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Metadiffusers: sound diffusers with deep-subwavelength dimensions

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Abstract **– We present deep-subwavelength diffusing surfaces based on acoustic metamaterials, namely *metadiffusers*. Sound diffusers are surfaces whose acoustic scattering distribution is uniform. Here, we achieve sound diffusion by using acoustic metamaterials composed by rigidly backed slotted panels, each slit being loaded by an array of Helmholtz resonators. Both, strongly dispersive propagation and slow sound speed are observed inside the slits, shifting their quarter wavelength resonances to the deep-subwavelength regime. Thus, the reflection coefficient of each slit can be tailored to obtain either customized reflection phase, moderate or even perfect absorption. By using a set of different slits with tuned geometry we designed surfaces with spatially-dependent reflection coefficients having uniform magnitude Fourier transforms, presenting good diffusion performance. First, various sub-wavelength diffusers based on known number-theoretical sequences such as quadratic residue or primitive root sequences are presented. Second, accurate designs for binary, ternary and index sequence diffusers are presented making use of perfect acoustic absorption. Finally, a 3 cm thick metadiffuser (1/46 times smaller than the wavelength) was designed working efficiently for frequencies ranging from 250 Hz to 2 kHz, i.e., 3 octaves.**

I. INTRODUCTION

Sound diffusers are surfaces whose scattering function is uniform, i.e., the reflected waves by these surfaces are dispersed in many different directions. Thus, the far-field polar pressure distribution characterizes the performance of the diffuser. This far-field polar pressure distribution, $p_s(\theta)$, of a locally-reacting reflecting surface with a spatially dependent reflection coefficient, $R(x)$, can be calculated using the Fraunhofer integral [1] as $p_s(\theta) = \int R(x) e^{jk_0 x \sin\theta} dx$, where θ is the polar angle and k_0 is the wavenumber in air. Note the far-field pressure is essentially a Fourier transform of the reflected field along the surface. Therefore, structures whose reflection coefficient distribution present a uniform magnitude Fourier transform will exhibit good sound diffusion properties [2]. The generation of spatially dependent reflecting surfaces is commonly achieved by using phase grating diffusers, also known as Schroeder's diffusers [2], that are rigid-backed slotted panels where each slit acts as a quarter wavelength resonator [3, 4], as shown in Fig. 1 (a). Due to the different resonance frequencies of each slit, the phase of the reflection coefficient locally depends on the wavenumber and depth of each slit. Thus, the depth of each slit is designed in such a way that the spatially-dependent reflection coefficient follows a numerical sequence whose Fourier spectrum magnitude is flat. The maximum phase shift of the reflection coefficient achieved by a single slit in a phase grating diffuser occurs at its quarter wavelength resonance, i.e., $L = c_0/4f$ where f is the frequency, L is the depth of the slit and c_0 is the speed of sound in air. Therefore, Schroeder diffusers are limited by their depths, which becomes large at low design frequencies. This results in thick and heavy panels, limiting the use of phase grating diffusers for low-frequencies where the wavelength of sound in air is of the order of several meters.

An approach to design deep-subwavelength thickness resonators is the use of local resonances to introduce strong dispersion in acoustic metamaterials, e.g. to make use of Helmholtz resonators (HR) to induce slow sound. Metamaterials based on slow sound condition have been widely used to design acoustic absorbers [5, 6, 7, 8]. Using slow sound results in a decrease of the cavity resonance frequency and, hence, the structure thickness can be drastically reduced to the deep-subwavelength regime [7]. Here, we present deep-subwavelength thickness diffusers based on acoustic metamaterials to reduce the thickness of Schroeder diffusers. The system works as follows: first, we consider a rigid panel of finite length with a set of N slits. Second, we modify the dispersion

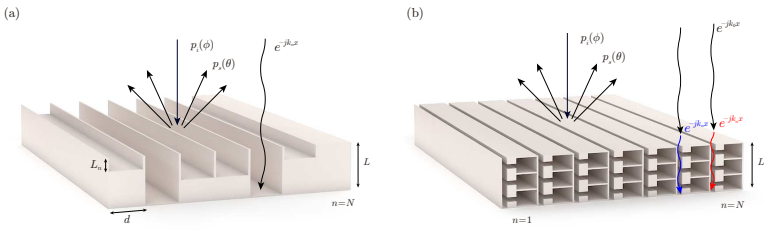


Fig. 1: (a) Scheme of a QRD Schroeder diffuser composed by $N = 7$ slits or quarter wavelength resonators. (b) Metadiffuser composed of $N = 7$ sub-wavelength slits, each of them loaded by $M = 3$ Helmholtz resonators, with slightly different geometry.

relations inside each slit by loading one of their walls with a set of HRs, as shown in Fig. 1 (b). The sound propagation becomes strongly dispersive in each slit and the resulting sound speed, c_p, is drastically reduced. Therefore, each slit behaves as a deep-subwavelength resonator. As a consequence, the effective depth of the slits can be strongly reduced as $L = c_p/4f$ holds. By tuning the geometry of the HRs and the thickness of the slits, the dispersion relations inside each slit can be modified. As a result, the phase of the reflection coefficients can be tailored to those of an Schroeder phase grating diffuser.

II. Quadratic residue metadiffusers

The quadratic residue sequence is given by $s_n = n^2 \text{mod} N$, where mod is the least non-negative remainder of the prime number N. If the phase grating diffuser is based on quarter wavelength resonators, the depth of the slits is given by $L_n = s_n \lambda_0 / 2N$, where λ_0 is the design wavelength. Here, we use optimization methods to tune the geometry of the metamaterial for its spatially-dependent reflection coefficient to match the one of regular QRD at single frequency. Figure 2 (a) shows the phase of the reflection coefficient along the surface for a $N = 5$ QRD with a design frequency of 500 Hz and a total thickness of $L = 27.4$ cm, and a QR-metadiffuser of $L = 2$ cm thickness and $M = 2$ HRs of same dimensions, calculated using the transfer matrix method (TMM). Perfect agreement is found between the reflection coefficients of the QR-metadiffuser and the targeted phase grating QRD. Figure 2 (b) shows the far-field calculation for both structures considering 6 repetitions of the unit cell in order to clearly generate the characteristic N diffraction grating lobes of the QRD in the far-field. Excellent agreement is obtained with the polar response using the TMM and a full-wave numerical solution using the finite element method (FEM) accounting for the thermo-viscous losses. The near field pressure distributions are shown in Figs. 2 (c-d) for the QR-metadiffuser, the QRD and a reference flat surface of the same width, respectively. Excellent agreement is observed between both diffusers, where it is clear how the field is scattered in other directions rather than specular one. Notice that the presented QR-metadiffuser is 17.1 times thinner than the usual QRD (34 times smaller than the QRD design wavelength).

III. Broadband optimal metadiffusers

To obtain a metadiffuser useful for room acoustics applications, its diffusion must be broaden in frequency. Thus, we extend the bandwidth of the optimization procedure. In particular, we look for deep-subwavelength

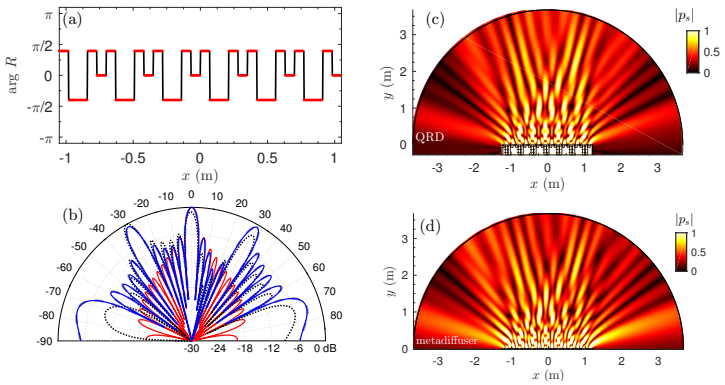


Fig. 2: Phase of the spatially-dependent reflection coefficient of a QRD (black line) and the QR-metadiffuser (red doted). (b) Far-field polar distribution of the QR-metadiffuser obtained by TMM (continuous blue) and FEM (dotted black), the reference QRD (dashed-grey), and a plane reflector with same width of the diffusers (continuous red). Near field pressure distribution at 2 kHz of a (c) phase grating QRD of thickness $L = 27.4$ cm (d) QR-metadiffuser with $N = 5$ slits and thickness $L = 2$ cm.

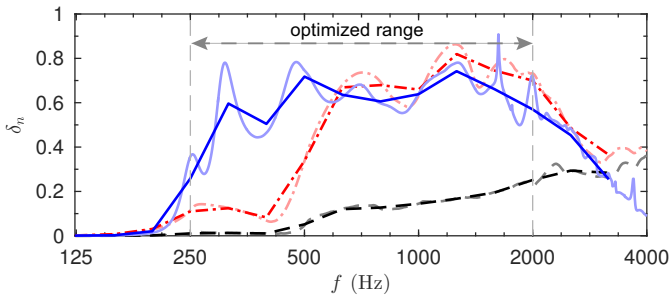


Fig. 3: (a) Normalized diffusion coefficient of a 3 cm QRD (dashed black), 56 cm QRD (dashed-dotted red) and optimized metadiffuser using TMM (blue) integrated in third of octaves. The third octave integration is shown in thick lines according to ISO 17497-2:2012.

thickness metadiffusers presenting maximum normalized diffusion coefficient in the frequency range from 250 Hz to 2 kHz. A set of $N = 11$ slits separated by $d = 12$ cm was used, and the thickness of the panel was constrained to $L = 3$ cm. Figure 3 (a) shows the frequency dependent diffusion coefficient calculated according to ISO 17497-2:2012 normalized to a flat reflector of same dimensions for a thick QRD with a design frequency of 250 Hz ($L_{\text{QRD}} = 56$ cm), a thin QRD with the same thickness of the metadiffuser $L_{\text{QRD,thin}} = 3$ cm, and the optimized metadiffuser. Over the optimized frequency range, the diffusion coefficient of the metadiffuser takes a mean value of $\delta_n \approx 0.65$, with peaks of $\delta_n = 0.91$. When compared to the thick QRD, its frequency band is extended of one octave below while the metadiffuser thickness is 46 times smaller than the wavelength.

IV. Conclusions

Metadiffusers that are novel acoustic diffusers with tailored scattering distributions are presented. These new structures are based on metamaterials comprising slotted panel, with slits loaded by a set of Helmholtz resonators. The propagation inside each slit presents strong dispersion and the sound speed can be significantly reduced so that each slit effectively behaves as a deep-subwavelength resonator. Different designs were presented based on number-theoretical sequences as quadratic residue and primary root sequences. It was shown that the structures can be optimized to work in a broadband frequency range covering 3 octaves. In particular, we presented a 3 cm thick diffuser efficient from 250 to 2000 Hz, demonstrating the potential of the metadiffusers to be used in critical listening environments due to their deep-subwavelength nature: the thickness of the panel was 1/46 times the impiginig strcutre wavelength, i.e., about a twentieth of the thickness of traditional designs.

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