On the Nonlinear Effects in Focused Ultrasound Beams with Frequency Power Law Attenuation

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Abstract

When finite amplitude ultrasound propagation is considered, changes in spatial features of focused ultrasound beams can be observed. These nonlinear effects typically appears in thermoviscous fluids as **nonlinear focal displacements**, **beam-width variations** *or* **gain changes**. *However, in soft-tissue media, the frequency dependence of the attenuation doesn't obeys a squared law. In this way, these complex media response leads to weak dispersion that prevents the cumulative processes of energy transfer to higher harmonics. In this work we explore the influence of different frequency power law attenuation responses and its influence on the* **self-defocusing** *effects in both plane wave propagation and focused ultrasound beams. Thus, we numerically explore the spatial field distributions produced by focused ultrasound devices radiating trough different soft-tissue media.*

Model

Model for relaxation media, **mass conservation** *(continuity eq.)*

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

Momentum conservation *(Navier-Stokes equation)*

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \eta\nabla^2\mathbf{v} + \left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\mathbf{v})$$

Thermodynamic **state** *equation*

$$p = c_\infty^2\rho' + \frac{c_0^2}{\rho_0}\frac{B}{2A}\rho'^2 - \sum_{n=1}^{N} S_n$$

Relaxation *state for each relaxation process (non-convolutive formulation)*

$$\frac{\partial S_n}{\partial t} = -\frac{1}{\tau_n}S_n + \frac{\eta_n c_0^2}{\tau_n}\rho'$$

Where $\mathbf{v}$ *is the particle velocity,* ρ *is the perturbation density over the stationary density* ρ_0 *,* ζ *and* η *are the bulk and shear viscosities,* S_n *is the relaxation variable,* τ_n *and* η_n *the relaxation characteristic time and modulus.*

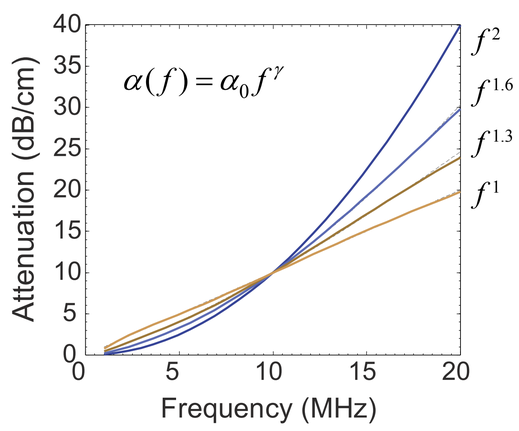
Power Law Attenuation

Solver

The constitutive equations are solved by finite-differences in time-domain method in cylindrical axisymmetric coordinate system. PML, are placed in the limits of integration area.

Optimization

Frequency power laws are obtained by a sum of 2 relaxation processes, where the values for relaxation coefficients (relaxation times and modulus) are obtained trough minimization techniques.

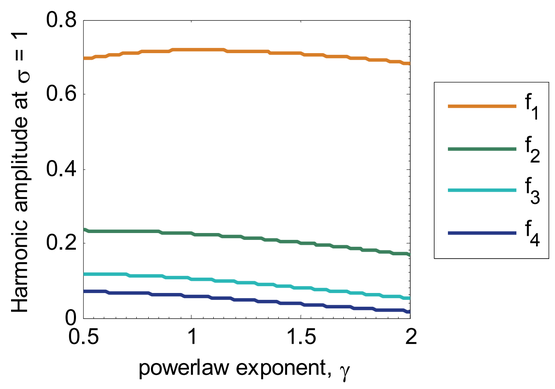


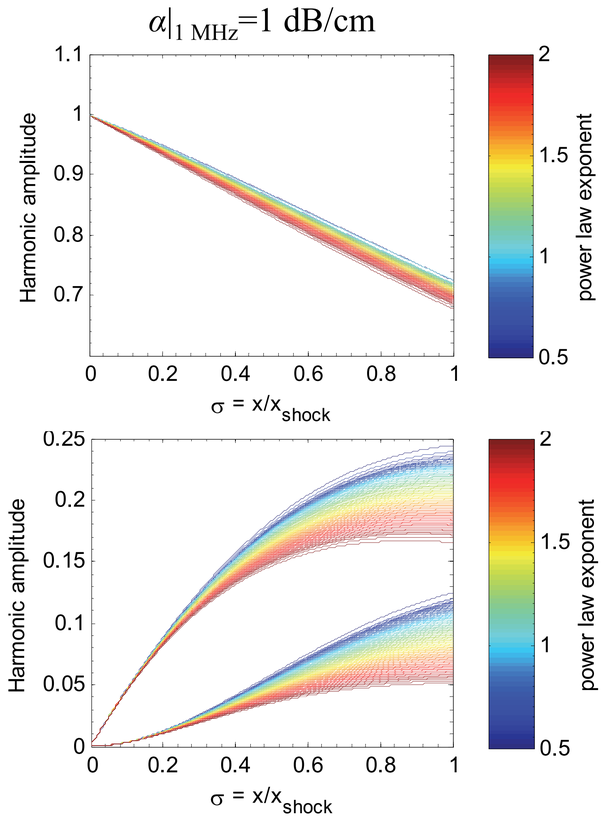
Results

1. *Harmonic generation (1D)*

Progressive harmonic generation is power-law dependent, final wave spectrum depends on two main factors:

- **Phase matching**> *The inclusion of different power law leads to the inclusion of weak dispersion: nonlinear efficiency is decreased by dispersion.*
- **Attenuation magnitude**> *Damping of each harmonic is modified by selected frequency power-law.*



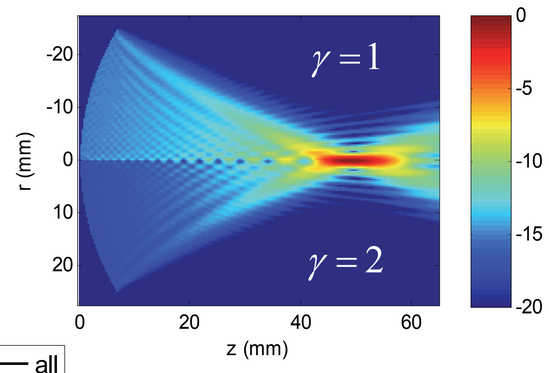


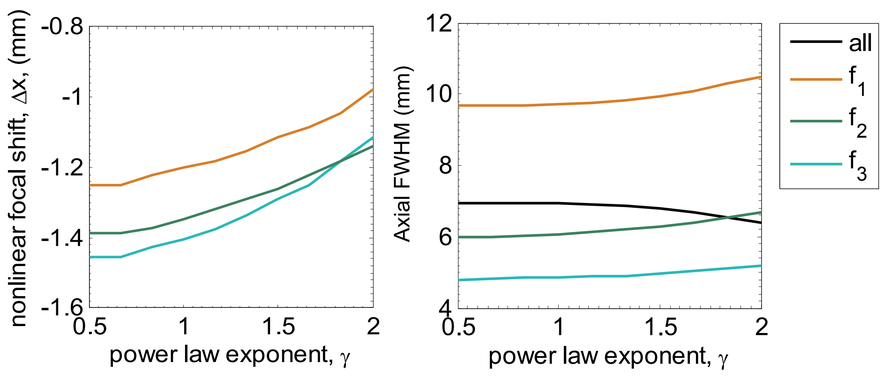
2. *Focused ultrasound* — *G=30, A=50 mm, F=50 mm*

Spatial effects

Nonlinear Focal Displacements

Due to different phase and magnitude of the progressive generation and attenuation of harmonics, focus position and spatial distribution for each harmonic is also modified. Thus, self-defocusing dependence on power law shows the different location of focus for each harmonic.





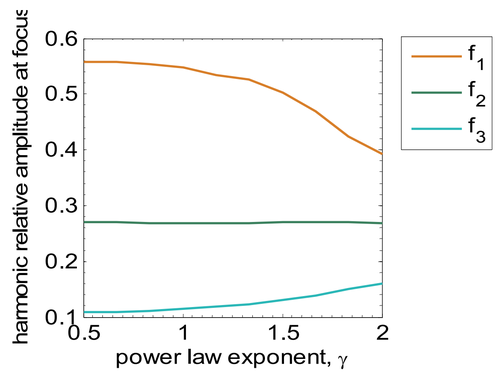
Beam-width

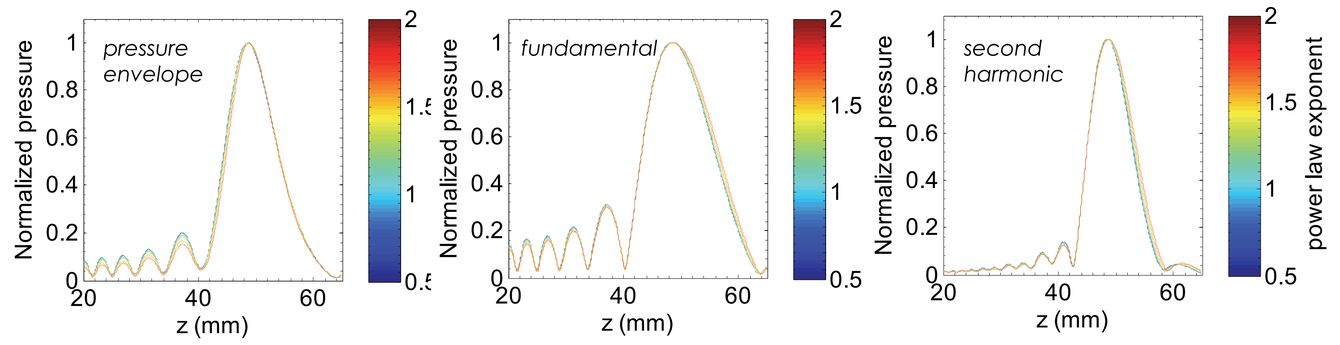
On the other hand, power law dependence of the beamwidth or FWHM is studied. Here, we show a nonlinear dependence of the effective sonificated volume, where the focusing is substantially changed.

Amplitude effects

Once again, the peak pressure for the fundamental and the first harmonics is modified, and the combined effects of phase matching (dispersion) and absorption modifies the amplitude at focus. Different power law attenuation leads to inclusion of different (weak) dispersion, son harmonic phase is modified. Thus, asymmetry of the waveforms at focus is modified.

The effective gain (P_{max}/P_0) *dependence on frequency power law shows that peak pressure exhibit a nonlinear dependence on the power law exponent.*





Conclusions

(1) *Developed* **model** *and* **solver** *for nonlinear acoustics in relaxing media,* **optimization** *for frequency power law attenuation as a model for* **soft tissues**

(2) *Nonlinear harmonic generation in 1D depends on phase matching (weak dispersion)and attenuation*

(3) *Characterization of nonlinear effects for focused beams for* **Gain**, **FWHM**, *and* **Nonlinear Focal Shifts**