Nonlinear waves in a Discrete Magnetic Lattice

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Abstract

We investigate the propagation of nonlinear waves in a periodic lattice of magnetically coupled oscillators. Experiments show that higher harmonics of the forcing frequency are generated and propagate along the chain. It is shown that at high frequencies harmonic generation is strongly influenced by the dispersive properties of the chain.

Introduction

Consider 1D Pendulum Chain oscillate through nonlinear contact forces. The equations of motion which governing the system are

$$\frac{\mathrm{d}^2 u_n(t)}{\mathrm{d}t^2} = \kappa_1\left(u_{n+1} - 2u_n + u_{n-1}\right) - \kappa_2\left(u_{n+1} - 2u_n + u_{n-1}\right)\left(u_{n+1} - u_{n-1}\right)$$

Where *k* is constant

Analytic Solution

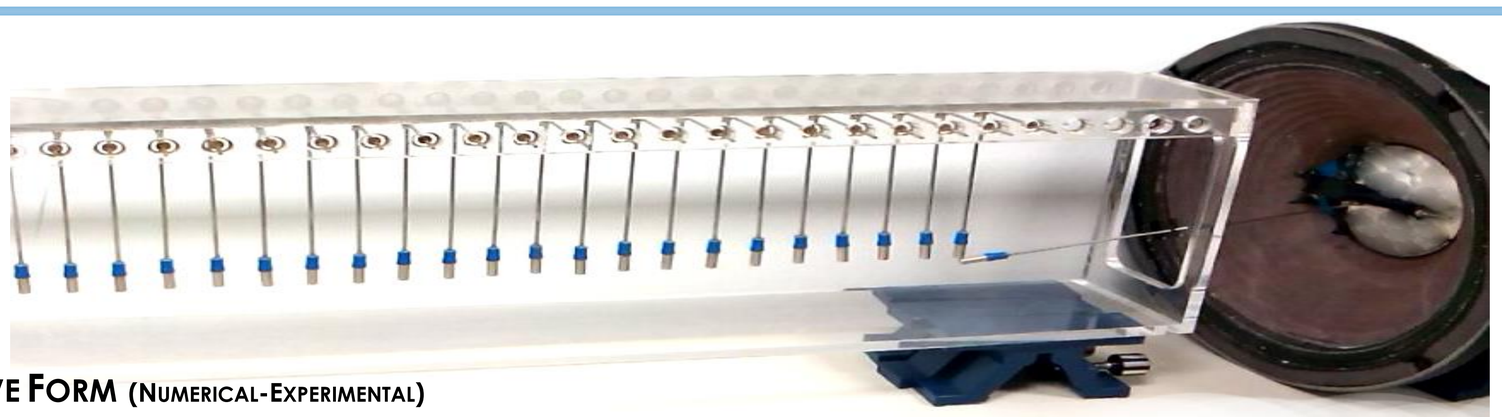
The analytical solution For displacement in two different cases:

Propagative second harmonic

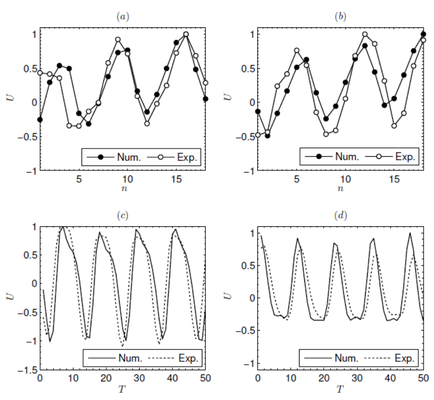
$$U_n = [1 + i\eta \sin\left(\frac{\Delta k\, n}{2}\right) e^{\frac{-i\Delta k\, n}{2}}]e^{i\theta_n} + \frac{\epsilon}{4}\cot\left(\frac{k}{2}\right)\sin\left(\frac{\Delta k\, n}{2}\right)e^{\frac{-i\Delta k\, n}{2}}e^{2i\theta_n} + c.c.$$

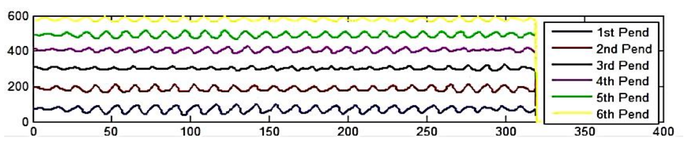
Evanescent second harmonic

$$U_n = [1 + i\frac{\eta}{2}\left(1 - e^{-k''n}e^{k'n}\right)]e^{i\theta_n} - \frac{\epsilon}{8}\cot\left(\frac{k}{2}\right)\left(1 - e^{-k''n}e^{k'n}\right)e^{2i\theta_n} + c.c.$$



Wave Form (Numerical-Experimental)





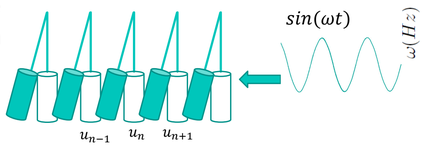
The video: QR code

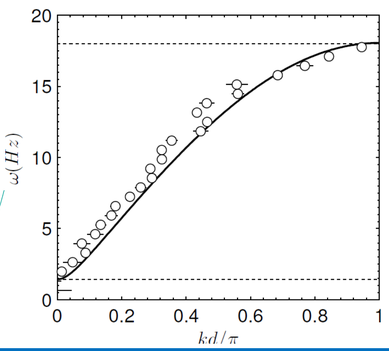
Dispersion relation

Mono-atomic

The dispersion Relation of Mono-Atomic Molecule can be written in the form

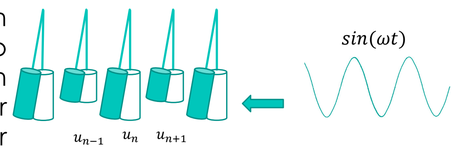
$$\omega^2 = \omega_0^2 + A\, sin^2(\frac{k}{2})$$



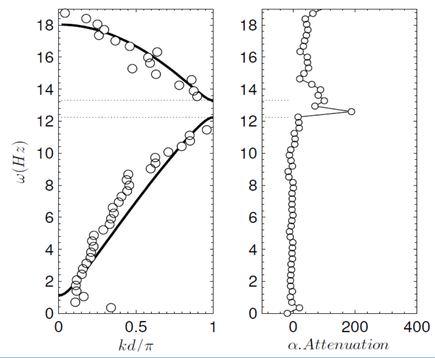


Di-atomic

For this model the dispersion relation for linear wave for Di-Atomic has two different branches : **Acoustic** branch for heavy Mass Pendulum (lower part) and **Optic** branch for lighter mass (Upper part).



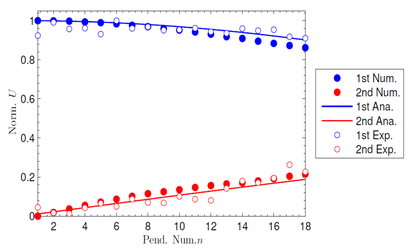
$$\omega^2 = \omega_0^2 + \frac{A}{m_1 m_2}(m_1 + m_2 \pm \sqrt{m_1^2 + m_2^2 + 2m_1 m_2 cos(kd)})$$



Harmonics Generation

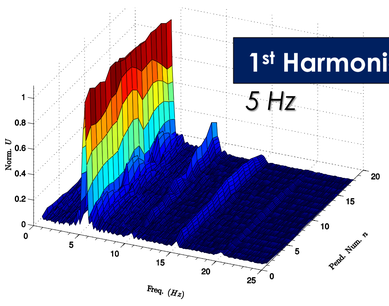
Mono-atomic

The system is excited from the right by using Sin-wave. With specific period of time, the propagating wave is just forward wave . There is no backward wave. Numerical solution based on FPU solution.



1st Harmonic Pass-2nd Pass Band

5 Hz



When the 2nd Harmonic lies close to the edge of the Band-Gap, Two second harmonic components with different wavenumbers are present:

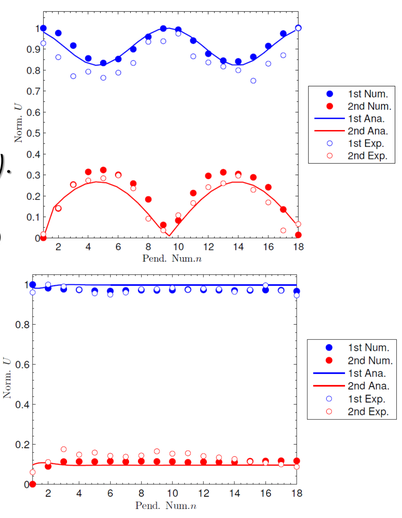
- **forced wave** at *k(2w)*
- **propagating wave** at *2k(w)*.

$\Delta k = k(2\omega) - 2k(\omega)$

Which define

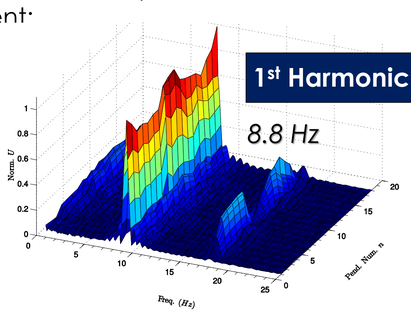
$l_c = 2\pi/\Delta k$

When 2nd Harmonic in Gap :



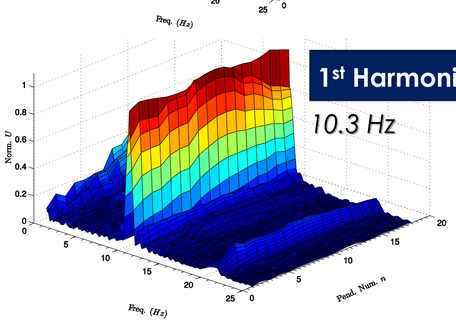
1st Harmonic Pass-2nd Pass (beating)

8.8 Hz



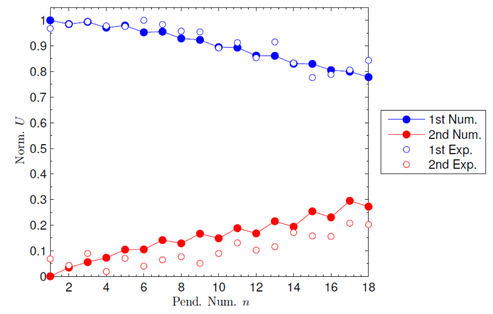
1st Harmonic Pass-2nd Gap Band

10.3 Hz

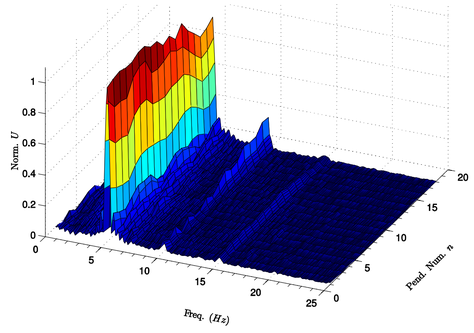


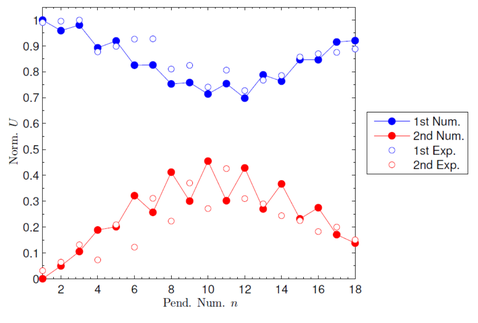
Di-atomic

In the case of Di-atomic molecule , generation of the harmonics are the same but the different is Harmonic oscillations.

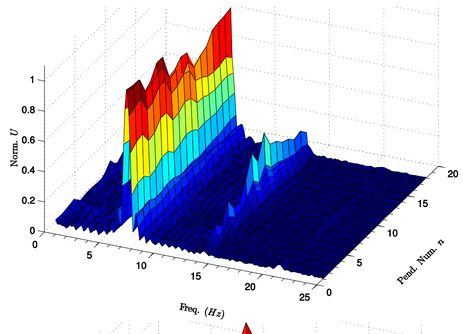


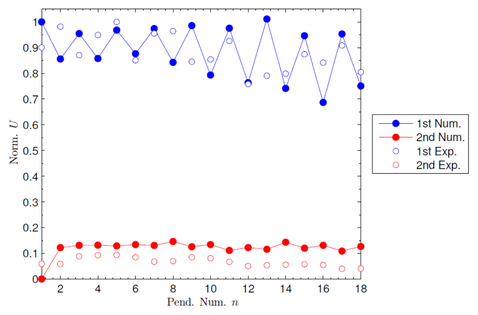
5 Hz



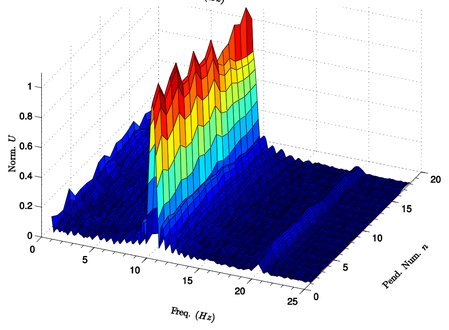


7 Hz



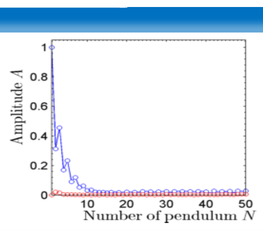


10.1 Hz



Conclusion

This work illustrate the type and the form of nonlinear behavior of the chain around the Band-Gap. At low amplitude excitation , the system is in linear regime. As the excitation level is increased , the nonlinear behavior become apparent . The Gap In Di-Atomic Molecule is proportional to the mass difference. When the 1st Harmonic lies within the band-gap, small amplitude waves propagate evanescently as shown in the next figure . And the 2nd Harmonic will follow fundamental to decay . The experiment compared with simulation, and we have a good agreement.



References

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Acknowledgements

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