2}$$

- Cost function (to minimize)

$$\varepsilon = \int_{f_1}^{f_2} \left(\frac{\alpha - \alpha_{goal}}{\alpha_{goal}} \right) df$$



Biological tissue modeling

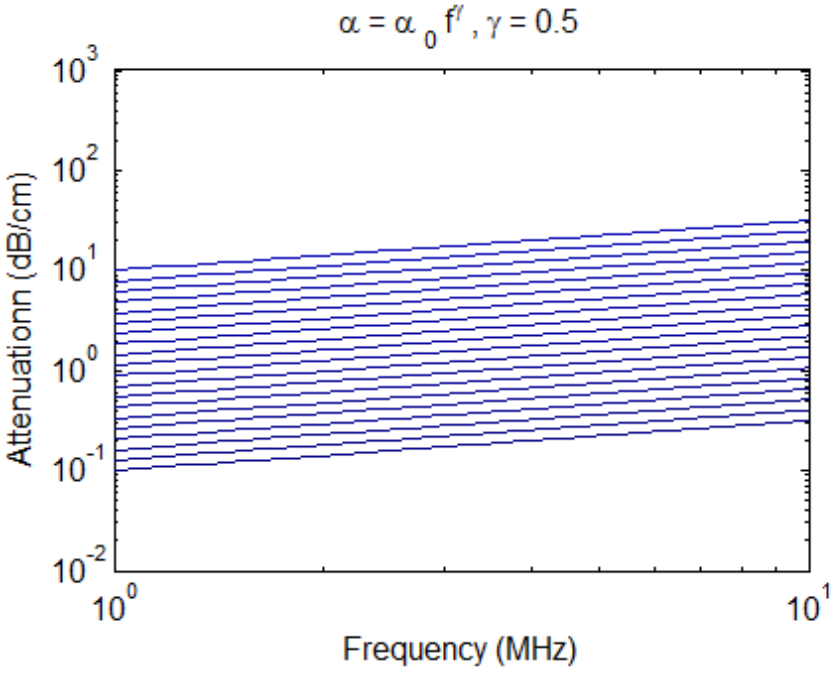
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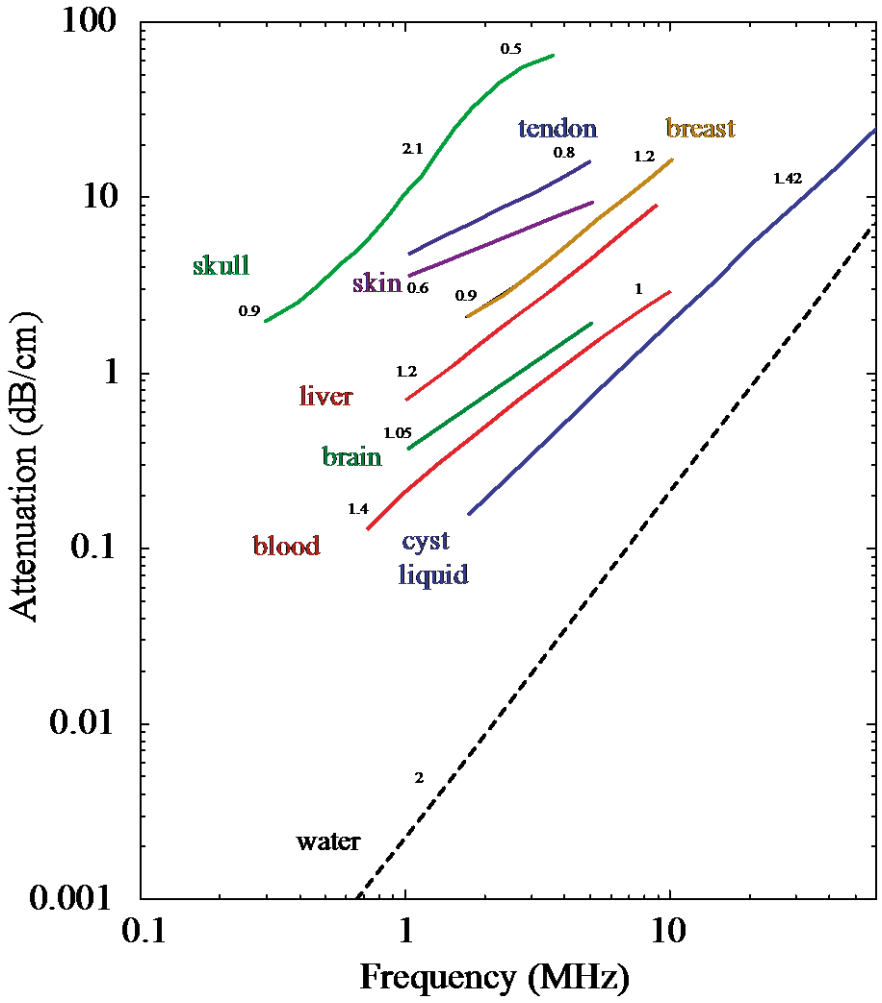
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Biological tissue modeling

- Optimization

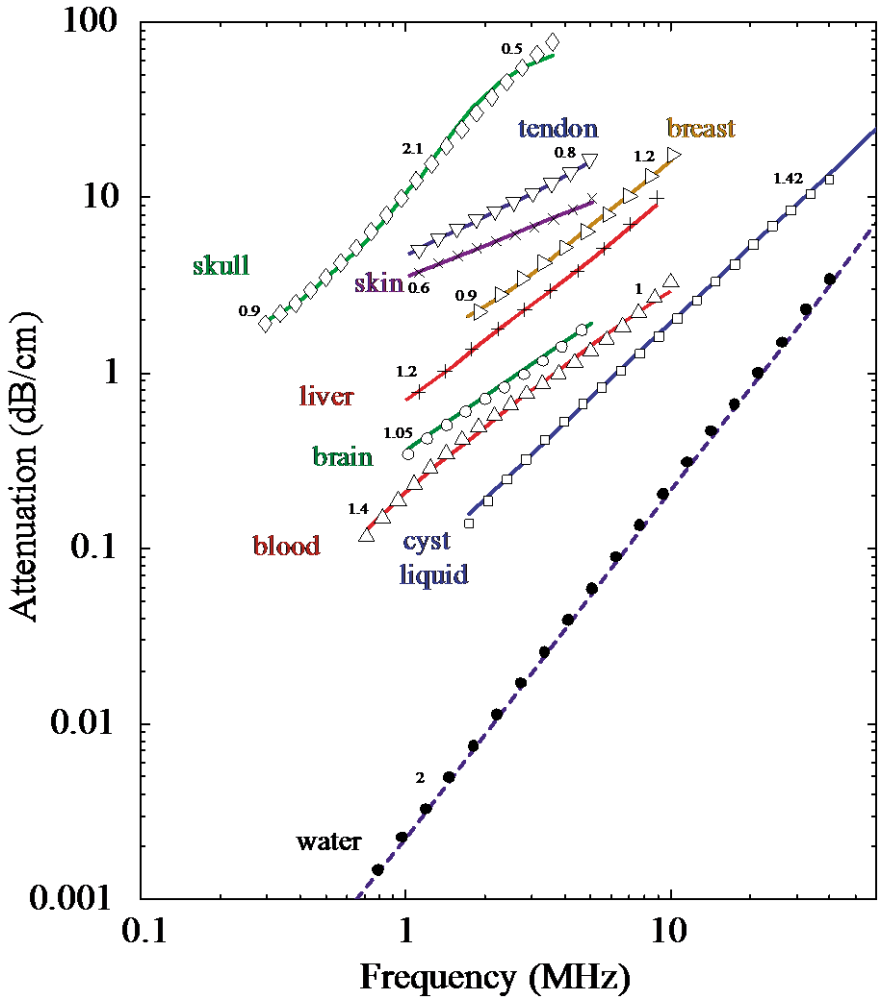


Biological tissue modeling

- Optimization

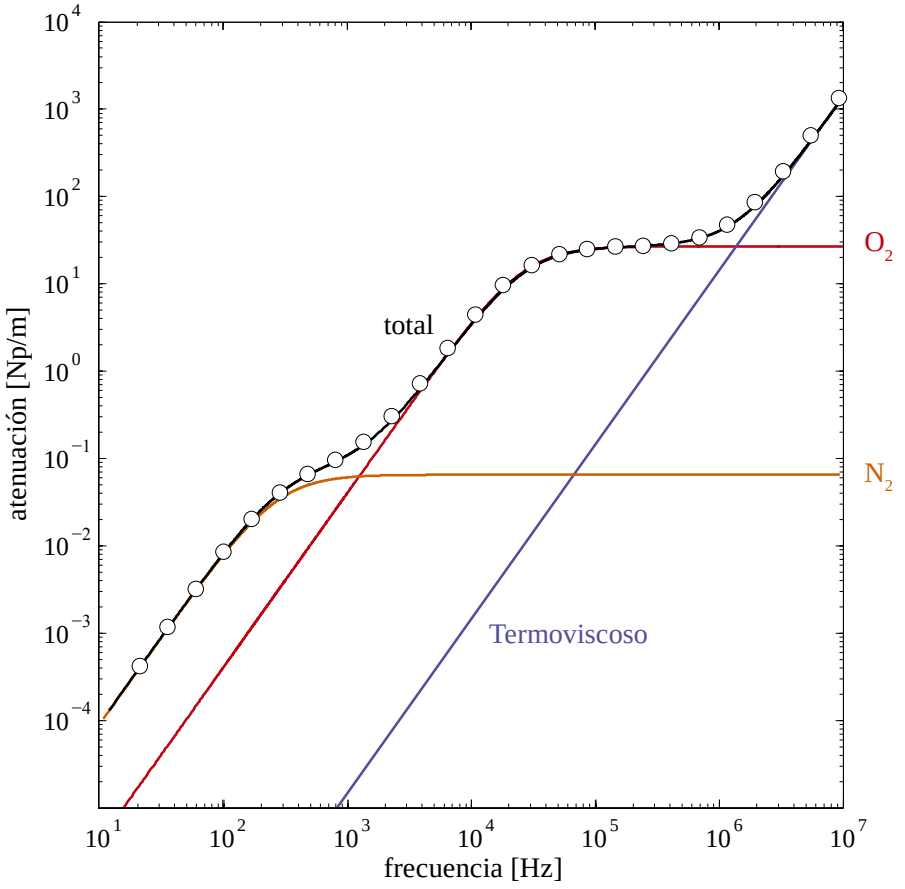
Relative error (%)

Tissue	N=1	N=2	N=4
Skin: $f^{0,6}$	6,67	0,17	0,12
Liver: $f^{1,2}$	7,62	0,52	0,16
Blood: $f^{1,4}$, f^{1}	8,34	0,35	0,31
Breast: $f^{0,9}$, $f^{1,2}$	5,20	0,22	0,20
Bone: $f^{0,9}$, $f^{2,1}$, $f^{0,5}$	10,6	10,54	5,19



Air relaxation modelling

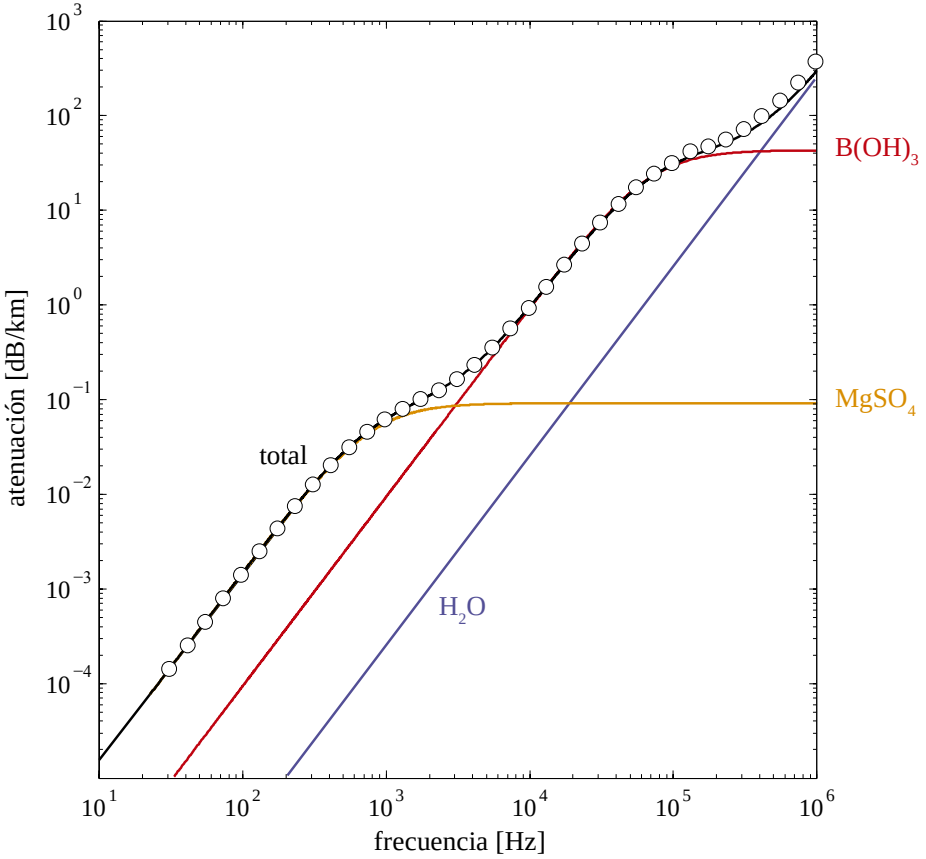
- Relaxation
  - Humidity 34%,
  - Temperature = 20°C
  - P = 1 atm
  - Nitrogen [N_2]
    - f_n = 300 Hz
  - Oxygen [O_2]
    - f_n = 26,5 kHz
- Viscosity
  - $b = 4.86 \cdot 10^{-5}$ Pa·s



P. V. Yuldashev. "Nonlinear Spherically Divergent Shock Waves Propagating in a Relaxing Medium". Acoustical Physics. 54(1), 32-41, 2008

Sea-water relaxation modeling

- Relaxation
  - Salinity = 35%
  - pH = 8.0
  - Temp = 4ºC
  - P = 1 atm
  - Boric Acid [$B(OH)_3$]
    - f_n = 0,78 - 1.7 kHz
  - Magnesium Sulfate [$MgSO_4$]
    - f_n = 69 - 240 kHz
- Viscosity
  - ~ Distilled water



F. H. Fisher. *Sound absorption in sea water.* J. Acoust. Soc. Am. 62(3). 558-564 (1977)

Conclusions

- Generalized Non-convolutive formulation
  - Complete system, (ρ, p, $\mathbf{v}$) time domain
  - Nonlinear
  - Relaxation
  - Non-convolutive operators
- Numerical solution by FDTD → *PSTD*, *k-space*…
- Optimization
  - Attenuation / dispersion fit to experimental data
- Validation
  - Air / Seawater
  - Soft-Tissues

Work lines

- Beams in Soft tissue
  - Nonlinearities vs. Non-thermoviscous attenuations
  - Beam changes: *Gain / Beamwidth / Focal position*
  - Experimental validations

- HIFU monitoring by HMI elastography
  - Determine Nonlinear limits of **Acoustic Radiation Force**
  - Temperature (Bio-heat Penne's equation)
    - c_0 (T) → spatial changes in Acoustic Radiation Force fields

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Thanks for your attention

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