NONLINEAR WAVES IN PHONONIC LATTICES AND SUPERLATTICES

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Overview

- Introduction:
  - Nonlinear Acoustic waves: harmonic generation and distortion
- Lattices (coupled oscillators)
  - FPU lattice. Long wavelength limit
  - Harmonic generation
  - Experiments with magnetic lattices
- Superlattices or 1D crystals
  - Dispersion relation
  - Harmonic nonlinear effects

Harmonic distortion in Nonlinear Acoustics

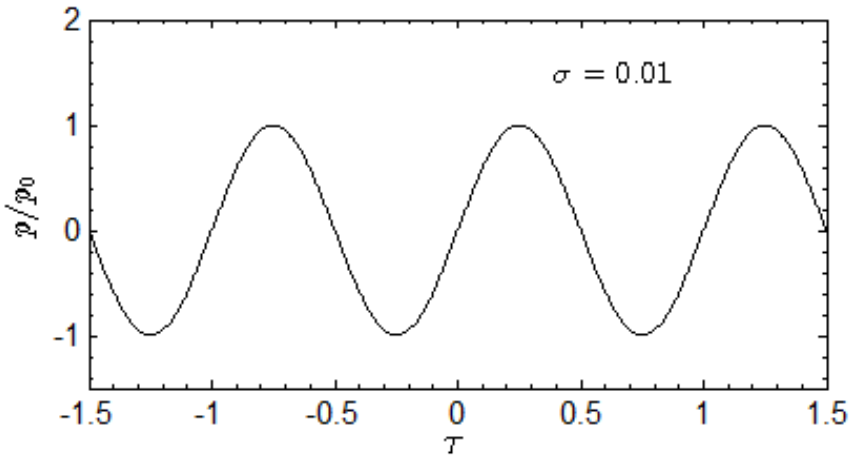
Plane waves in homogeneous medium

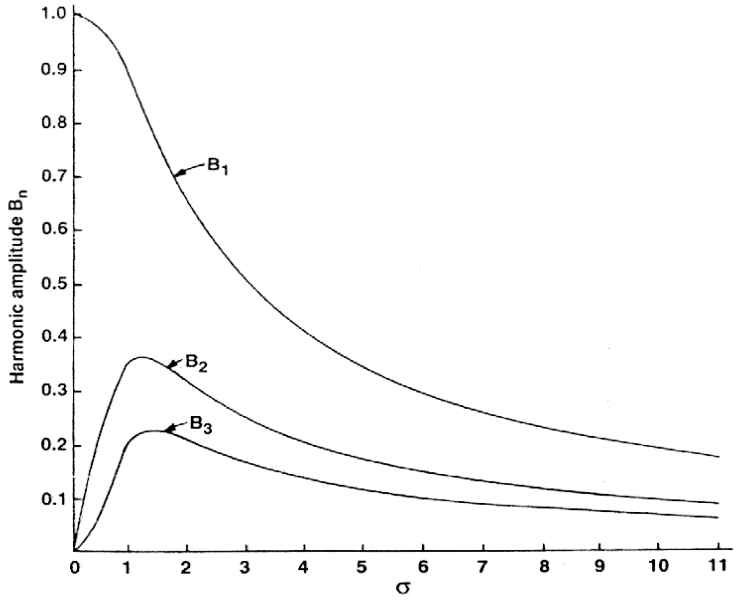
Nonlinear and

$$p = p_0 \sum_{n=1}^{\infty} B_n(s)\sin(n\omega t')$$

$$B_n(x) = \frac{2J_n(ns)}{ns}, \quad s = \frac{x}{x_s}$$

Shocks at σ=1





Lattices

The general form (for nearest neighbors coupling)

$$m\frac{\mathrm{d}^2 u_n(t)}{\mathrm{d}t^2} = V'(u_{n+1} - u_n) - V'(u_n - u_{n-1})$$

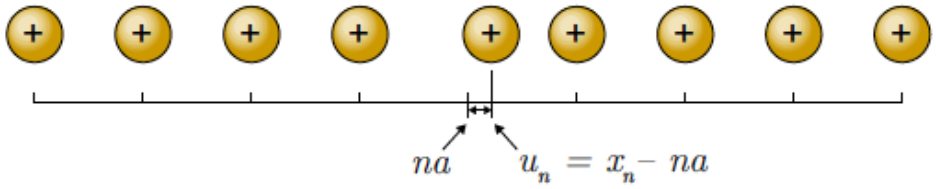
Many possible interaction potentials

$V'(r) = r^{3/2}$ Hertz

$V'(r) = r^{-2}$ Coulomb

$V'(r) = e^{-r} - 1$ Toda

$\vdots$



Most of them approximate to quadratic (FPU) potential

$$V'(r) = kr + ar^2$$

FPU Lattices

Consider the lattice with quadratic nonlinearity (α-FPU)

$$\frac{\mathrm{d}^2 u_n(t)}{\mathrm{d}t^2} = k\left(u_{n+1} - 2u_n + u_{n-1}\right) - a\left(u_{n+1} - 2u_n + u_{n+1}\right)\left(u_{n+1} - u_{n-1}\right)$$

At long wavelengths, the continuum limit applies, and the evolution equation reads

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}$$

The nonlinear acoustics wave (Boussinesq) equation!

$$b = 2 + \frac{B}{A}$$ For fluids

$$b = 3 + \frac{c_{111}}{r_0 c^2}$$ For solids (along particular directions)

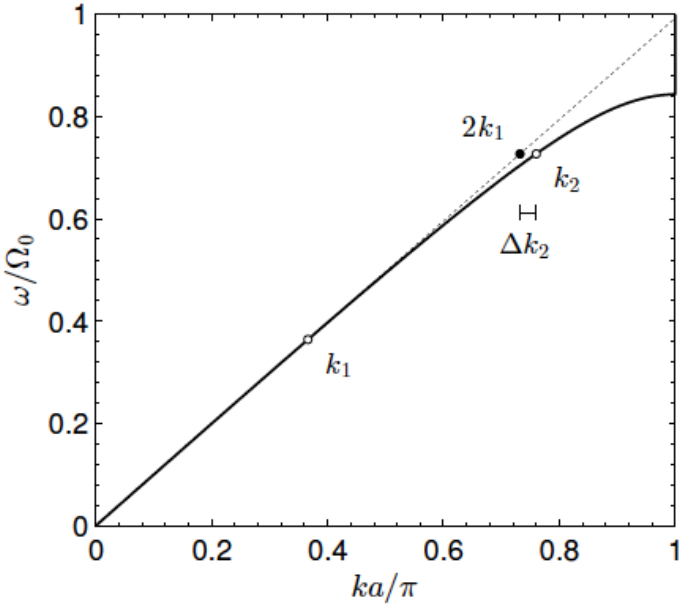
Harmonic generation in Lattices

Main results for FPU lattice *:

- **Nonlinearity:** Higher harmonic generation (ω, 2ω, 3ω,…)
- **Dispersion:** Two regimes
  - Harmonic in band: beatings at the half period given by the coherence length

$$x_c = \frac{p}{|\Delta k|} = \frac{p}{|k(2w) - 2k(w)|}$$

  - Evanescent harmonic: finite amplitude (forced) waves
- **Analytical solutions** by perturbative methods

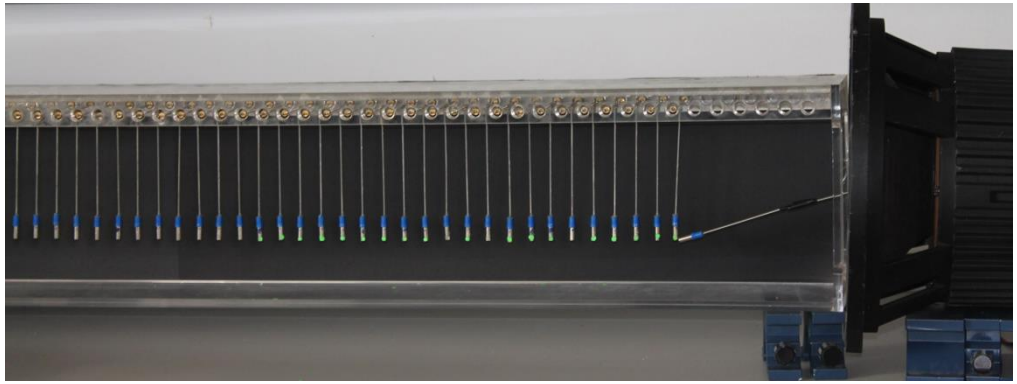


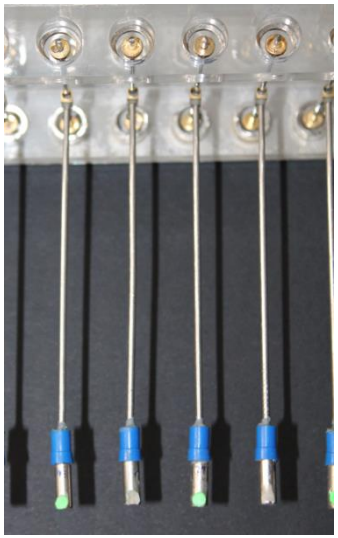
* V. J. Sanchez-Morcillo et al., **Second-harmonic generation for dispersive elastic waves in a discrete granular chain,** Phys. Rev. E **88**, 043203 (2013)

Experiment: Coulomb lattice

Array of 50 magnets, with magnetic dipoles vertically oriented in the same direction: repelling forces

Each magnet is the mass in a pendulum with very low friction





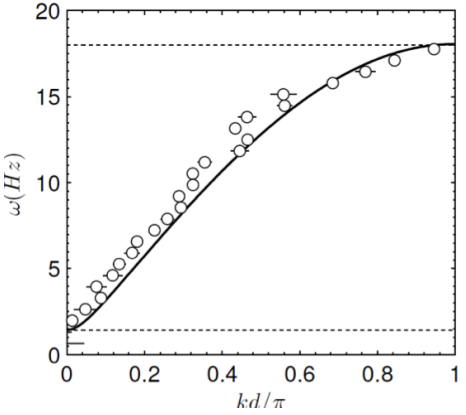


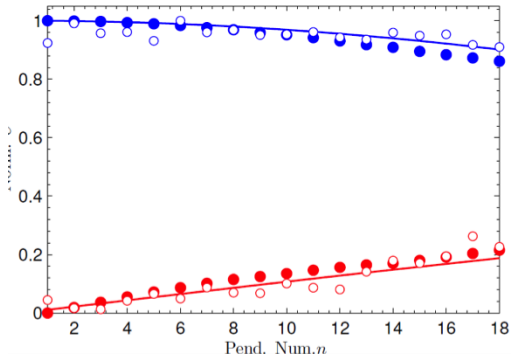
Equation of motion $$\frac{d^2 u_n}{dt^2} + w_0^2 \sin u_n = - \frac{1}{(1 + u_{n+1} - u_n)^2} + \frac{1}{(1 + u_n - u_{n-1})^2}$$

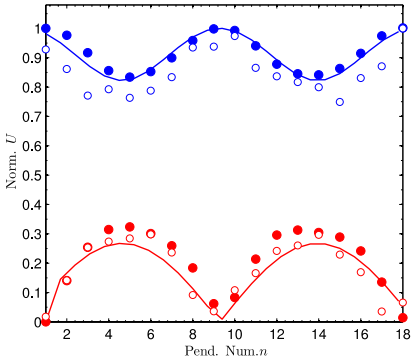
(FPU in low-amplitude limit)

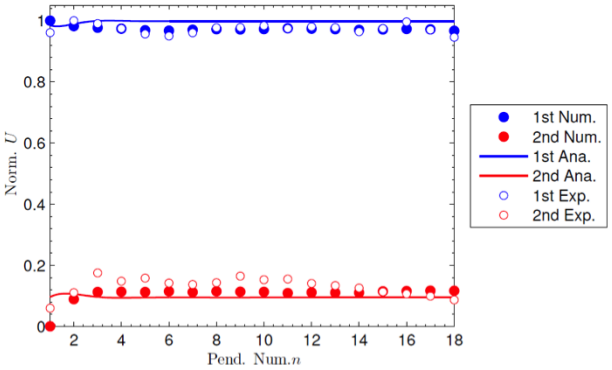
Harmonic generation in a Coulomb lattice

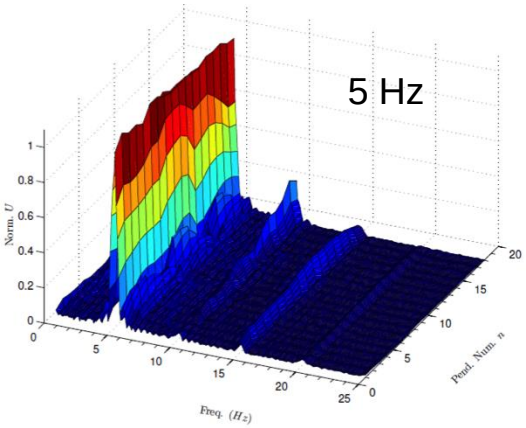
Dispersion relation: $\omega^2 = \omega_0^2 + \sin^2\left(\frac{ka}{2}\right)$

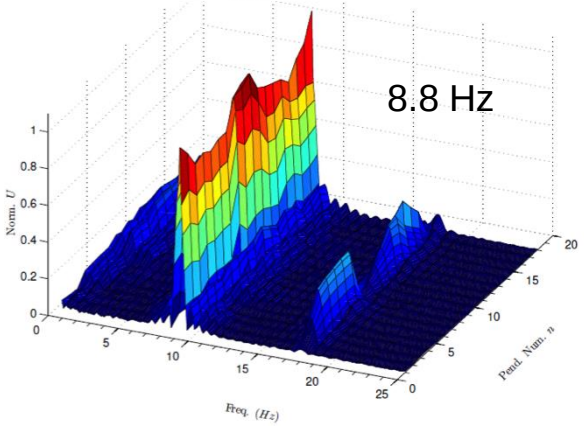


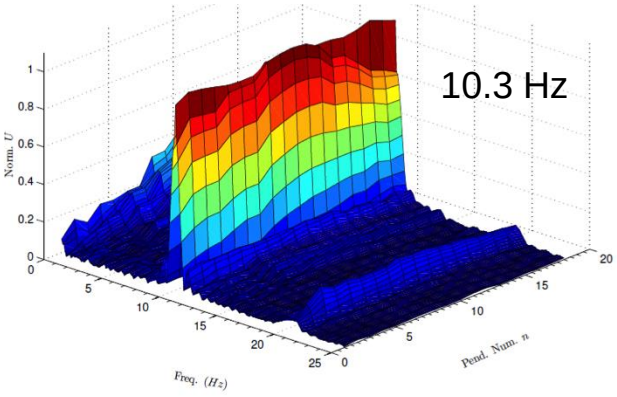




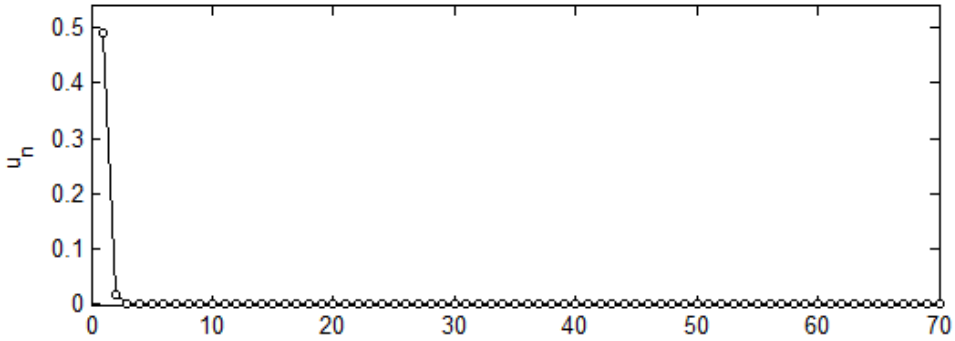








Localized modes: travelling kinks (solitons) and phonons in the lattice

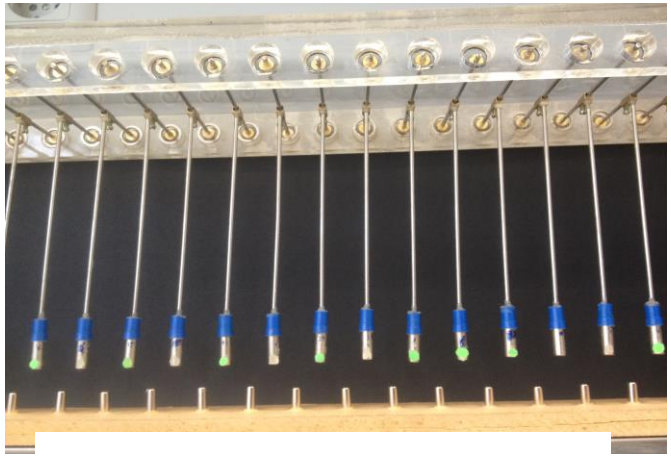


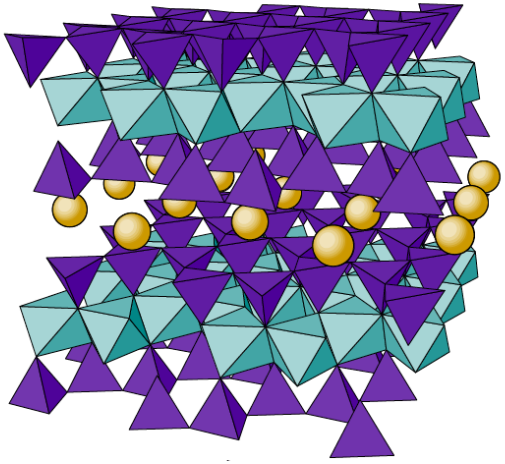
Numerical



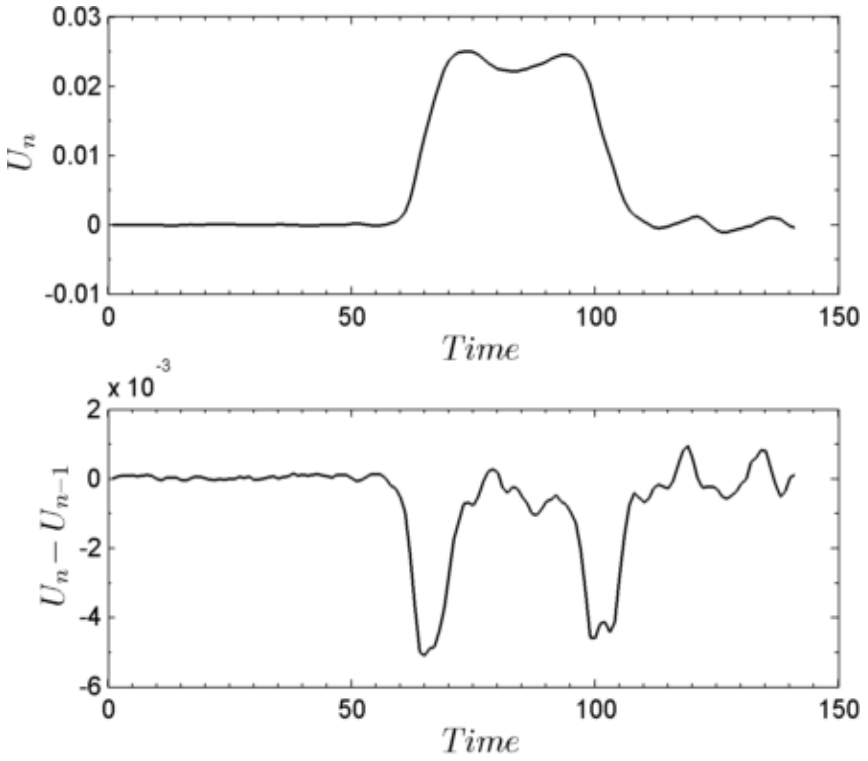
Experiment

Localized traveling waves: the effect of on-site potentials

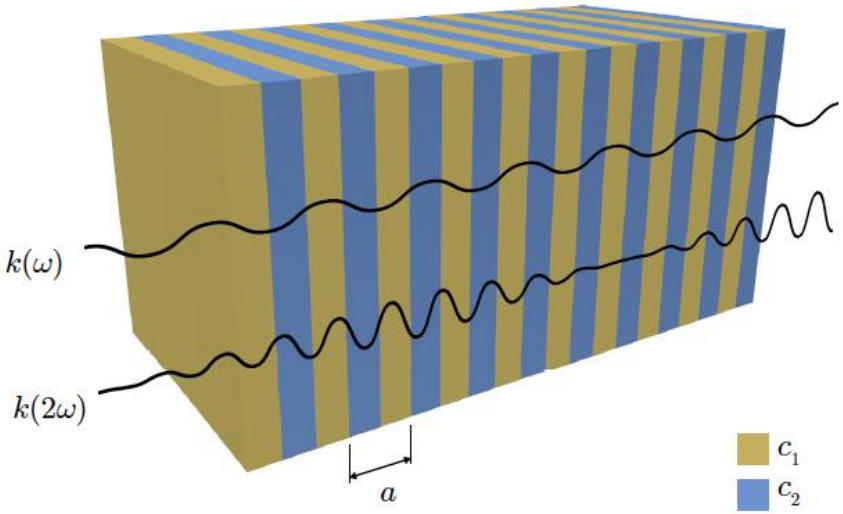




Experiment: displacement and strain



1D crystals (multilayers, superlattices)

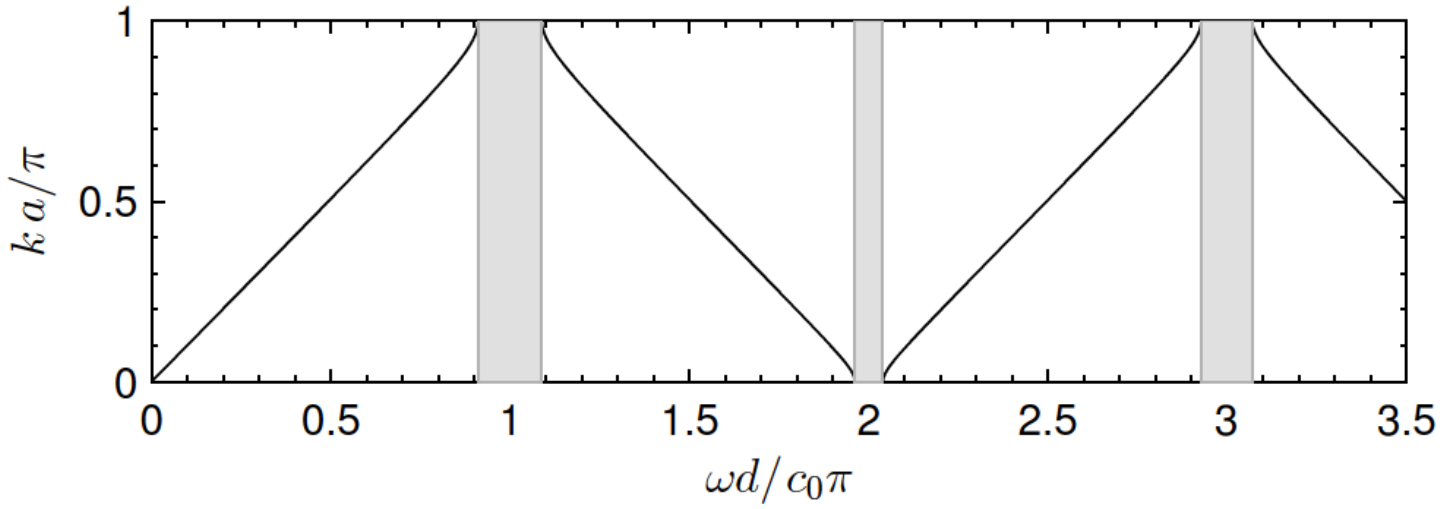


- The layers may differ in density, elasticity, nonlinearity,…
- May be fluid/fluid, fluid/solic or solid/solid
- Used as: phononic mirrors, phonon lasers,…
- Mostly studies in linear regime.

Dispersion and band structure

The dispersion relation (band structure) is **analytical**

$$\cos(kd) = \cos(k_1 d_1)\cos(k_2 d_2) - \frac{1}{2}\left(\frac{k_1}{k_2} - \frac{k_2}{k_1}\right)\sin(k_1 d_1)\sin(k_2 d_2)$$



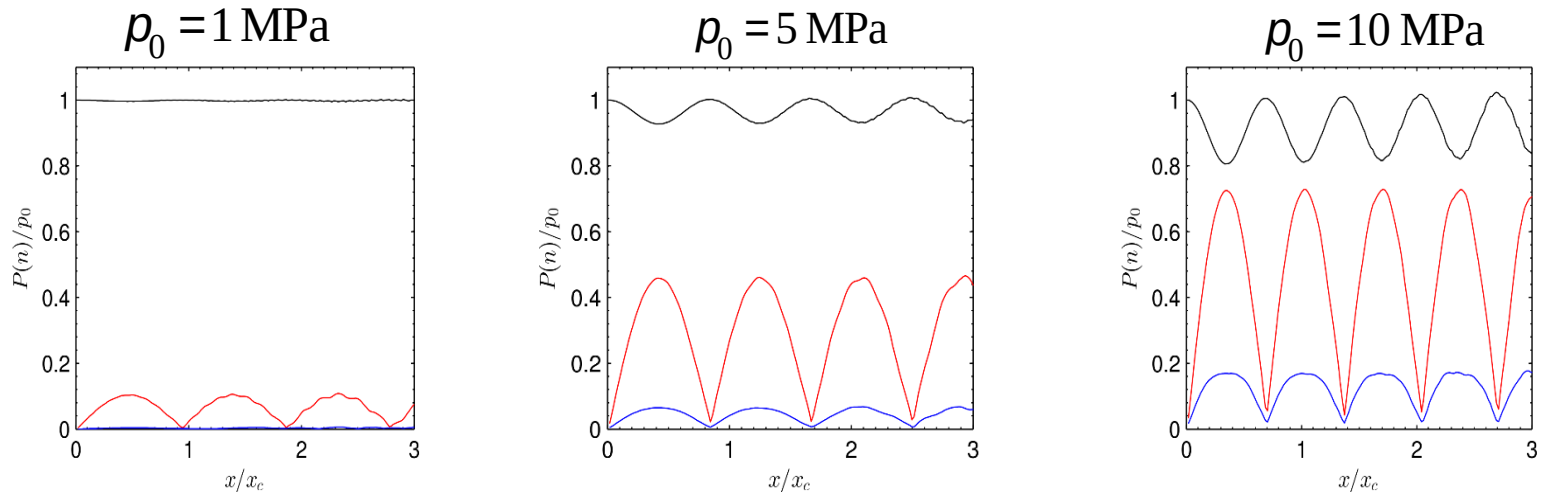
Nonlinear propagation effects

- Single-band (lattice-like) effects:
  - Asynchronous harmonics: Beatings. Nonlinear dispersion
  - Forbidden waves with finite amplitude in the bandgap
- Multi-band effects: Enhancement of particular harmonics
  - All waves in synchrony: nondispersive high frequency waves
  - Enhancement of 2nd harmonic: optic-like case
  - Enhancement of 3rd harmonic: cubic-like nonlinearity

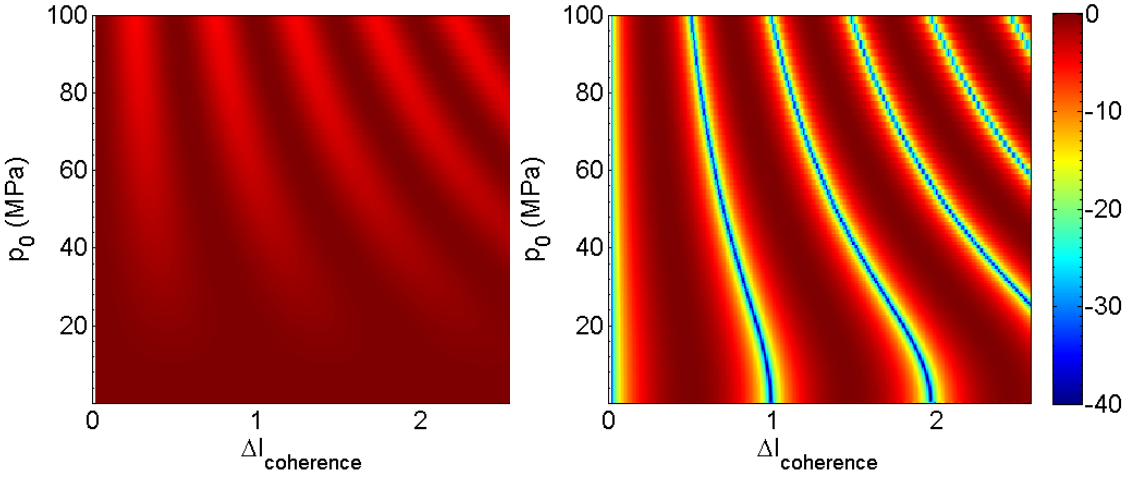
Numerical study by solving constitutive equations for (p, ρ, $\mathbf{v}$) using FDTD method

2nd harmonic: in-band case

- **Dispersion:** beatings at the half period given by the coherence length

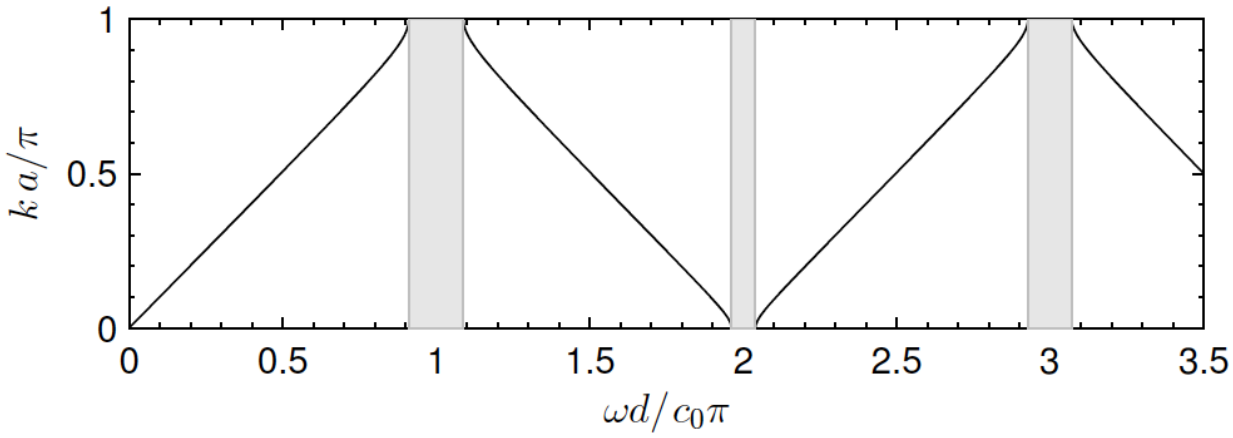


- The beating period depends o the amplitude
- Signature of **nonlinear dispersion**

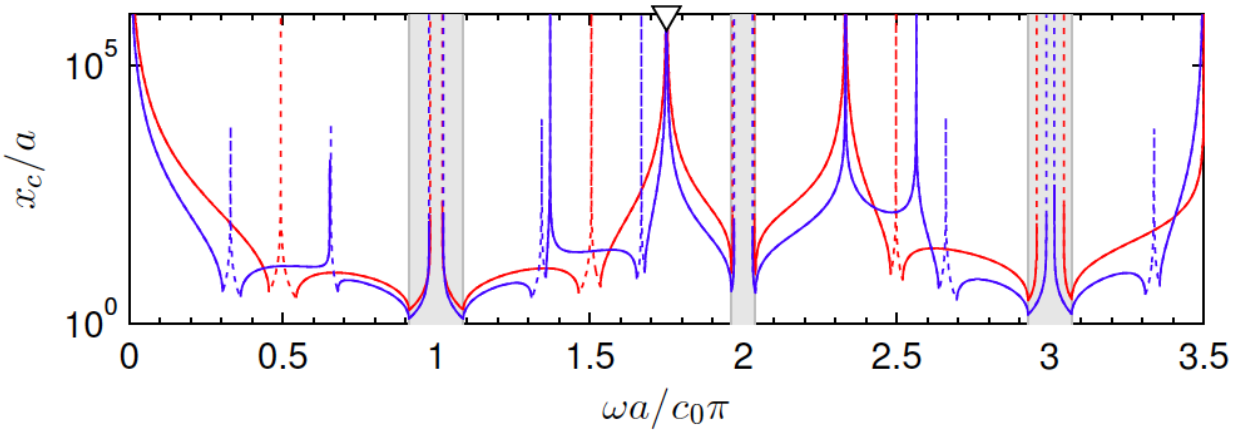


Managing coherence between harmonics

- Dispersion relation may be used to evaluate coherence



Dispersion relation



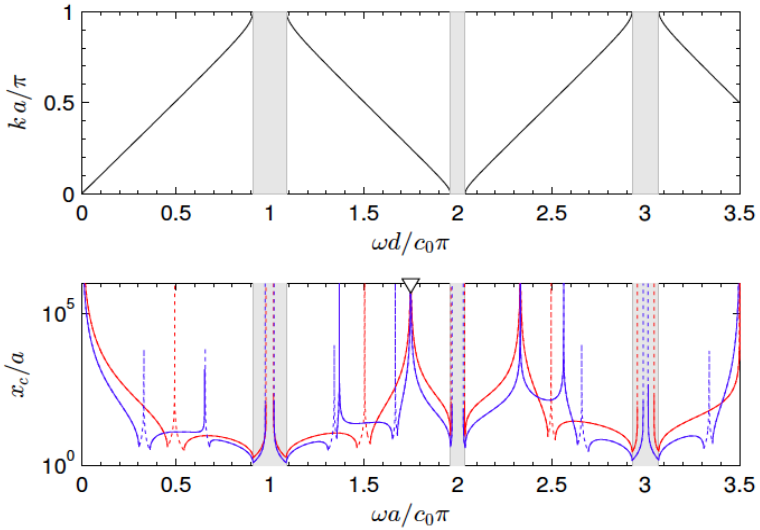
Coherence length:

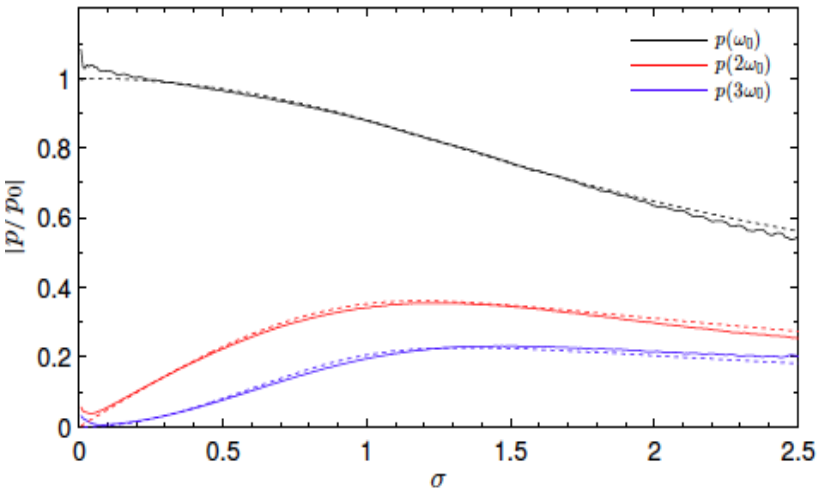
Red: ω-2ω
Blue: ω-3ω

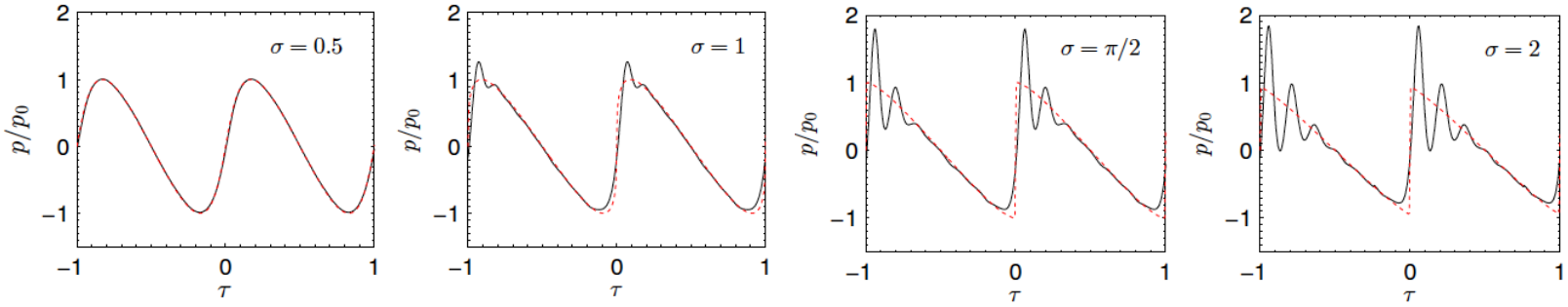
Quasi non-dispersive propagation

2nd and 3rd harmonics phase-matched with 1st.

Shock-type waveform



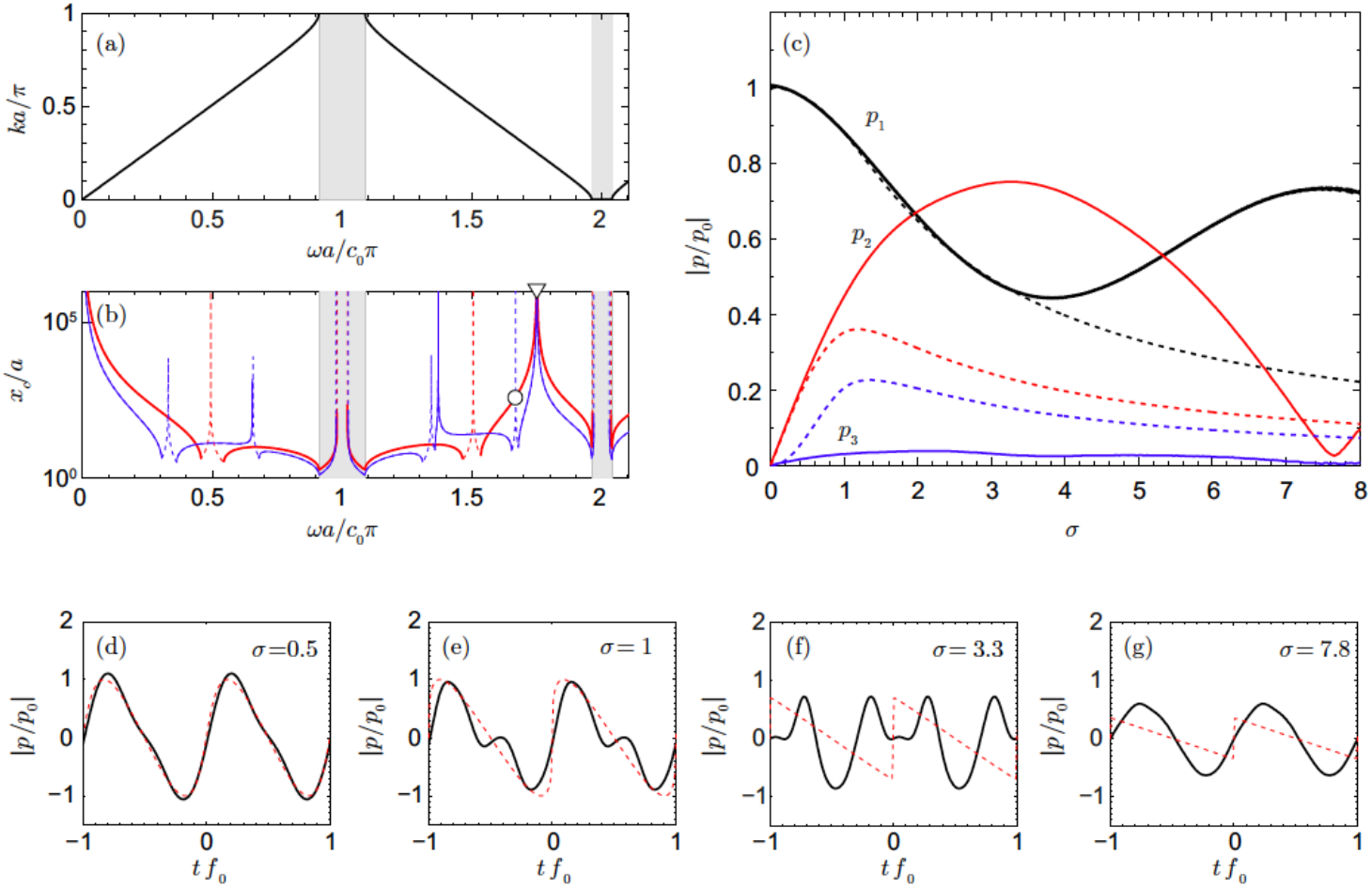




Enhancement of second harmonic

2nd harmonic phase-matched with 1st. 3rd harmonic in a BG.

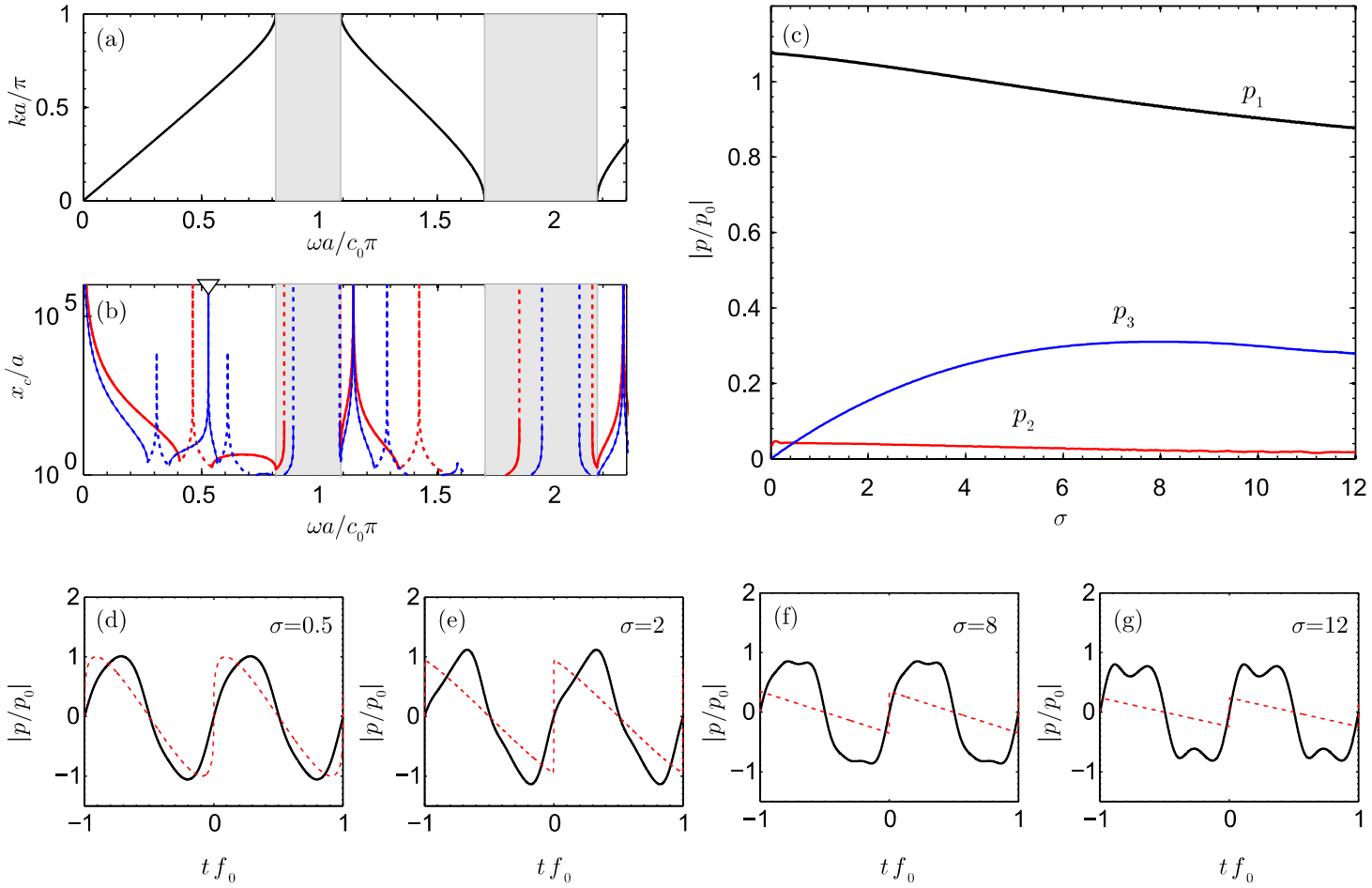
Second harmonic amplitude overcomes first harmonic



Enhancement of third harmonic

2nd harmonic falls in bandgap. 1st and 3rd harmonics phase-matched.

Result in an **effective cubic nonlinear medium**



Conclusions

The interplay between nonlinearity and periodicity offers new and interesting possibilities to control wave propagation in structured materials

1D systems allow for analytical predictions based on very simple models

Results are extendable to higher dimensions (2D, 3D)

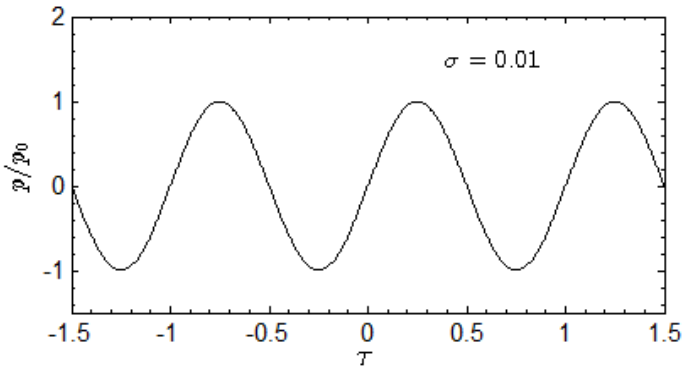
We have considered classical (weak, quadratic) nonlinearities. But there many other nonlinearity types to be explored

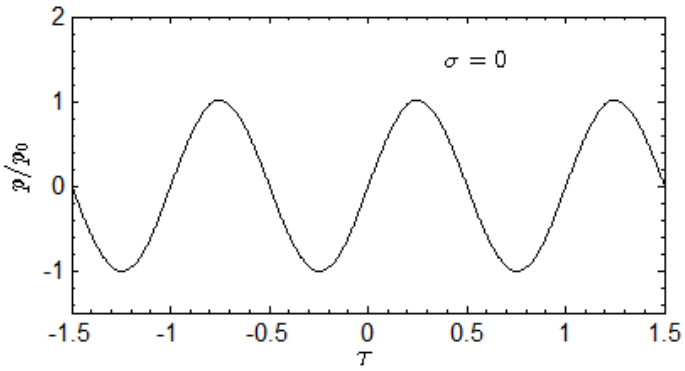
Experiments to confirm the predictions are in progress

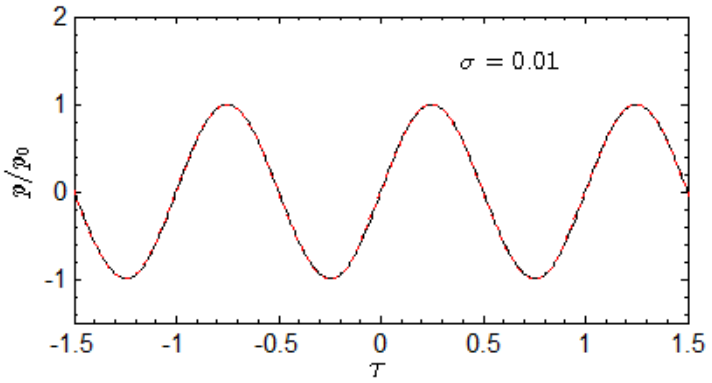
THANK YOU FOR YOUR ATTENTION

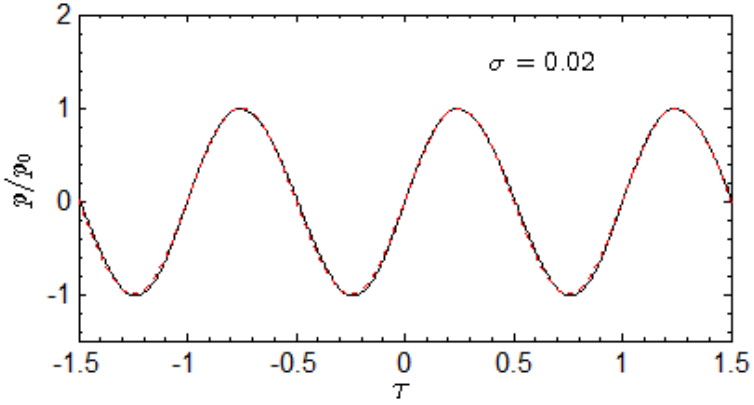
Time waveforms: comparison

…



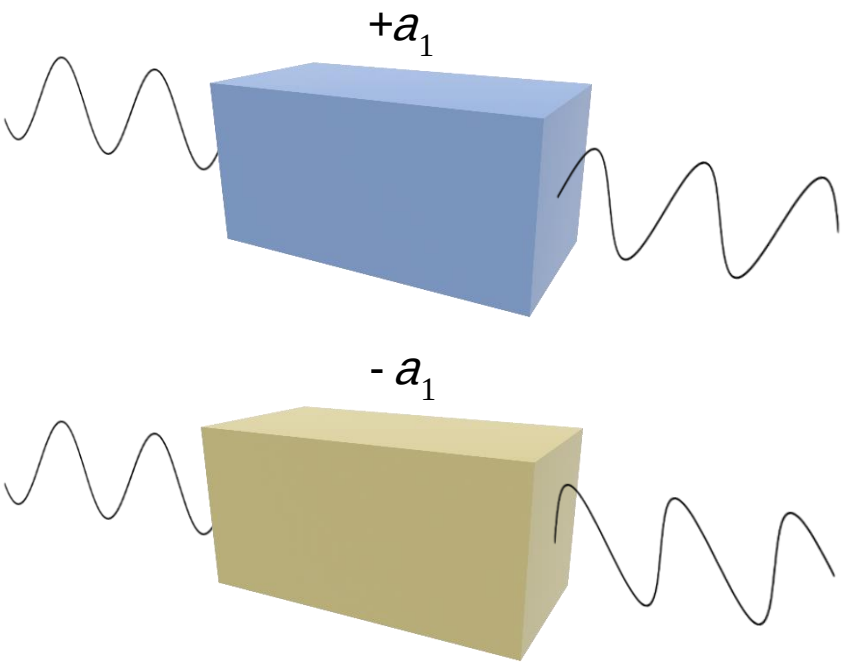


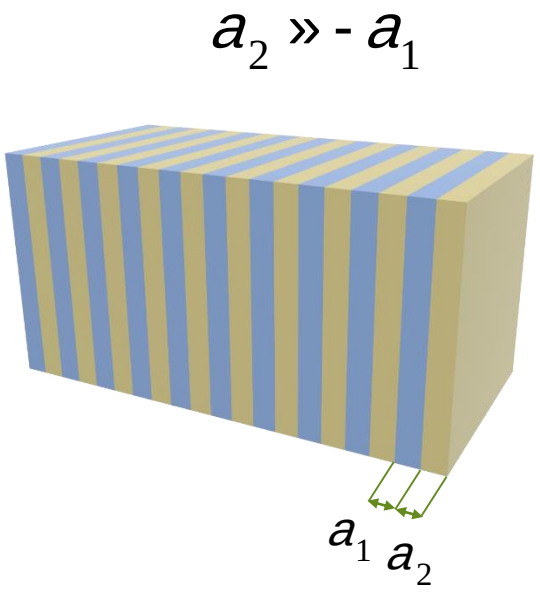




Modulated nonlinearity

- Alternate sign of nonlinearity $c_1 = c_2 \quad a = b$

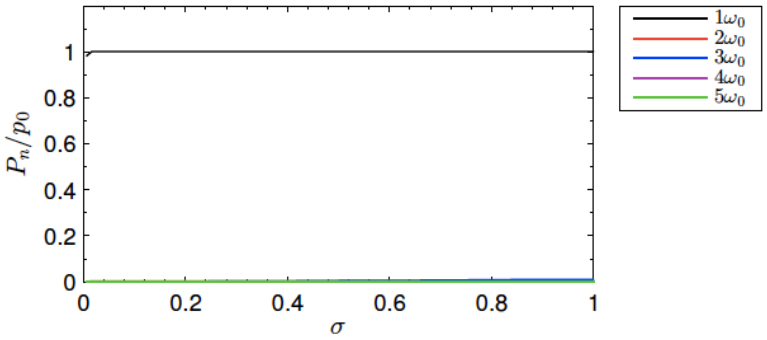


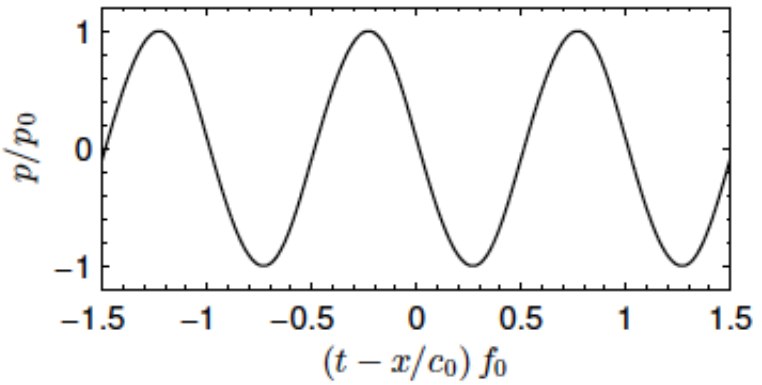


Distorsion compensation

- Nonlinear effects are compensated for $x \sim \sigma$

  An extraordinary linear medium!





- For σ>>1, cubic nonlinear effects appear

