



Time-domain simulation of the constitutive relations of nonlinear acoustics including relaxation for frequency power law attenuation media modelling

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Goal

- Prediction
  - Acoustic waves for Medical Applications
- Soft-tissue frequency power law attenuation
  - (High) Attenuation $\alpha|_{1\mathrm{MHz}} > 0.5\,\mathrm{dB/cm}$
  - (Weak) Dispersion $\mathbf{k_3} \neq \mathbf{k_1} + \mathbf{k_2}$
- High Intensity fields
  - Nonlinearity
- Complex Geometry
  - Inhomogeneities
  - Backscattering -> no one-way propagation models
  - No paraxial
  - Calculate full-wave acoustical magnitudes, $(\rho, p, \mathbf{v})$, Lagrangian density
- Formulation
  - Avoid convolutions in time domain

Soft-Tissue modelling

- Microscopic structure
  - Heterogeneous (*fl.-fl., fl.-sol., sol.-sol., gas)*
  - Quasi-periodic, random, layered
  - Complex dynamical response
    Relaxation, viscoelasticity, nonlinearity

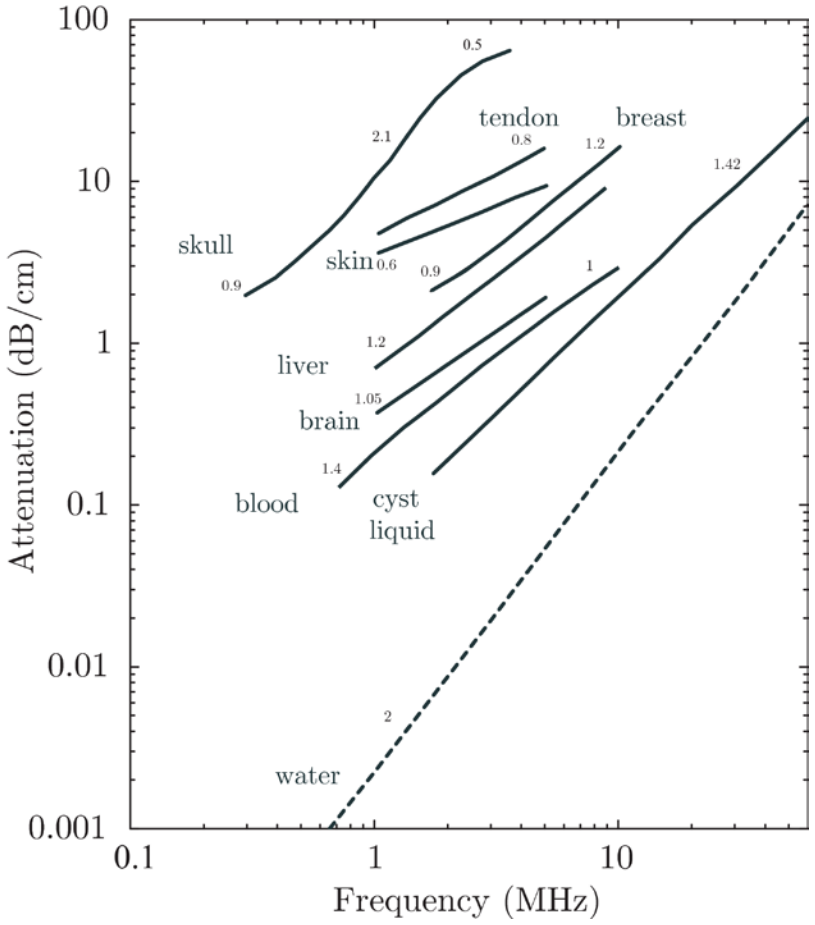
- Model
  - Micro-modeling
    λ << structure
  - Homogenization: US scale
    λ >> structure

- Soft-Tissue
  - Attenuation (frequency power law)

$$\alpha(\omega) = \alpha_0 \omega^{\gamma}$$

  - Dispersion

$$\frac{1}{c_p(\omega)} = \frac{1}{c(\omega_0)} + \alpha_0 \tan\left(\frac{\pi\gamma}{2}\right)\left(|\omega|^{\gamma-1} - |\omega_0|^{\gamma-1}\right)$$



C.R. Hill et. all. Physical Principles of Medical Ultrasonics. John Wiley & Sons, 2004

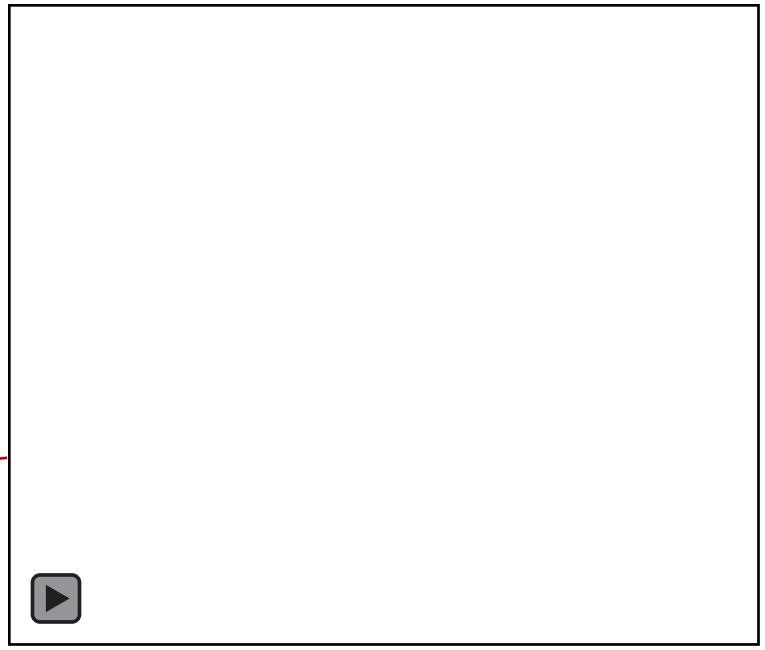
Soft-Tissue modelling

- Relaxation

$$\alpha(\omega) = \frac{\omega^2}{\rho_0 c_0^3}\left(\zeta + \frac{4}{3}\eta\right) + \sum_{n=1}^{N} \frac{\eta_n}{2c_0\tau_n}\frac{\omega^2\tau_n^2}{1+\omega^2\tau_n^2}$$

$$\alpha(\omega) = \alpha_0\omega^\gamma$$

$$0 \leq \gamma \leq 2$$



X. Yang and R. O. Cleveland, J. Acoust. Soc. Am. 117, 113-123 (2005)

Soft-Tissue modelling

- Conservative laws of *mass*, *momentum* and *state*

continuity
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

momentum
$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \underbrace{\eta \nabla^2 \mathbf{v} + \left(\zeta + \frac{1}{3} \eta \right) \nabla (\nabla \cdot \mathbf{v})}_{\alpha(\omega) = \alpha_0 \omega^2}$$

state
$$p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0} \frac{B}{2A} \rho'^2 - \underbrace{\int_{-\infty}^{t} G(t - t') \rho' dt}_{\alpha(\omega) = L(\omega)}$$

Soft-Tissue modelling

- Non-convolutive formulation
  - Multiple exponential relaxation

$$\int_{-\infty}^{t} G(t-t')\rho' dt = \sum_{n=1}^{N} G_n * \rho' \qquad G_n(t) = \frac{\eta_n c_0^2}{\tau_n} \mathrm{e}^{\frac{-t}{\tau_n}} H(t)$$

  - State variable

$$S_n = G_n * \rho'$$

  - Evolution

$$\frac{\partial S_n}{\partial t} = \left(-\frac{1}{\tau_n} \frac{\eta_n c_0^2}{\tau_n} \mathrm{e}^{-\frac{t}{\tau_n}} H(t) + \frac{\eta_n c_0^2}{\tau_n} \right) * \rho'$$

  - Finally...

$$\frac{\partial S_n}{\partial t} = -\frac{1}{\tau_n} S_n + \frac{\eta_n c_0^2}{\tau_n} \rho' \qquad p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0} \frac{B}{2A} \rho'^2 - \sum_{n=1}^{N} S_n$$

Soft-Tissue modelling

- Conservative laws of *mass*, *momentum* and *state* and *multiple relaxation*

continuity
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

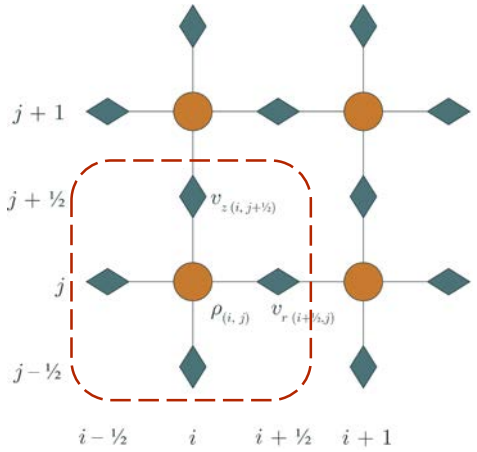
momentum
$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \eta \nabla^2 \mathbf{v} + \left(\zeta + \frac{1}{3} \eta \right) \nabla \left(\nabla \cdot \mathbf{v} \right)$$

state
$$p = c_\infty^2 \rho' + \frac{c_0^2}{\rho_0} \frac{B}{2A} \rho'^2 - \sum_{n=1}^{N} S_n$$

relaxation
$$\frac{\partial S_n}{\partial t} = -\frac{1}{\tau_n} S_n + \frac{\eta_n c_0^2}{\tau_n} \rho'$$

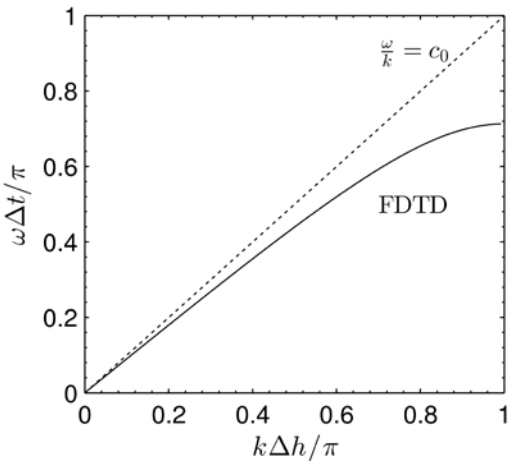
FDTD Dispersion

- (2nd order) Finite-Differences in Time-Domain:
  - Centered finite differences
  - Staggered discretization in space/time
  - Reduced stencil -> less computations
  - Derivatives from local information: errors propagate locally (*k*-space methods needs global information)
  - Easy implementation of boundary conditions (hard, axisymmetric), sharp discontinuities in space, no Gibbs oscillations.
  - Dispersion!



FDTD Dispersion

- Centered finite differences: numerical dispersion
- Nonlinear coupled oscillators
  - Physical nonlinearity + numerical dispersion: nonphysical solutions
  - Oscillatory tails and shocks, solitons (supersonic), CFL instability, ...

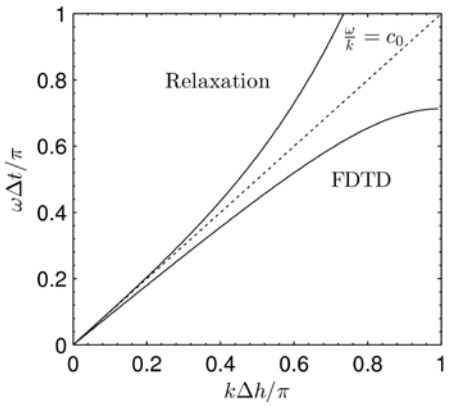


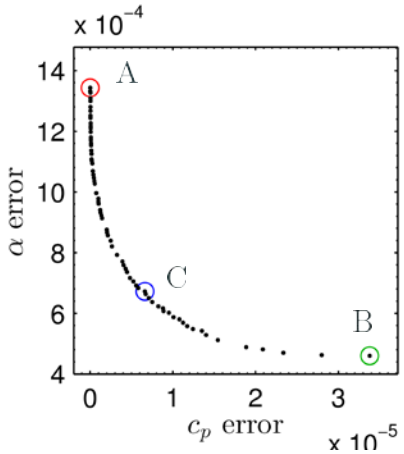
$S_{\mathrm{CFL}} = 0.95$

$S_{\mathrm{CFL}} = 0.3$

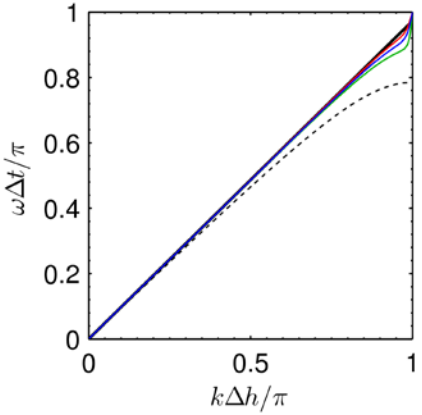
FDTD Dispersion

- Dispersion correction by artificial relaxation

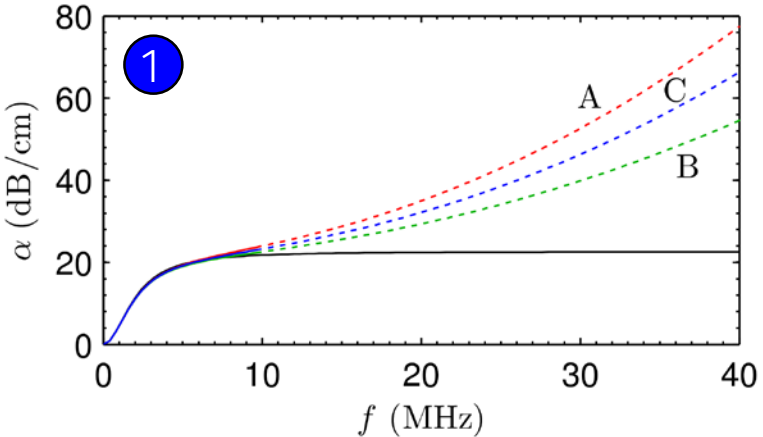


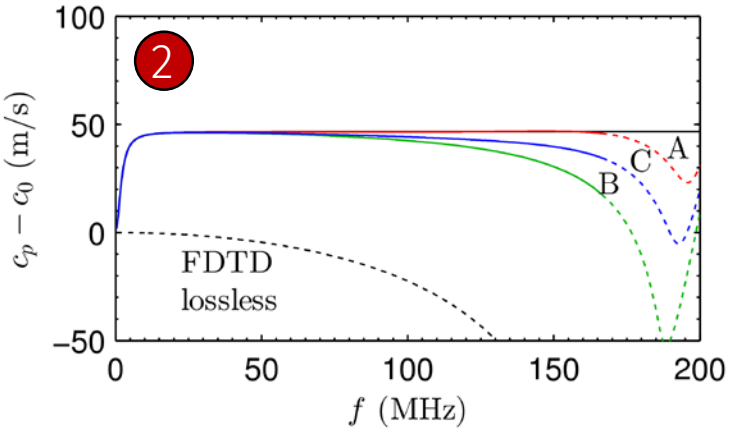


corrected



- Multi-objective genetic optimization
  - 1 (Low freq.) Physical attenuation and dispersion
  - 2 (High freq.) Lattice dispersion + artificial attenuation

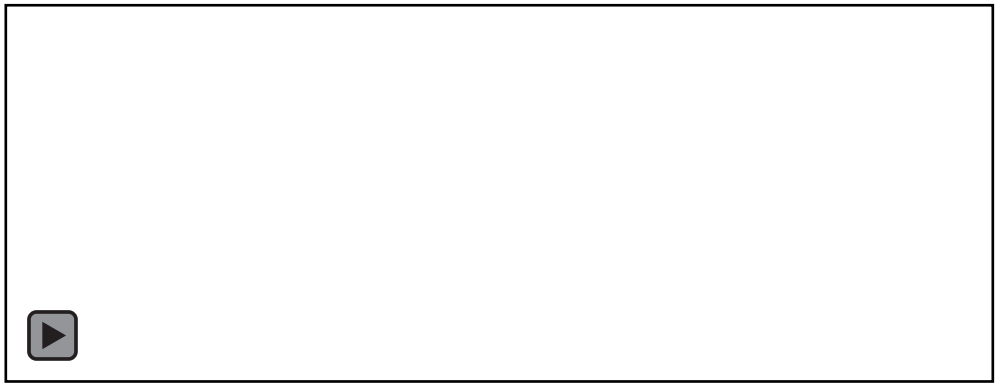




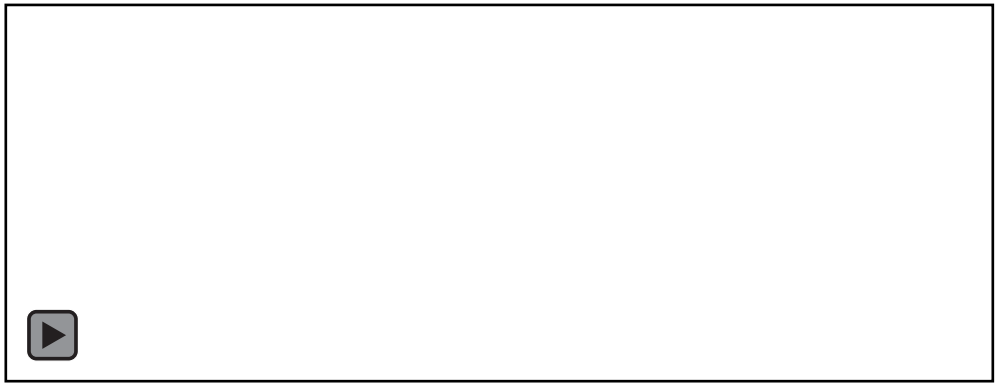
Validation

- Steady shock (inverted) solution for a monorelaxing media

$$\tau = \tau_n \ln \frac{(1 + p/p_0)^{D-1}}{(1 - p/p_0)^{D+1}} \qquad D = \eta_n \rho_0 c_0^2 / 2\beta p_0$$



(artificial viscosity)



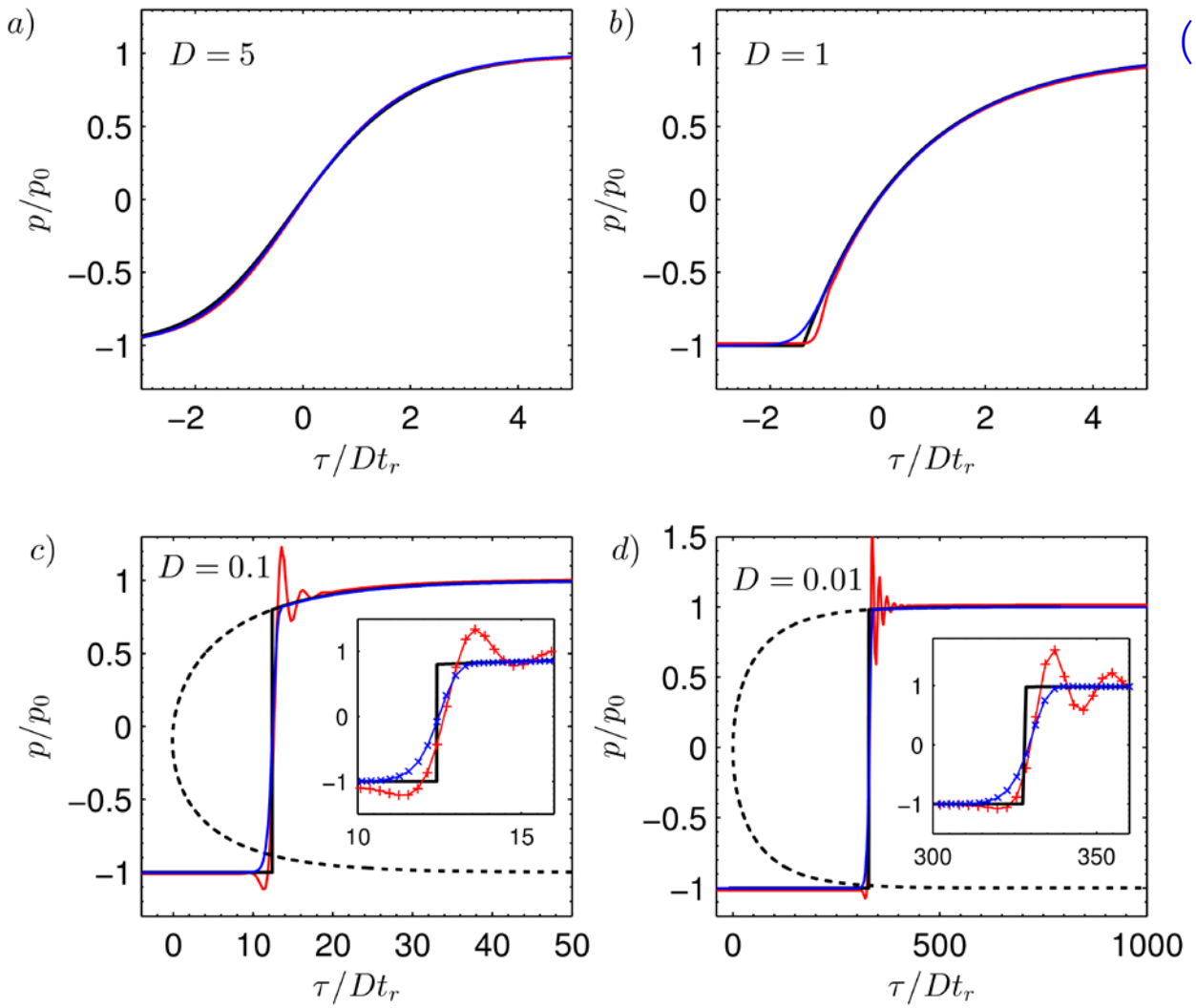
(artificial relaxation)

Validation

- Steady shock (inverted) solution for a monorelaxing media

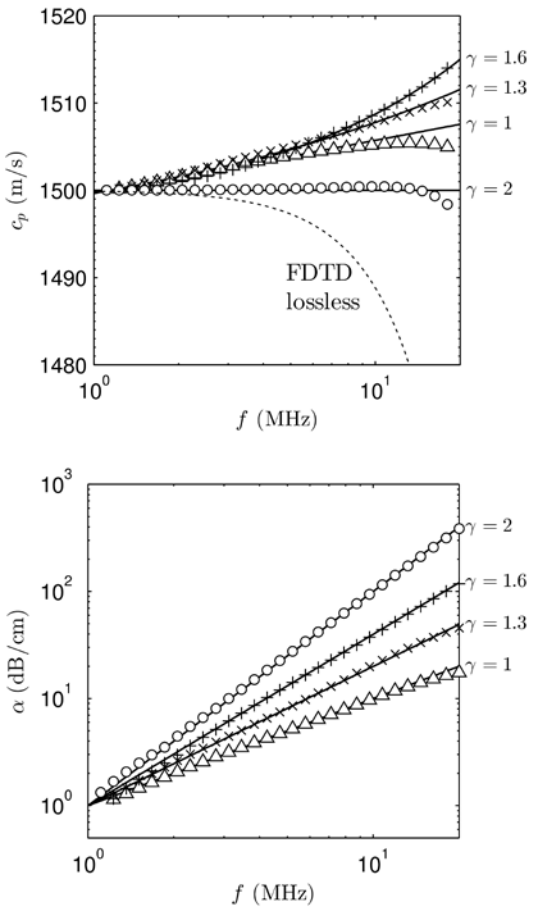
(artificial viscosity)

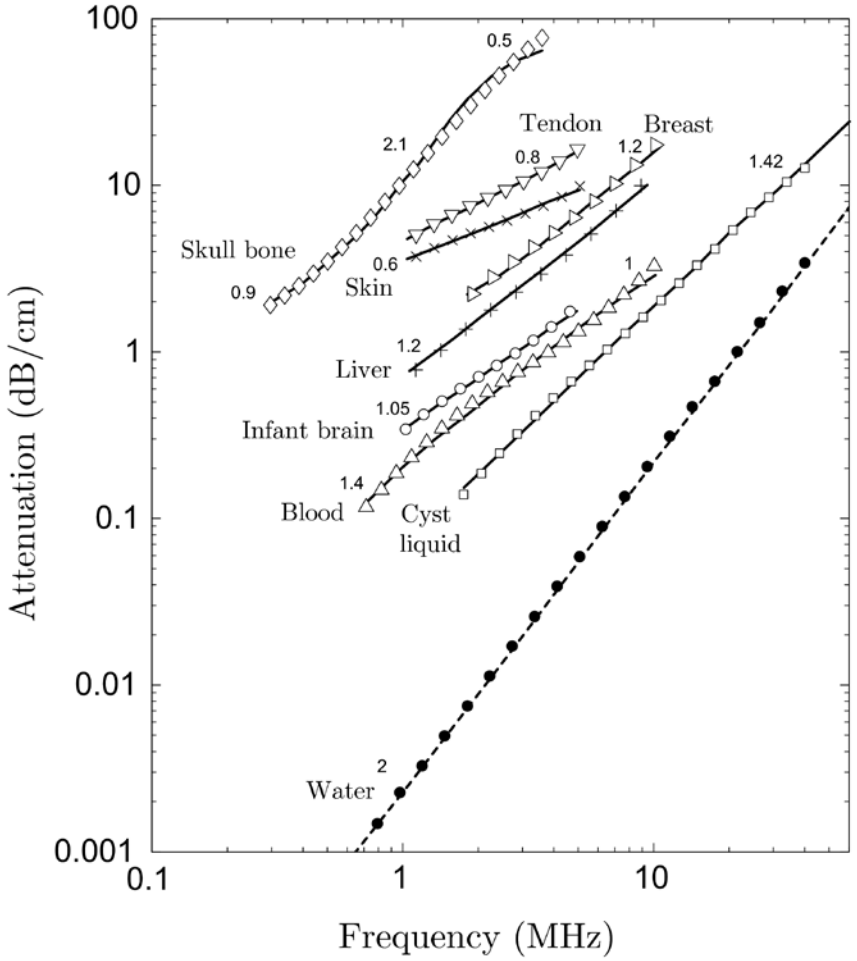
(artificial relaxation)



Validation

- Frequency power law attenuation media



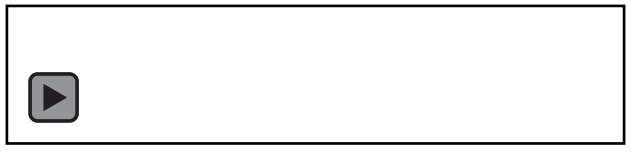


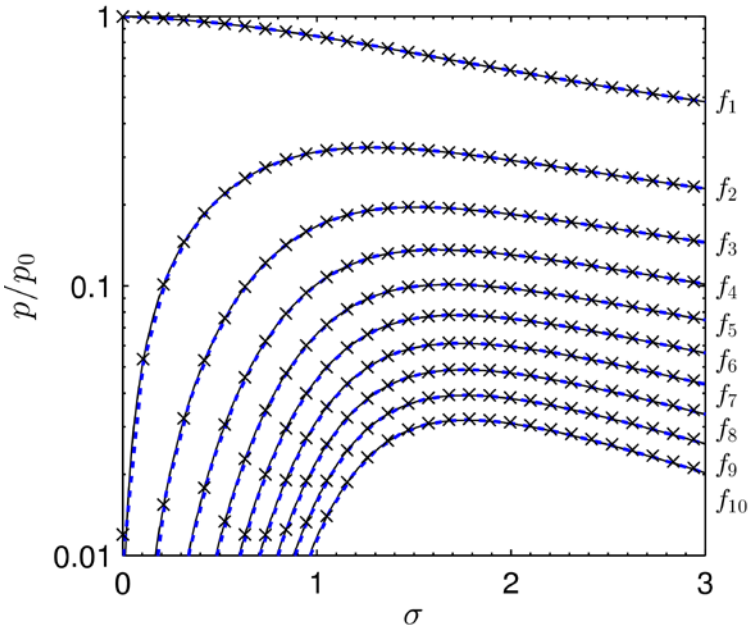
C.R. Hill et. all. Physical Principles of Medical Ultrasonics. John Wiley & Sons, 2004

Validation

- Frequency power law attenuation media

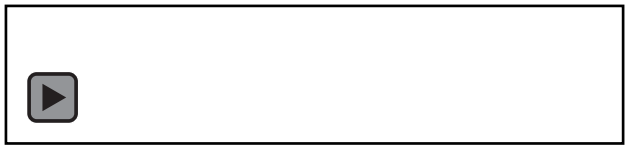
Thermoviscous ($\gamma = 2$)

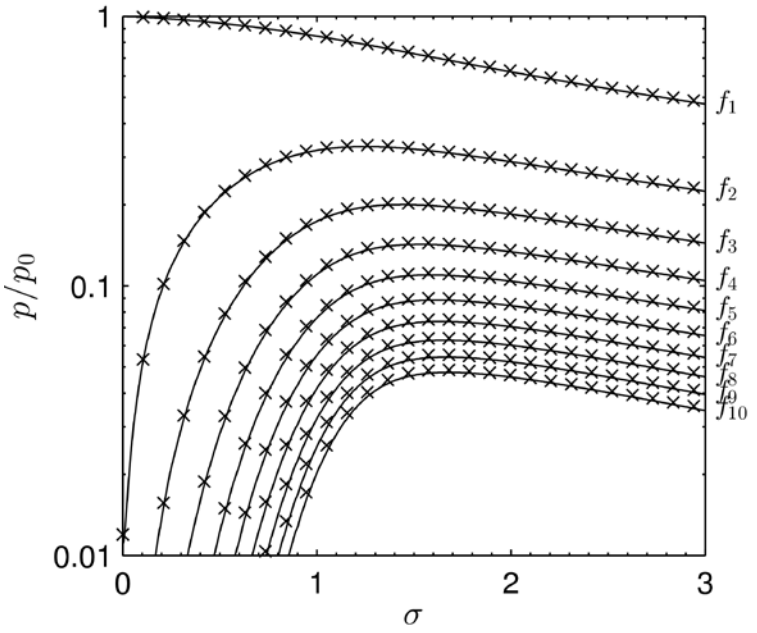




Analytic

Tissue ($\gamma = 1.6$)



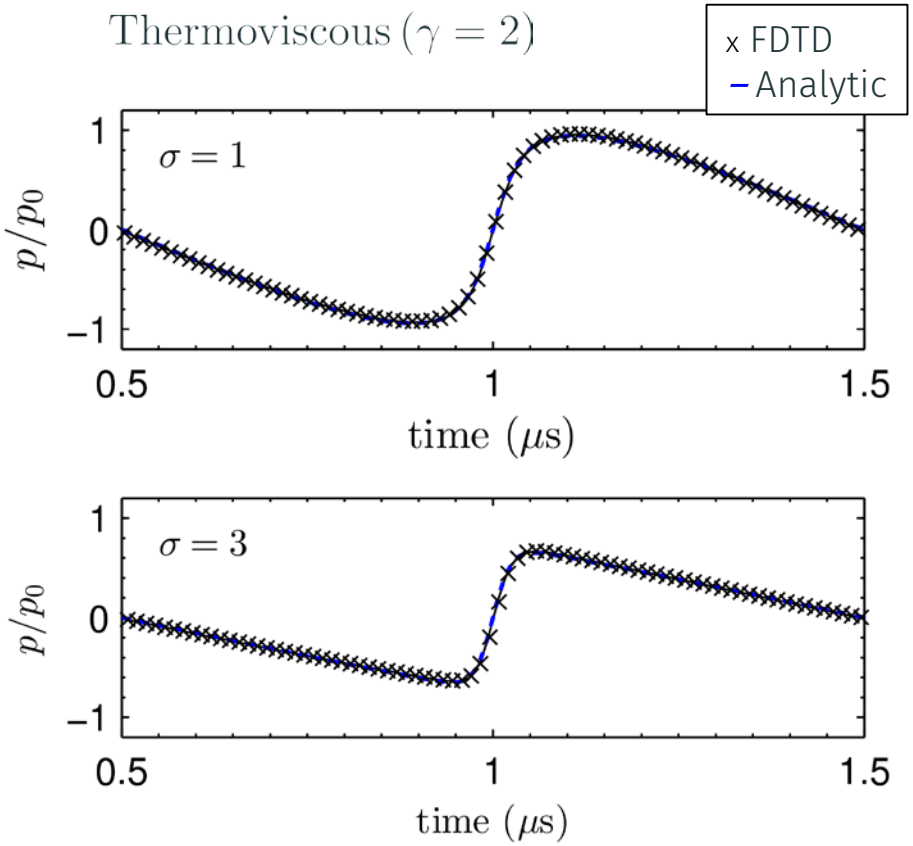


k-space method

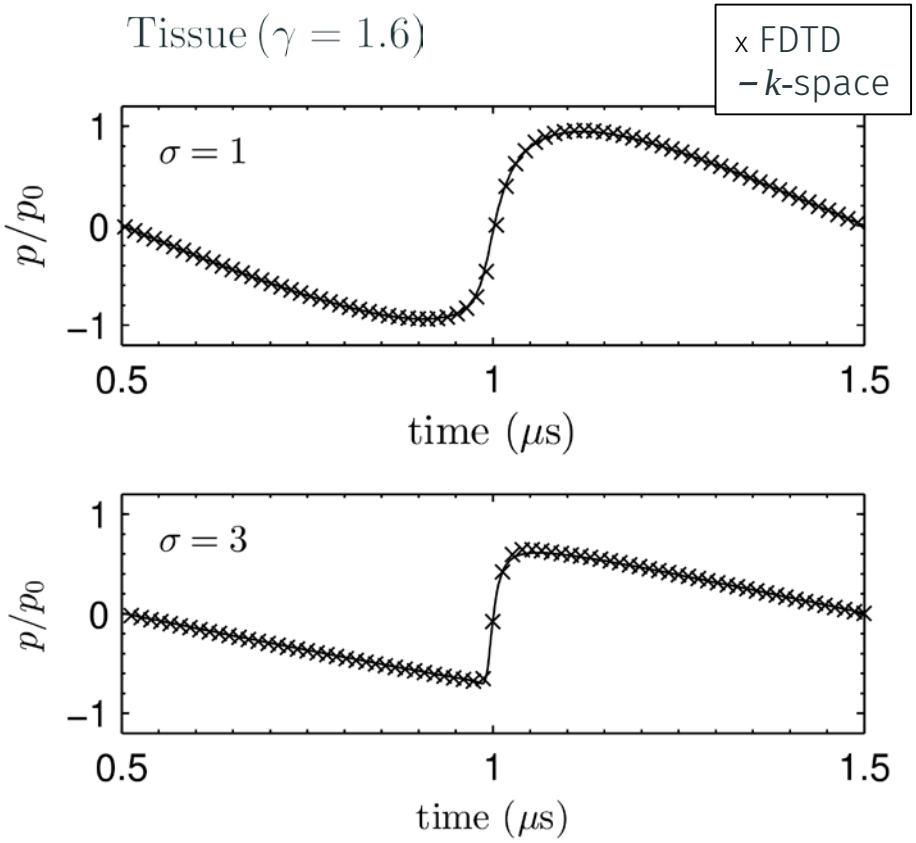
Validation

- Frequency power law attenuation media

Non-dispersive



weak dispersion



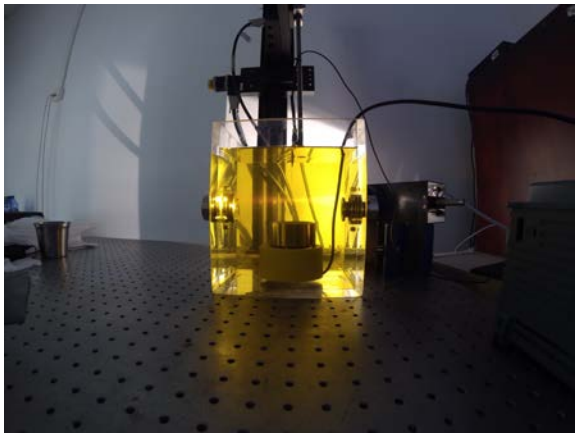
B. E. Treeby, J. Jaros, A. P. Rendell, and B. Cox. J. Acoust. Soc. Am. 131, 4324-4336 (2012).

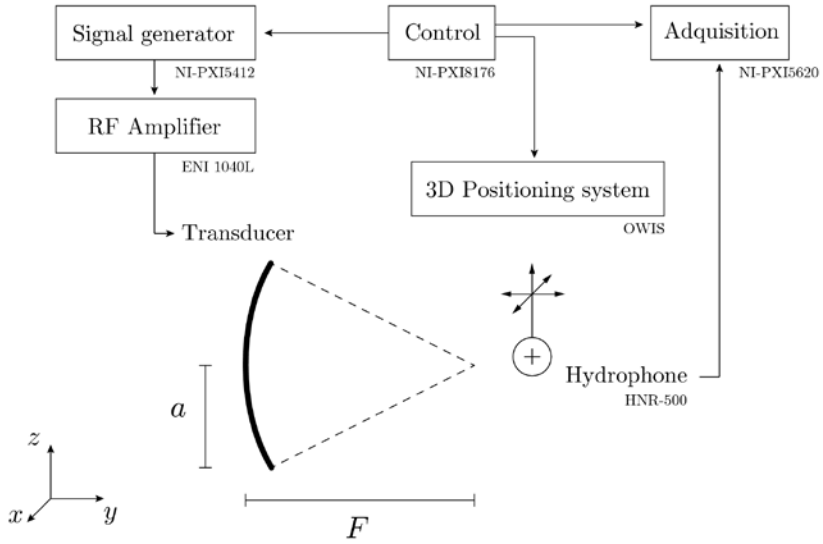
Validation: Diffraction

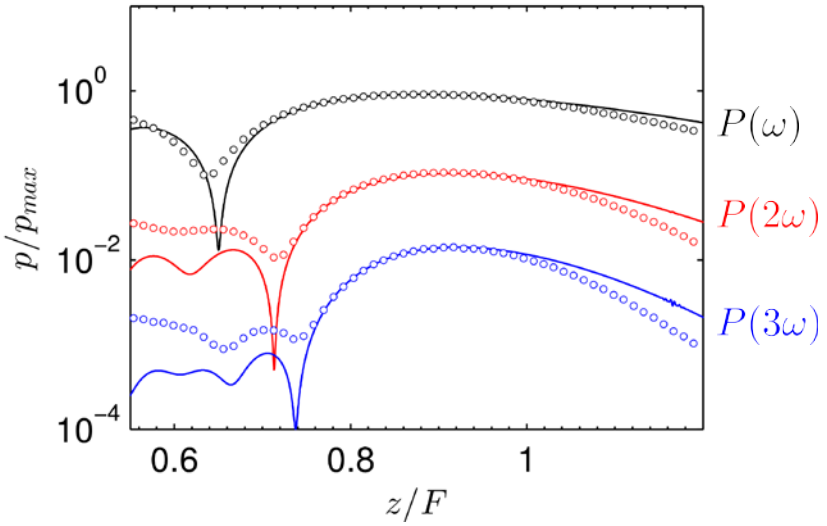
- Diffraction:
  - Beam measurements in castor oil

$$\gamma = 1.6, \quad \alpha = 0.4\,\mathrm{dB/cm/MHz}$$

$$a = 50\,\mathrm{mm}, \quad F = 110\,\mathrm{mm}, \quad f_0 = 1.1\,\mathrm{MHz}$$







B. E. Treeby, et.al. Ultrasonics, Ferroelectrics, and Frequency Control, IEEE Transactions on 56, 1666-1676 (2009)

Diffraction

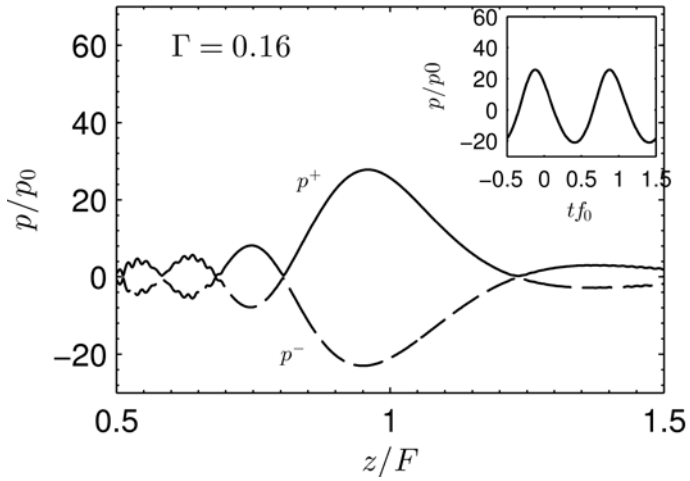
- Diffraction:
  - Simulations of 2 layer media

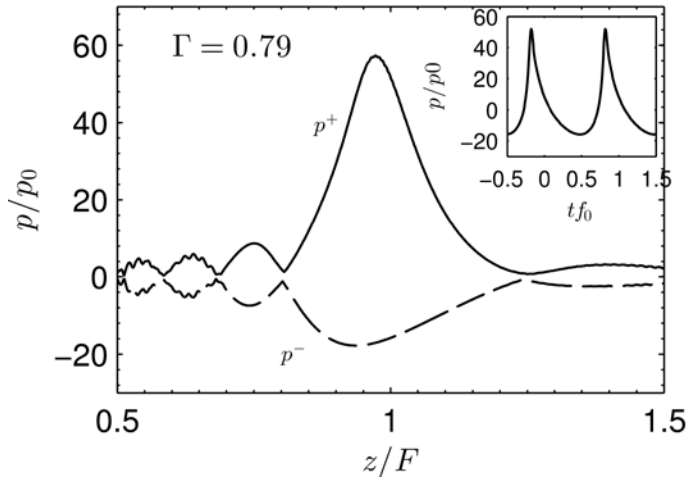
$\gamma_1 = 2, \;\; \alpha_1 = 0.002\,\mathrm{dB/cm/MHz}$

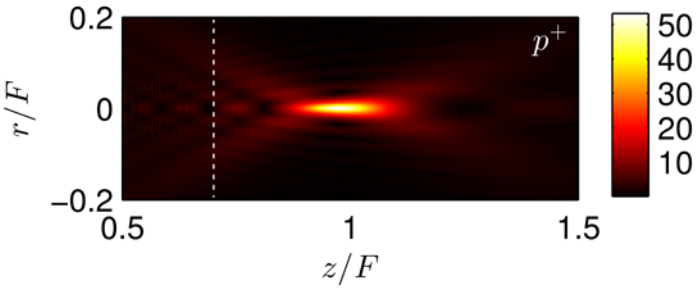
$\gamma_2 = 1, \;\; \alpha_2 = 0.75\,\mathrm{dB/cm/MHz}$

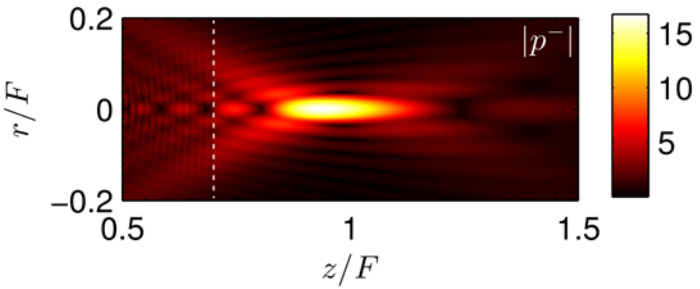
$a = 50\,\mathrm{mm}, \;\; F = 50\,\mathrm{mm}, \;\; f_0 = 1\,\mathrm{MHz}$

$G = 26.5\;, \;\; f - \mathrm{number} = 1$









Discussion and Conclusions

Developed method for intense waves in soft-tissue media:

- Formulation
  - Constitutive equations in a generalized formulation (not only power-law)
  - Calculation of full-wave acoustical magnitudes, rotational fields
  - (e.g. mean rate of momentum, streams)

- Numerical method (2^{nd} order FDTD)
  - Reduced stencil -> fast computations
  - Easy implementation of boundary conditions (hard, axisymmetric)
  - Dispersion corrected by genetic optimization (< 1 min / tissue)
  - FDTD dispersion is anisotropic! -> isotropic scheme

Thanks for your attention



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COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK

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