Complex frequency analysis of the scattering matrix for the design of **perfect absorbers** in reflection and transmission problems

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Introduction

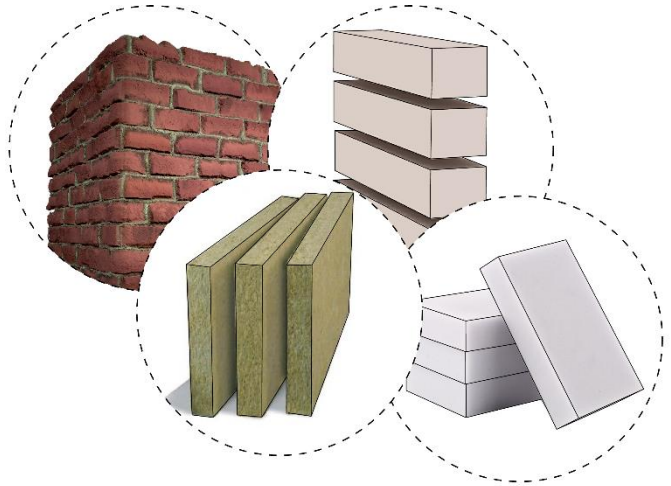
Scattering matrix:

- Relates the incoming and outgoing waves

$$\mathbf{S} = \begin{bmatrix} T & R^- \\ R^+ & T \end{bmatrix}$$

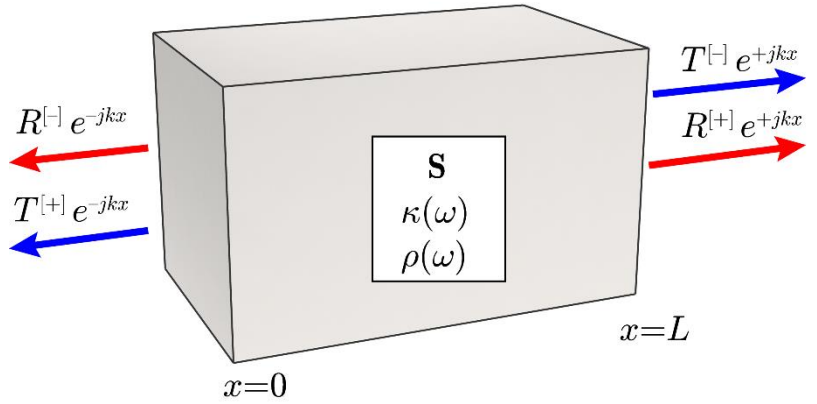
- Eigenvalues of the scattering matrix

$$\lambda_{1,2} = T \pm \sqrt{R^+ R^-}$$



Perfect absorption

- Perfect absorption is achieved when no waves are scattered by the structure
- Both eigenvalues of the **S**-matrix should vanish: $\lambda_{1,2} = 0$



From which direction are waves perfectly absorbed?

- Eigenvectors corresponding to λ_1 and λ_2

$$\vec{v}_1 = \left[R^-, -\sqrt{R^+ R^-}\right], \quad \vec{v}_2 = \left[\sqrt{R^+ R^-}, R^+\right]$$

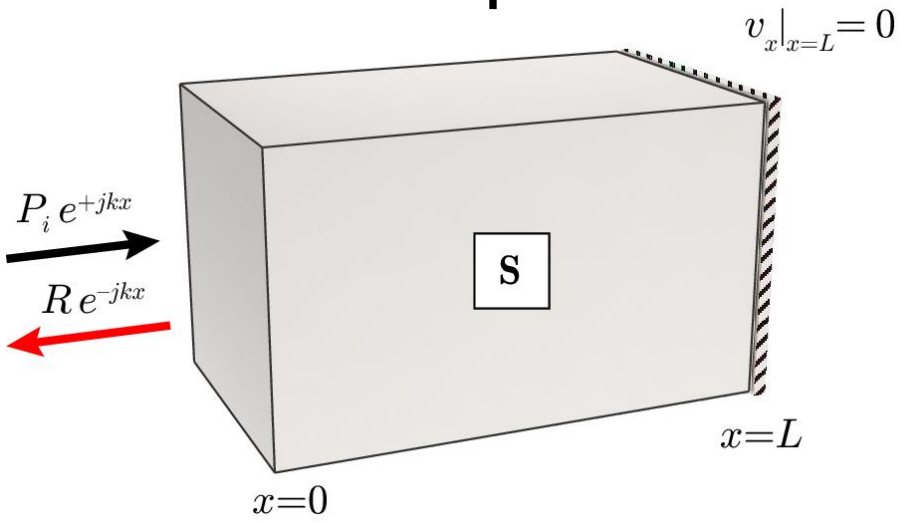
- Eigenvectors show the asymmetry of the scattering

Reflection and transmission problems

Slab of absorbing material

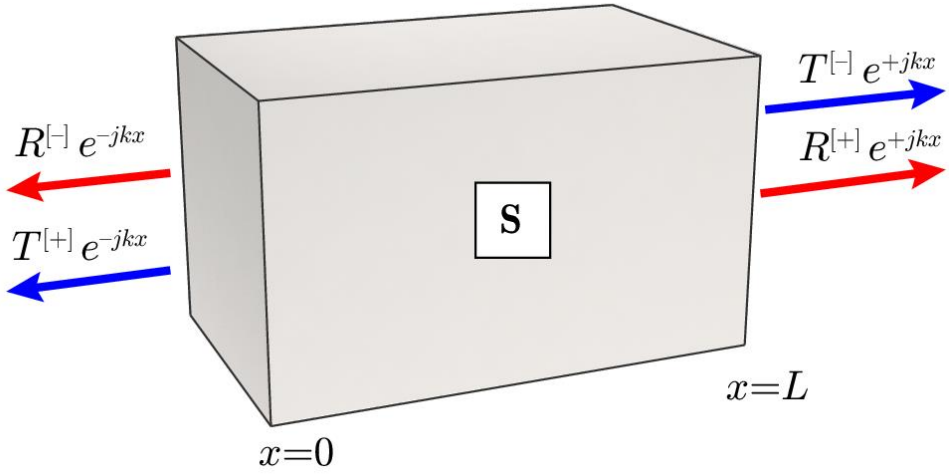
- Boundary conditions at $x = L$

Reflection problem



- Scattering matrix: $\mathbf{S} = \begin{bmatrix} 0 & R \\ R & 0 \end{bmatrix}$
- Eigenvalue(s): $\lambda_{1,2} = R$

Transmission problem



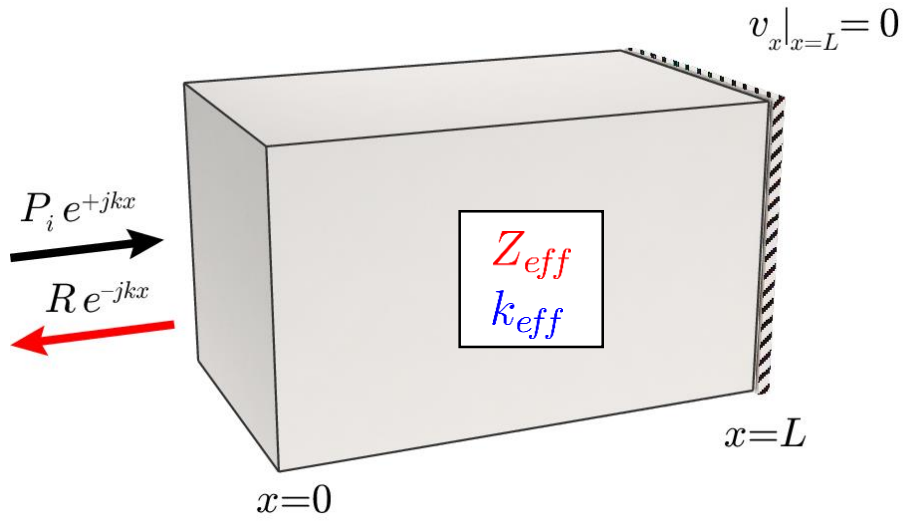
- Scattering matrix: $\mathbf{S} = \begin{bmatrix} T & R^- \\ R^+ & T \end{bmatrix}$
- Eigenvalue(s): $\lambda_{1,2} = T \pm \sqrt{R^+ R^-}$

Reflection problems

Slab of absorbing material

- Scattering matrix and eigenvalue

$$\mathbf{S} = \begin{bmatrix} 0 & R \\ R & 0 \end{bmatrix} \qquad \lambda_{1,2} = R$$



- Characteristic impedance Z_{eff} and wavenumber k_{eff}
- Acoustic impedance: $Z = -iZ_{eff}\cot(k_{eff}L)$
- Reflection

$$R = \frac{Z(\omega)/Z_0 - 1}{Z(\omega)/Z_0 + 1}$$

- **Perfect absorption**: $\alpha = 1 - |R|^2 \to 1 \begin{cases} \mathrm{Re}(Z) & = Z_0 \\ \mathrm{Im}(Z) & = 0 \end{cases}$ Impedance matching conditions

Which is the thinnest perfect-absorber we can built?

- $\mathrm{Re}(Z)$ is in general different from Z_0
- $\mathrm{Im}(Z)$ is generally complex, only can be $\mathrm{Im}(Z) = 0$ at resonance $L = \lambda_{eff}/4$

Reflection problems

Slab of absorbing material (e.g, rigidly-backed porous material)

To downshift the QWR

1 Change thickness

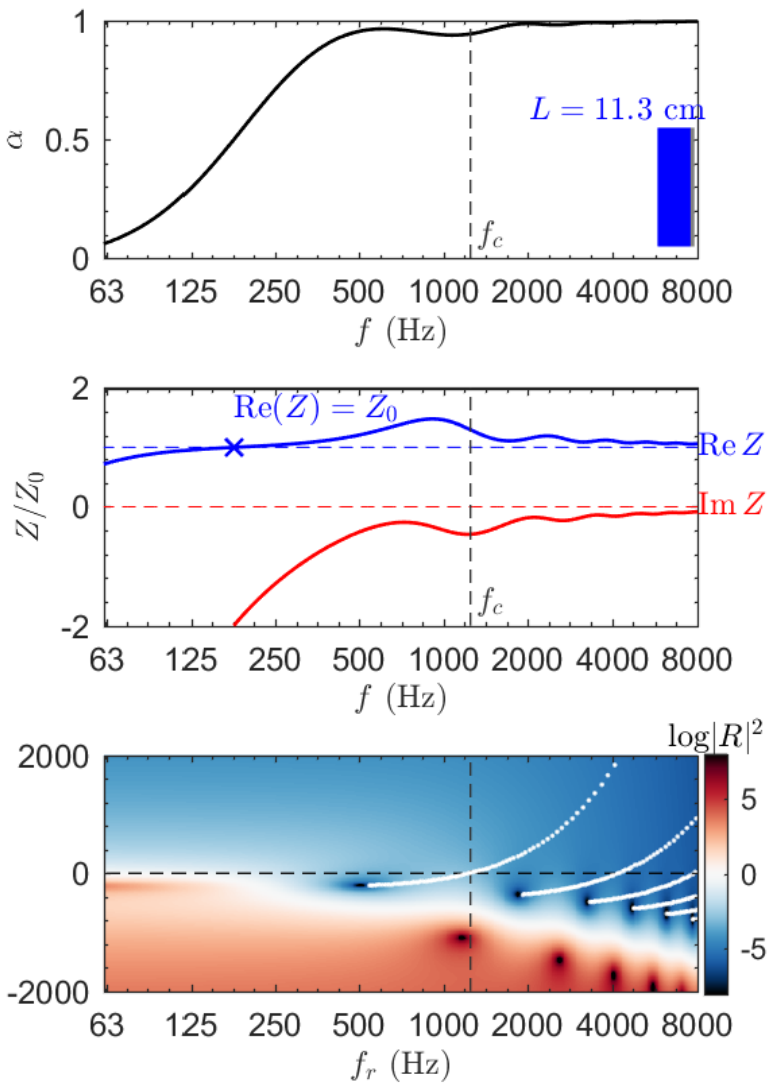
- Is it perfect absorption?

2 Look at the impedance matching

- Phase matching can be achieved
- For thick panels the QWR resonance enters in the viscous regime

3 Look at the scattering matrix: $\mathbf{S} = \begin{bmatrix} 0 & R \\ R & 0 \end{bmatrix} \rightarrow \lambda_{1,2} = R$

- No scattering for $\lambda_{1,2} = 0$
- Critical coupling condition
- Represent the eigenvalues in the complex frequency plane



Introduction

Goal: **to design subwavelength and perfect absorbers**

1 Perfect absorbers

- System must be **critically coupled** with the exterior media
- In general: $\lambda_{1,2} = 0$
- Intrinsic losses of the material exactly compensate the leakage of the structure
- We need some degree of freedom to tune the losses of the system

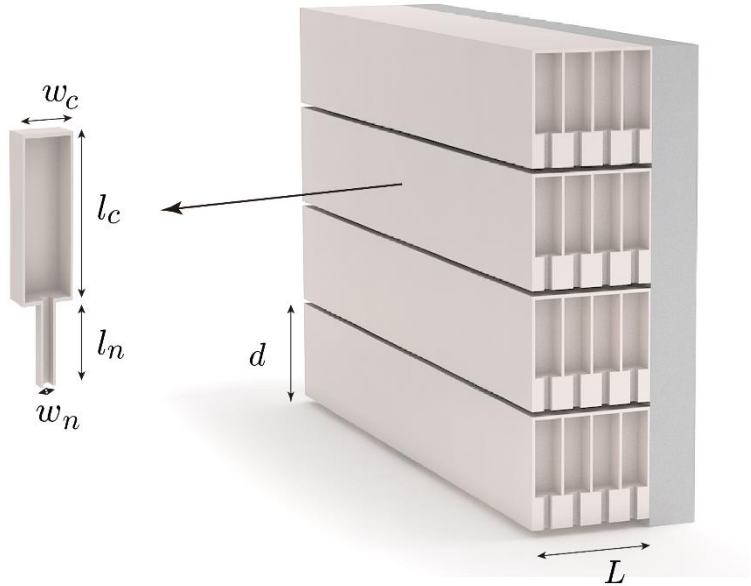
2 Subwavelength resonance

- Use of **subwavelength resonators**
- QWR, space coiling, Helmholtz, elastic plates and membranas, decorated membranes, bubbles ...
- Degree of freedom to tune the losses of the system

Reflection problems

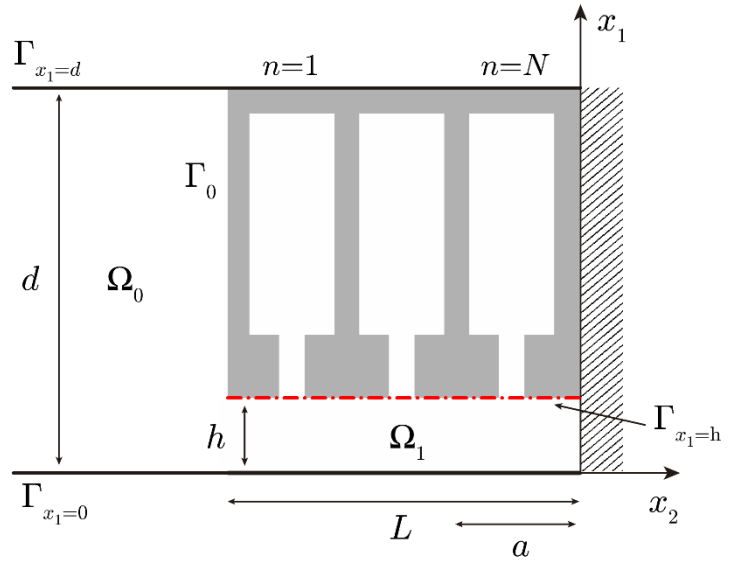
Locally resonant metamaterial:

- Slotted panel loaded by an array of Helmholtz resonators [1]
- Thermo-viscous losses are adjusted by tuning the geometry [2]
- Use **slow-sound:** deep-subwavelength resonator
- Use **critical coupling:** perfect absorption



Solution of the problem: methods

- ***Theoretical***: Modal expansion method (MEM)
- ***Theoretical***: Effective parameters are derived
- ***Theoretical***: Transfer matrix method (TMM)
- ***Numerical***: Finite-Element method (FEM) Comsol Multiphysics
- ***Experiment***: Experimental validation in impedance tube



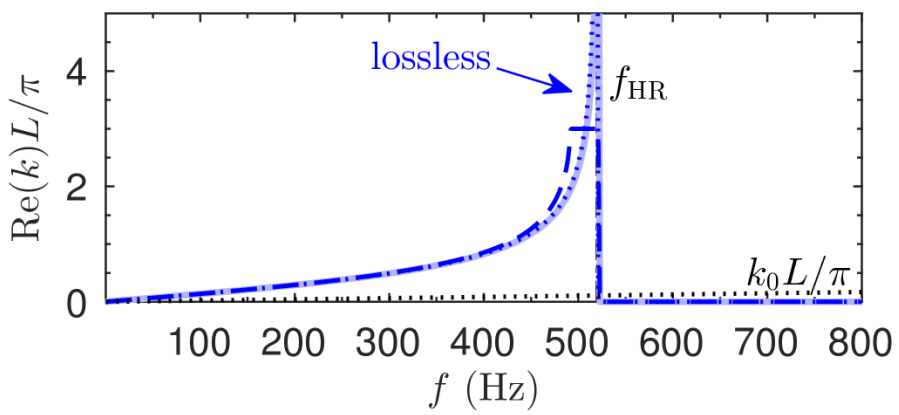
[1] N. Jiménez et. al. App Phys Lett 109(12) 121902 (2016)
[2] MR. Stinson, J Acoust Soc Am 89(2) 550 (1991)

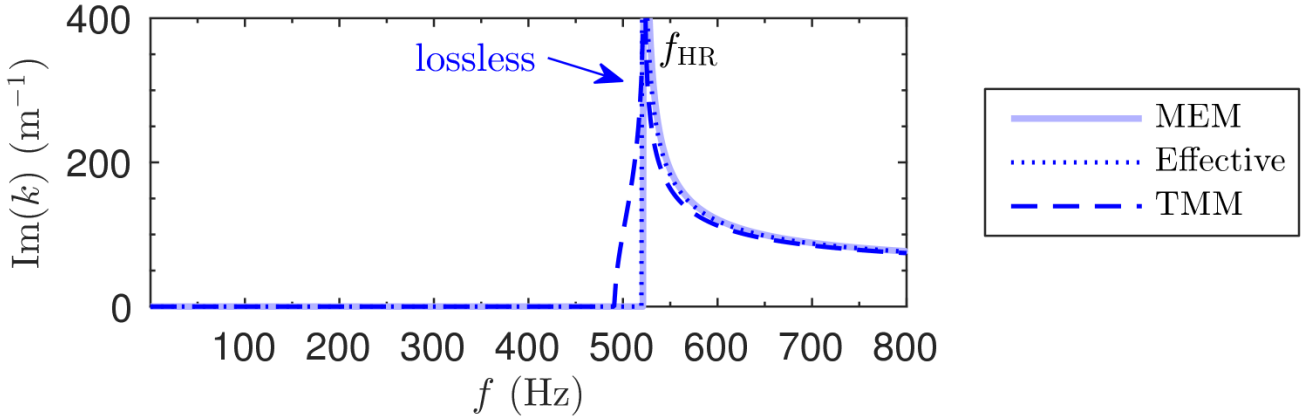
Reflection problems

Dispersion relations in the slit

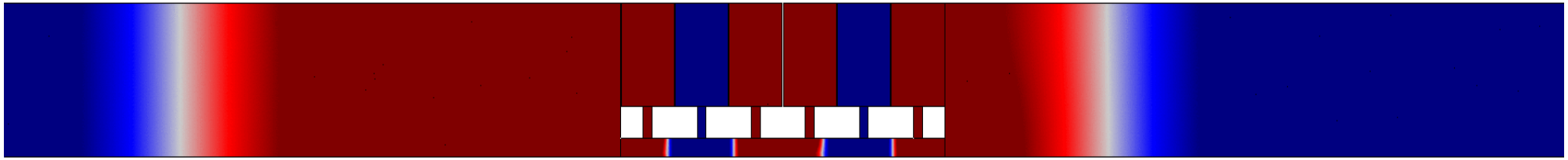
Lossless case

- Band-gap above the resonance of the HRs
- Highly dispersive propagation: slow-sound in the low frequency regime





- Cavity modes are down-shifted: The slit behaves as a deep-subwavelength resonator



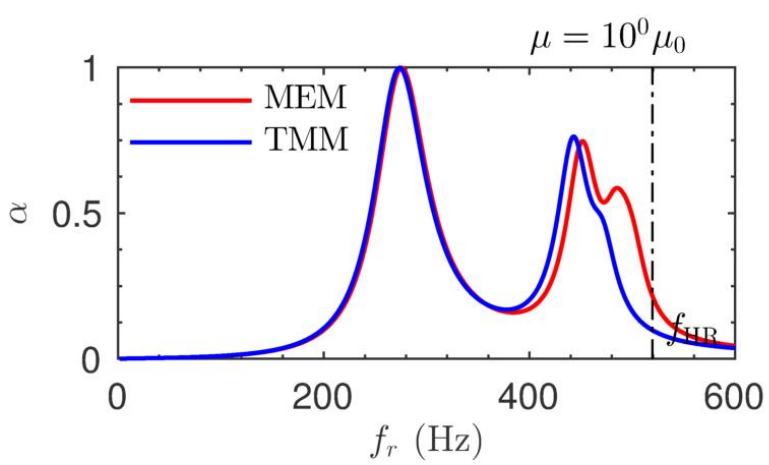
[1] N. Jiménez et. al. App Phys Lett 109(12) 121902 (2016)

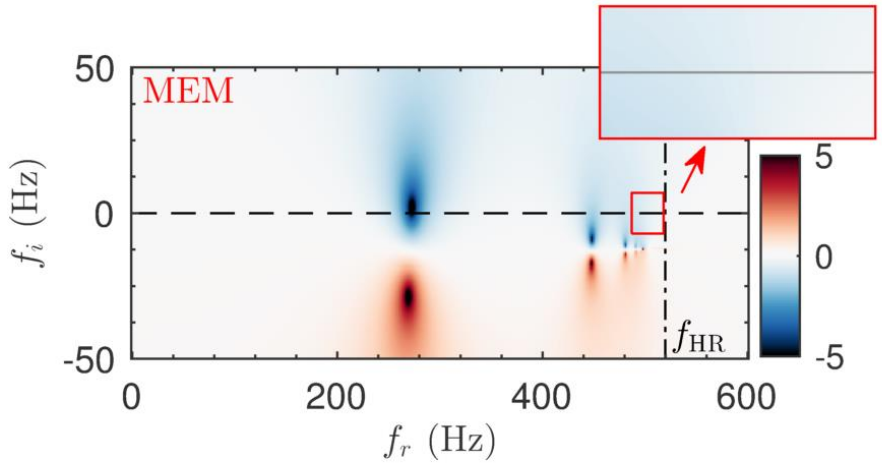
Reflection problems

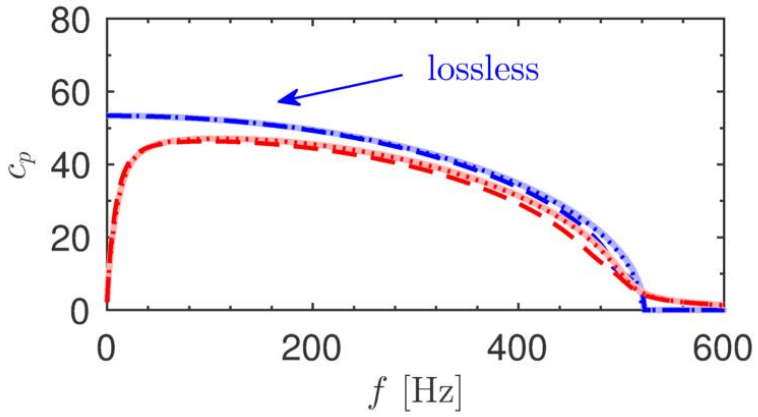
Critical coupling

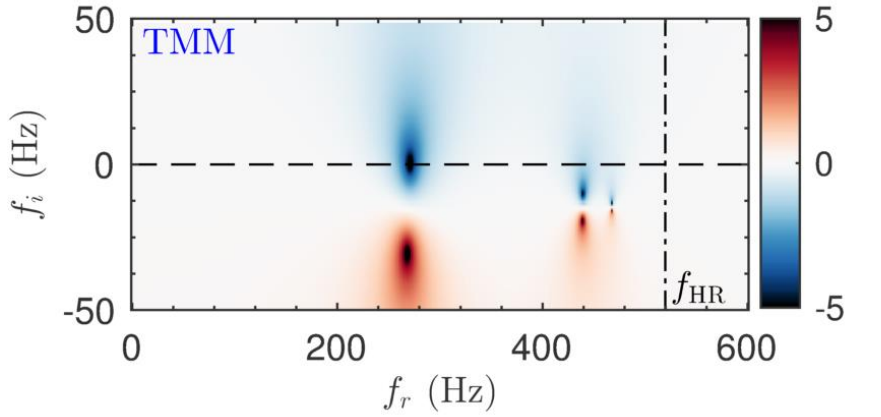
Lossy case

- Complex-frequency plane representation of the eigenvalues of the scattering matrix: N = 3 HRs









MEM: ∞ resonances:
Overestimate absorption

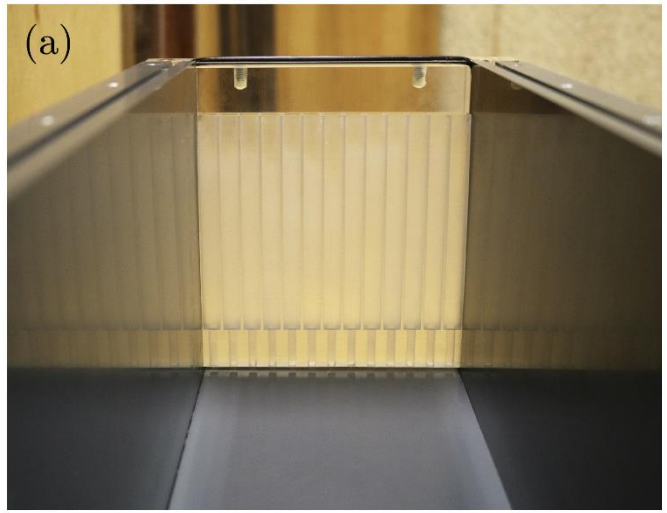
TMM: N resonances:
N peaks of absorption

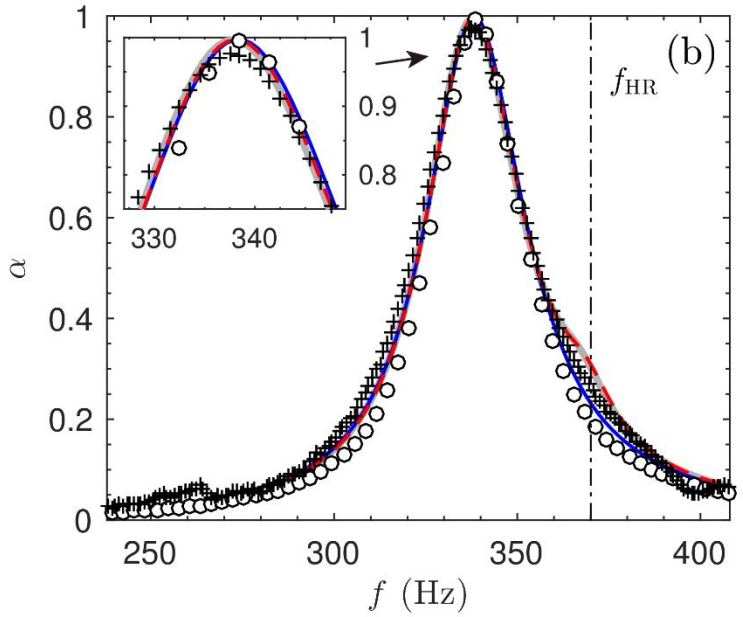
Reflection problems

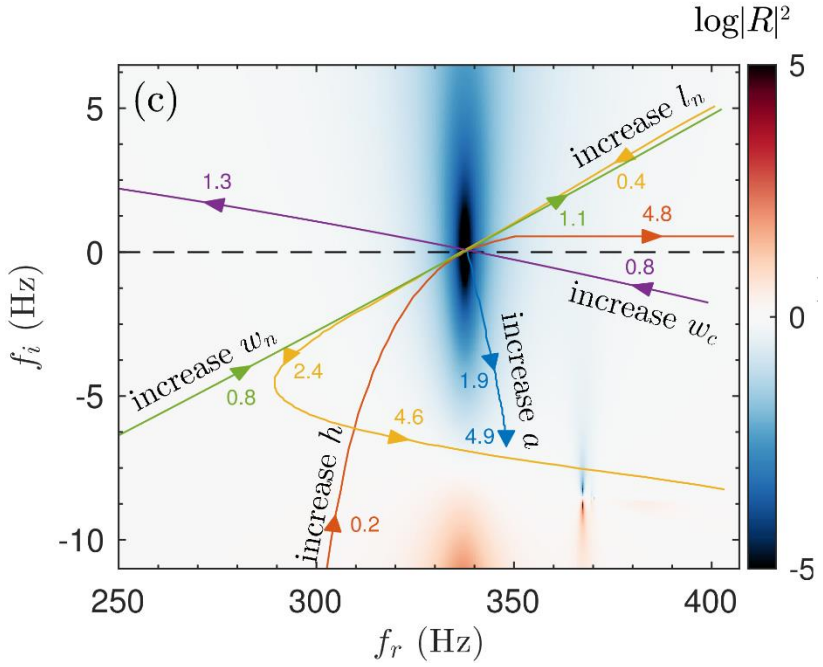
Ultra-thin panel

- Use optimization to design deep-subwavelength absorber, f = 350 Hz
- Experimental validation of a 3D printed panel
- Thickness L=λ/86, (L = 1 cm) (around L=λ/100 with thinner walls)
- **Is it perfect?** Look at the complex plane





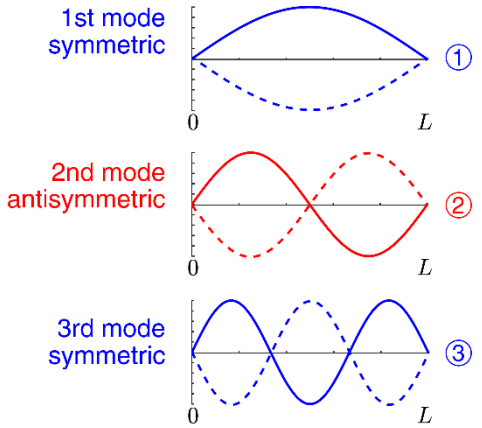


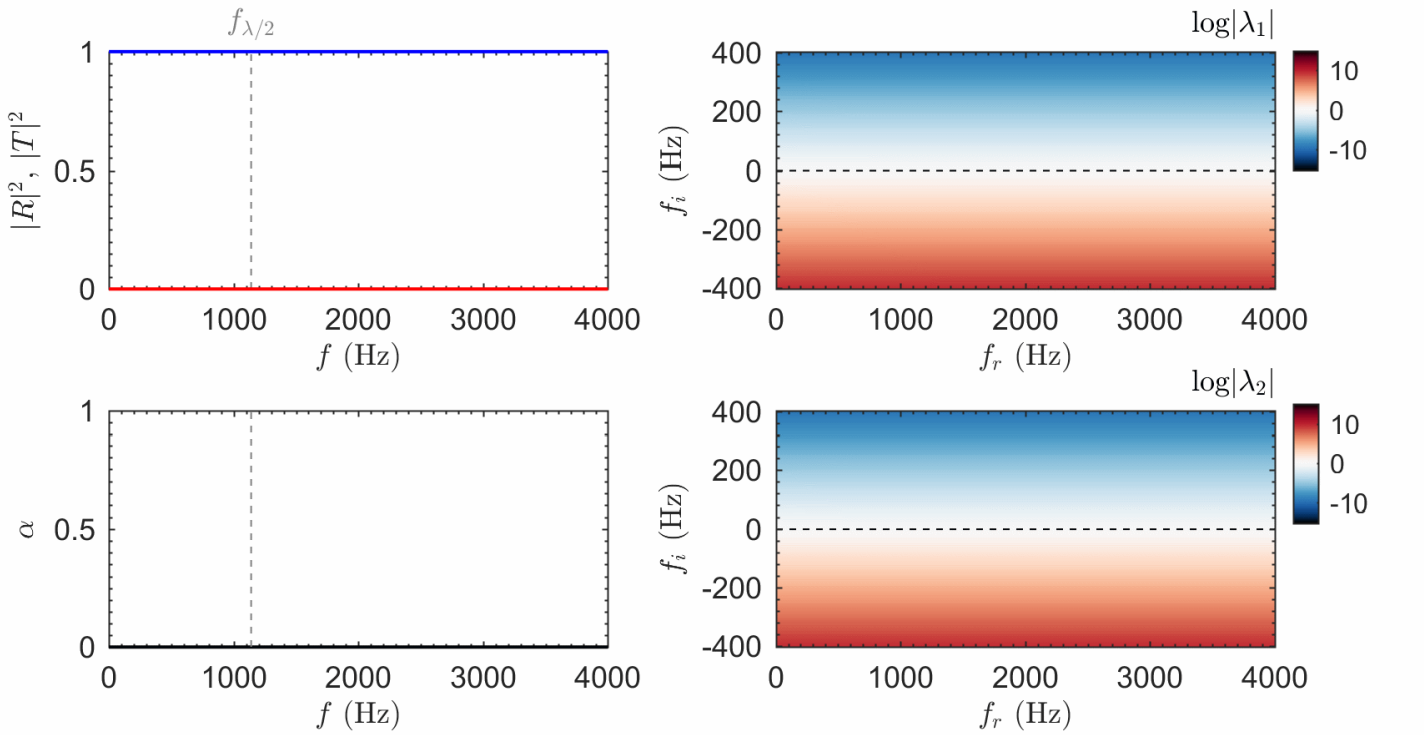


Transmission problems

Slab of absorbing material (Lossless case):

- Fabry-Pérot modes: perfect transmission / reflection
- Two eigenvalues $\lambda_1 = |R - T|$ and $\lambda_2 = |R + T|$
- The zeros of the eigenvalues are **staggered in frequency**

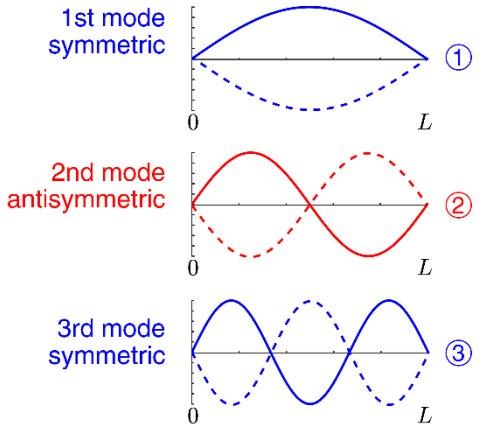


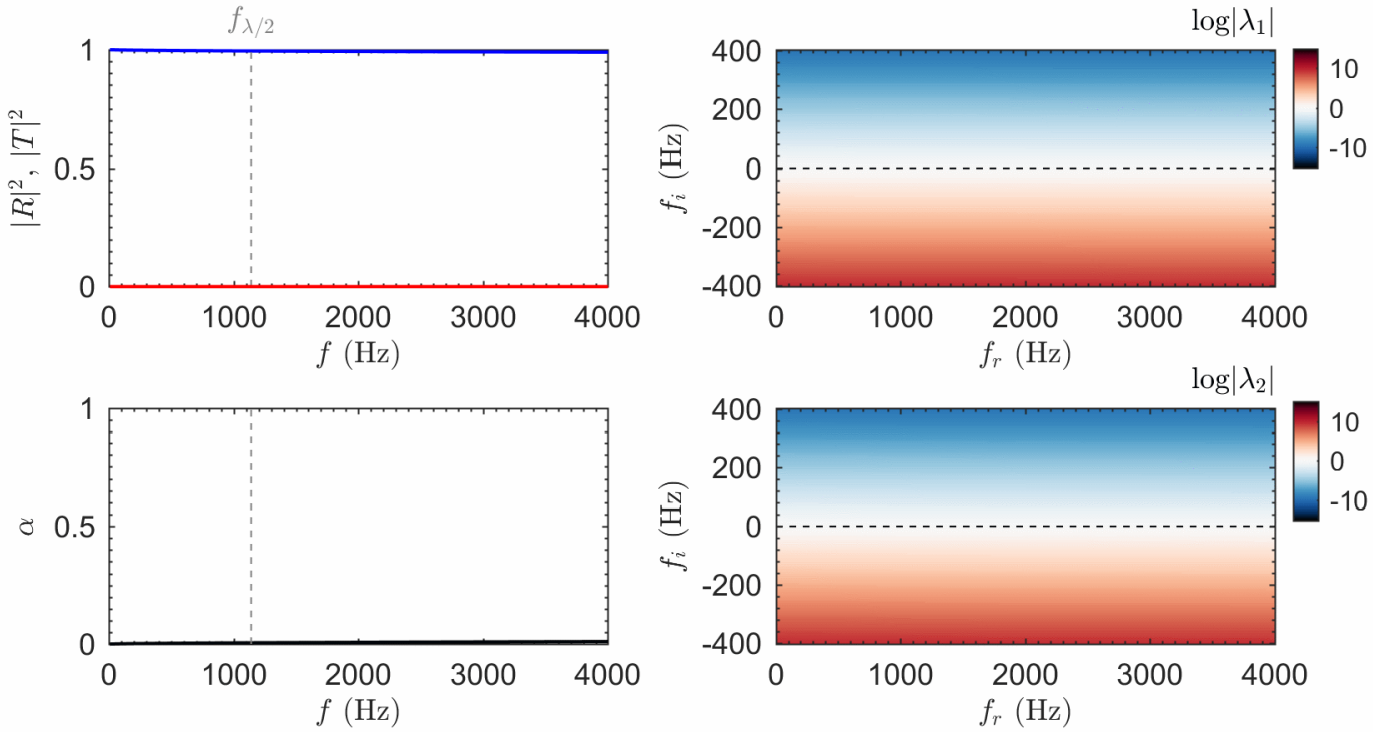


Transmission problems

Slab of absorbing material (Lossy case):

- Fabry-Pérot modes: perfect transmission / reflection
- Two eigenvalues $\lambda_1 = |R - T|$ and $\lambda_2 = |R + T|$
- The zeros of the eigenvalues are **staggered in frequency**

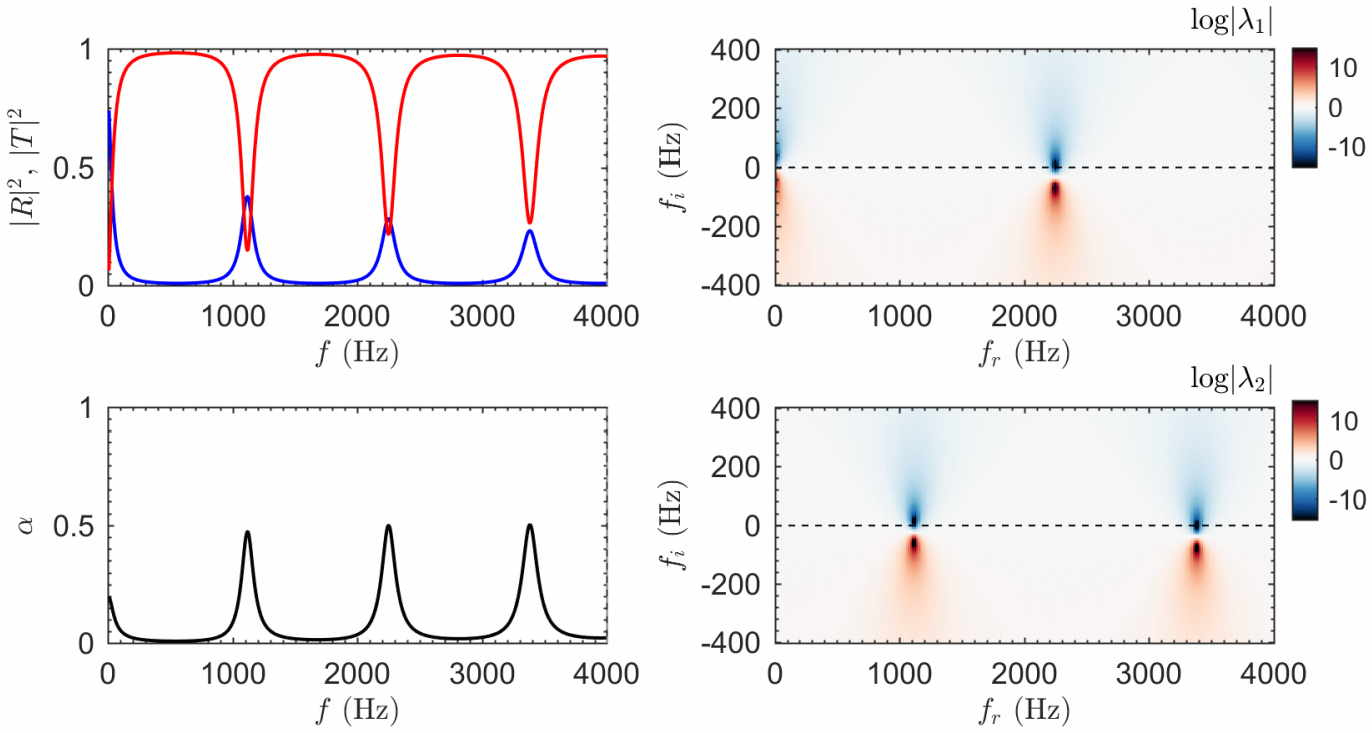


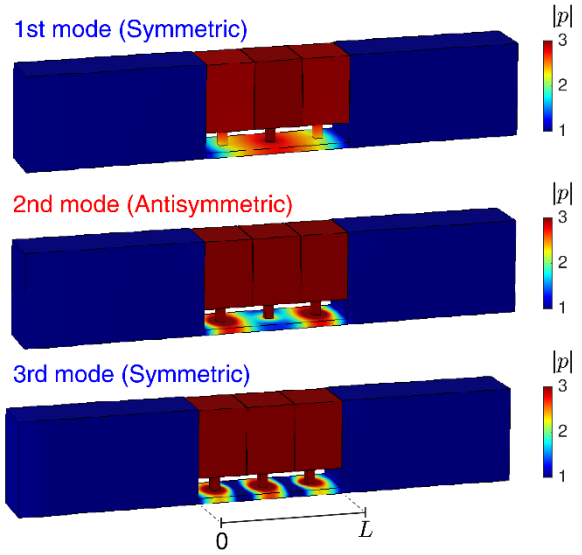


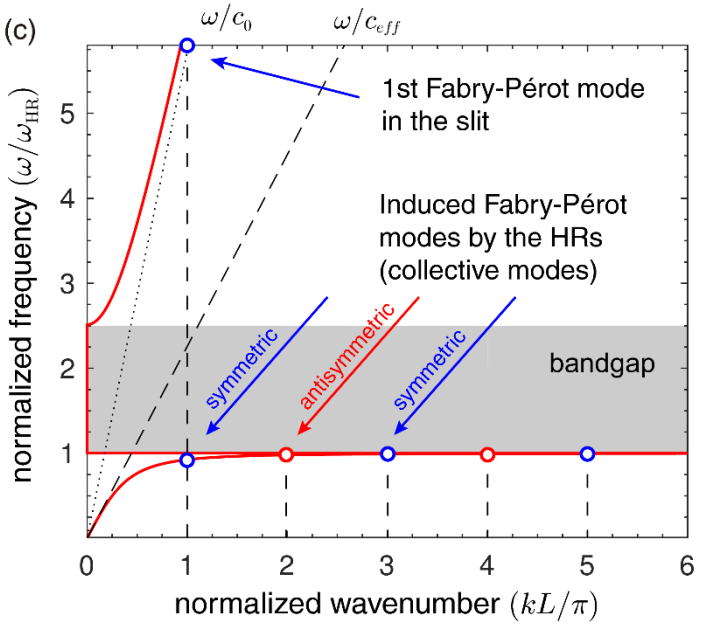
N. Jiménez, et al, Phys Rev B, 95 014205 (2017)

Slab of absorbing metamaterial

- Fabry-Pérot modes: perfect transmission / reflection
- Two eigenvalues $\lambda_1 = |R - T|$ and $\lambda_2 = |R + T|$
- The zeros of the eigenvalues still are **staggered in frequency**







Perfect absorption in transmission

Or… how to un-stagger the zero-pole structure?

1 **Use strong dispersion**

- Cavity modes can be accumulated around a flat-propagation band [1]
- Deep subwavelength monopolar resonators (or dipolar)
- Arrays of HRs, QWR, membranes and elastic plates, bubbles,

2 **Combinations of monopolar and dipolar resonances**

- Degenerate resonators [2]

3 **Symmetry breaking using non-identical resonators**

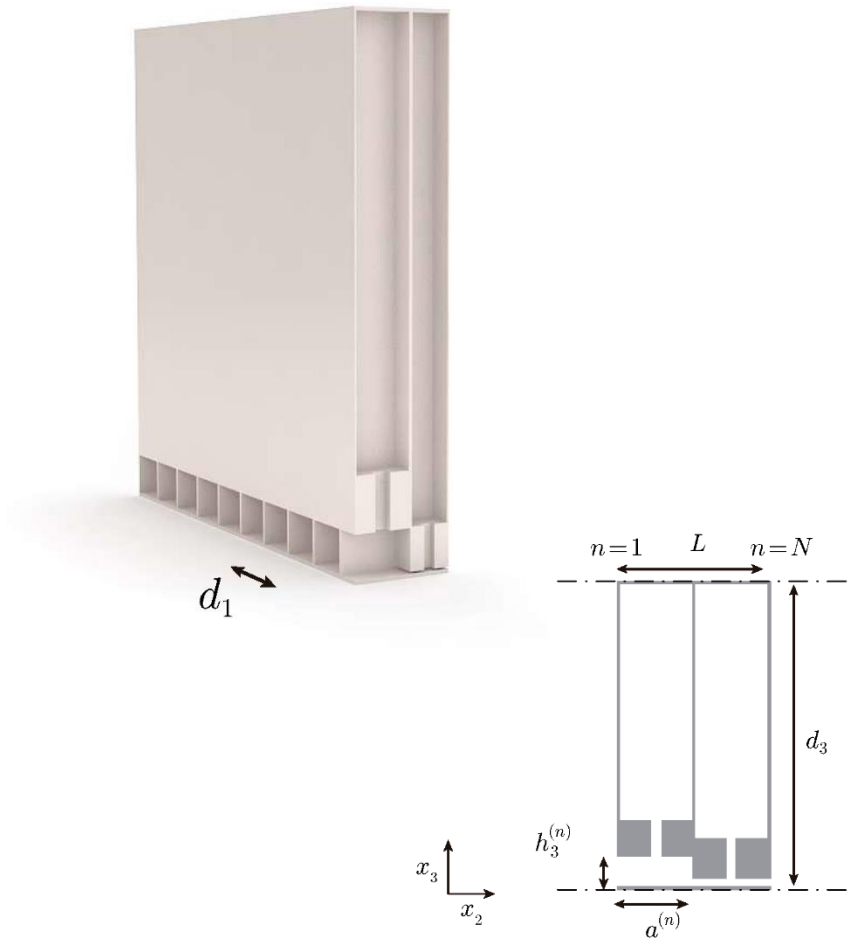
- Double-interacting resonators in waveguides [3]
- Rainbow Trapping Absorbers: broadband absorption

[1] N. Jiménez, et al, Phys Rev B, 95 014205 (2017)
[2] M. Yang, et al, Appl. Phys. Lett. 107 104104 (2015)
[3] A. Merkel, et al, Appl. Phys. Lett. 107 244102 (2015)

Symmetry breaking using non-identical resonators

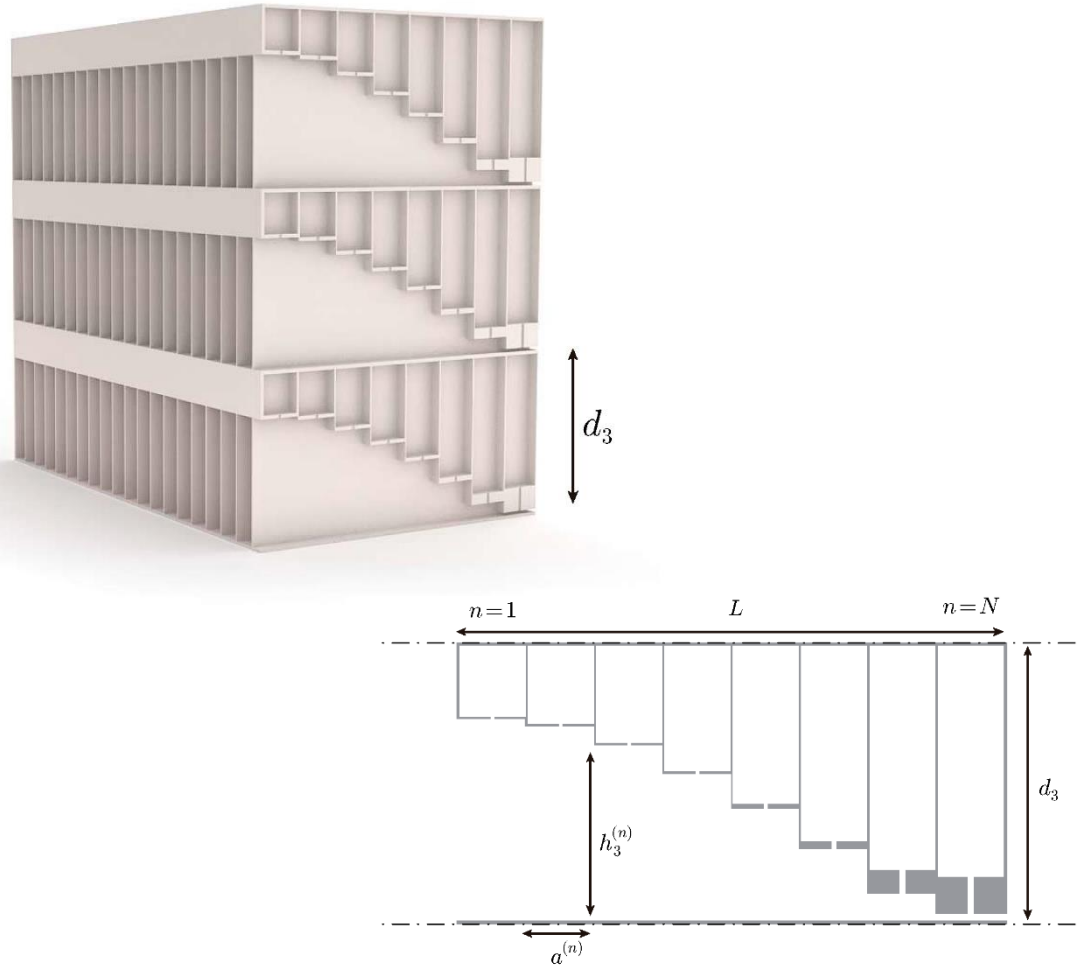
1 Subwavelength Asymmetric Panel

- Double-interacting HRs



2 Rainbow Trapping Absorber

- Array of multiple-interacting HRs



Subwavelength asymmetric panel

Panel with $N = 2$ resonators

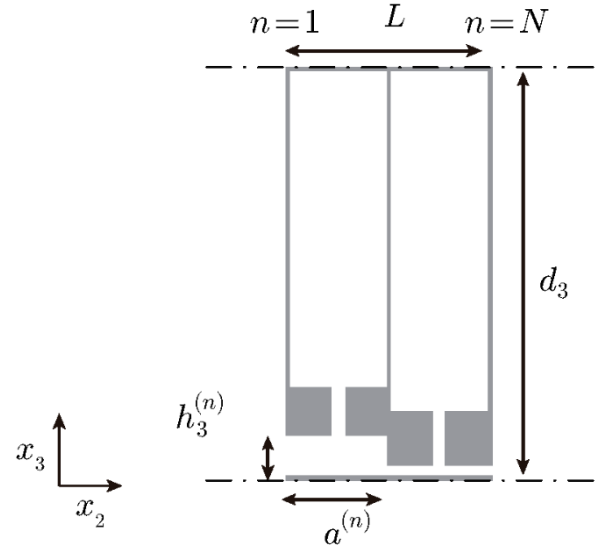
1 **Inner resonator:**

- Lower resonance frequency
- Induce a bandgap around its resonance frequency
- Transmission drops drastically: it acts as a rigid wall



2 **External resonator:**

- Slightly higher resonance frequency
- The losses are tuned to critically couple the reflection problem



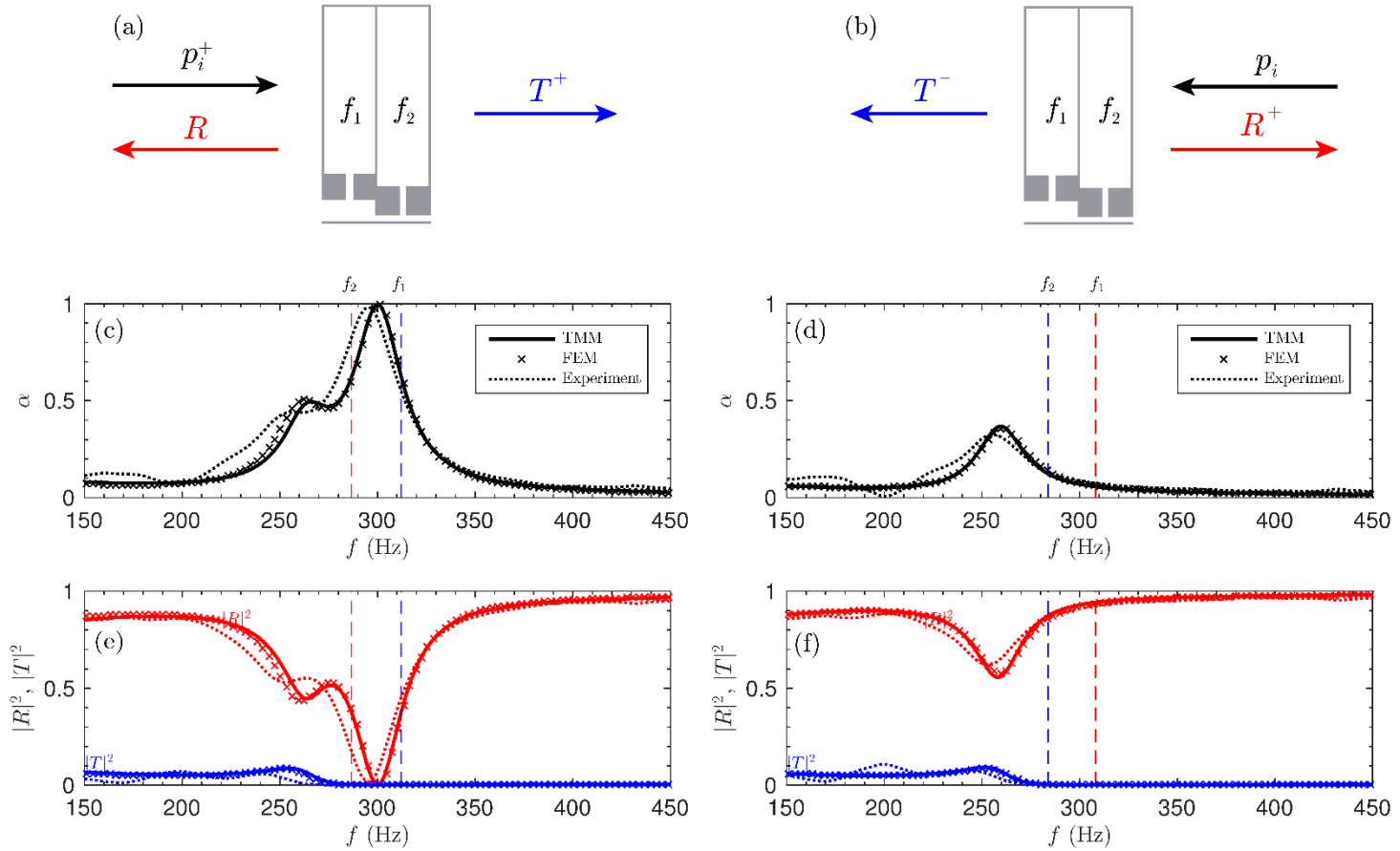
[1] Jimenez, et. al. Scientific Reports, 7, pp 13595, (2017)

Subwavelength asymmetric panel

Forward and backward propagation

[1] Jimenez, et. al. Scientific Reports, 7, pp 13595, (2017)

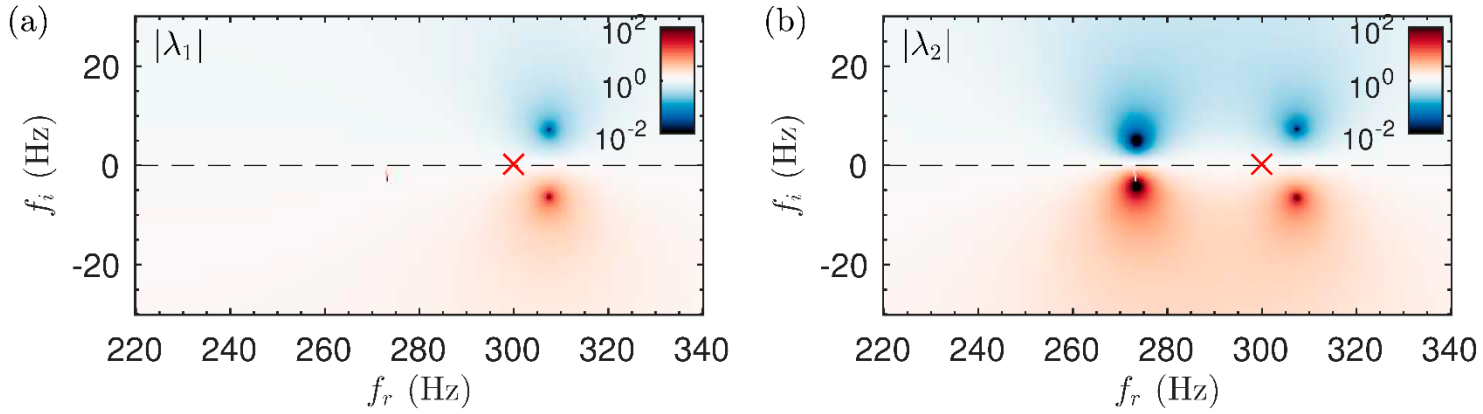
- Absorption becomes unidirectional
- Perfect absorption for a thickness of L = λ/ 40 (L = 2.5 cm)



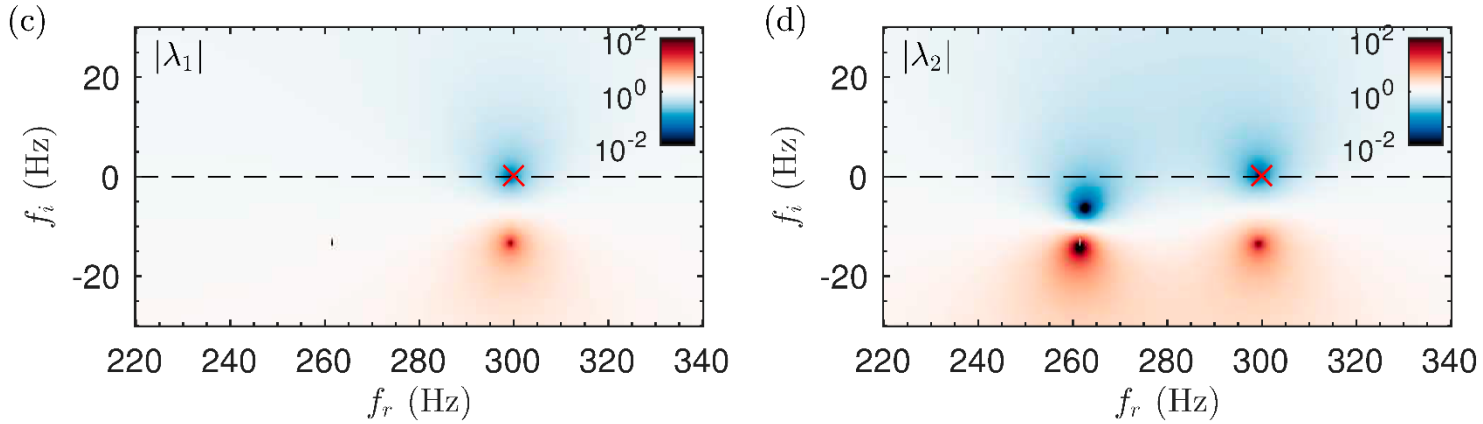
Subwavelength asymmetric panel

Complex frequency plane

- Lossless case



- Lossy case

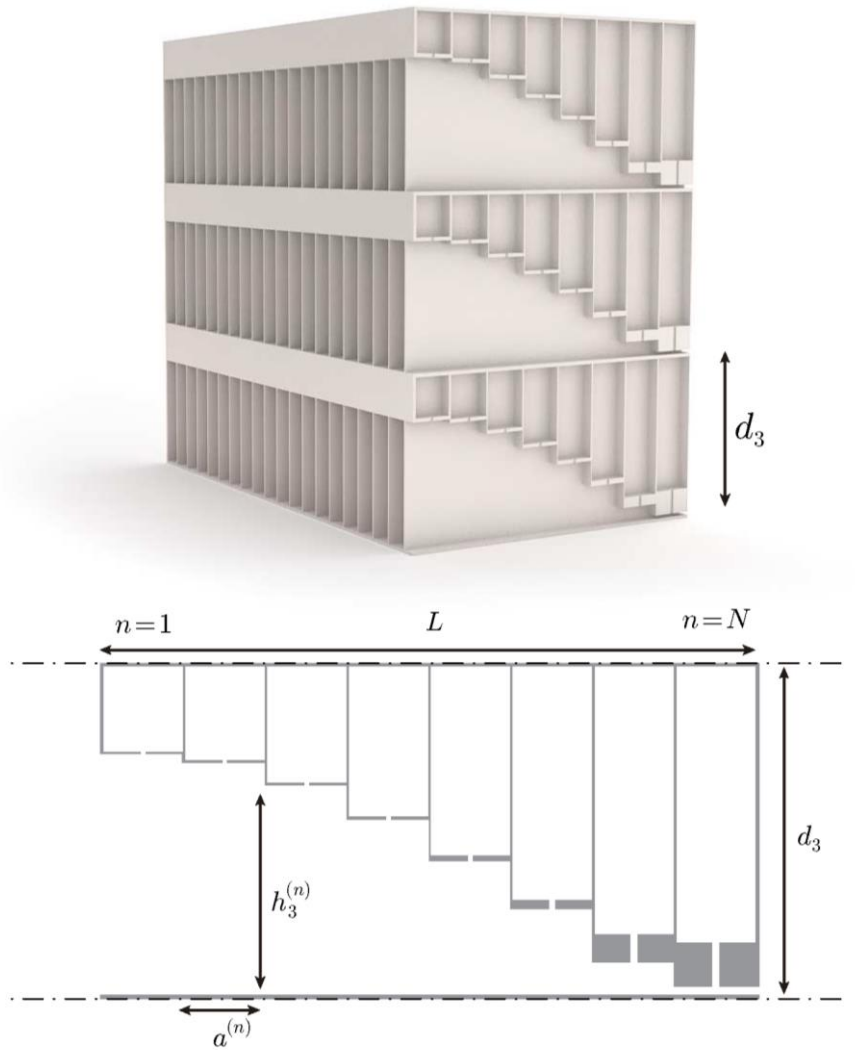


[1] Jimenez, et. al. Scientific Reports, 7, pp 13595, (2017)

Rainbow trapping absorber

Extend this idea, but in a cascade process:

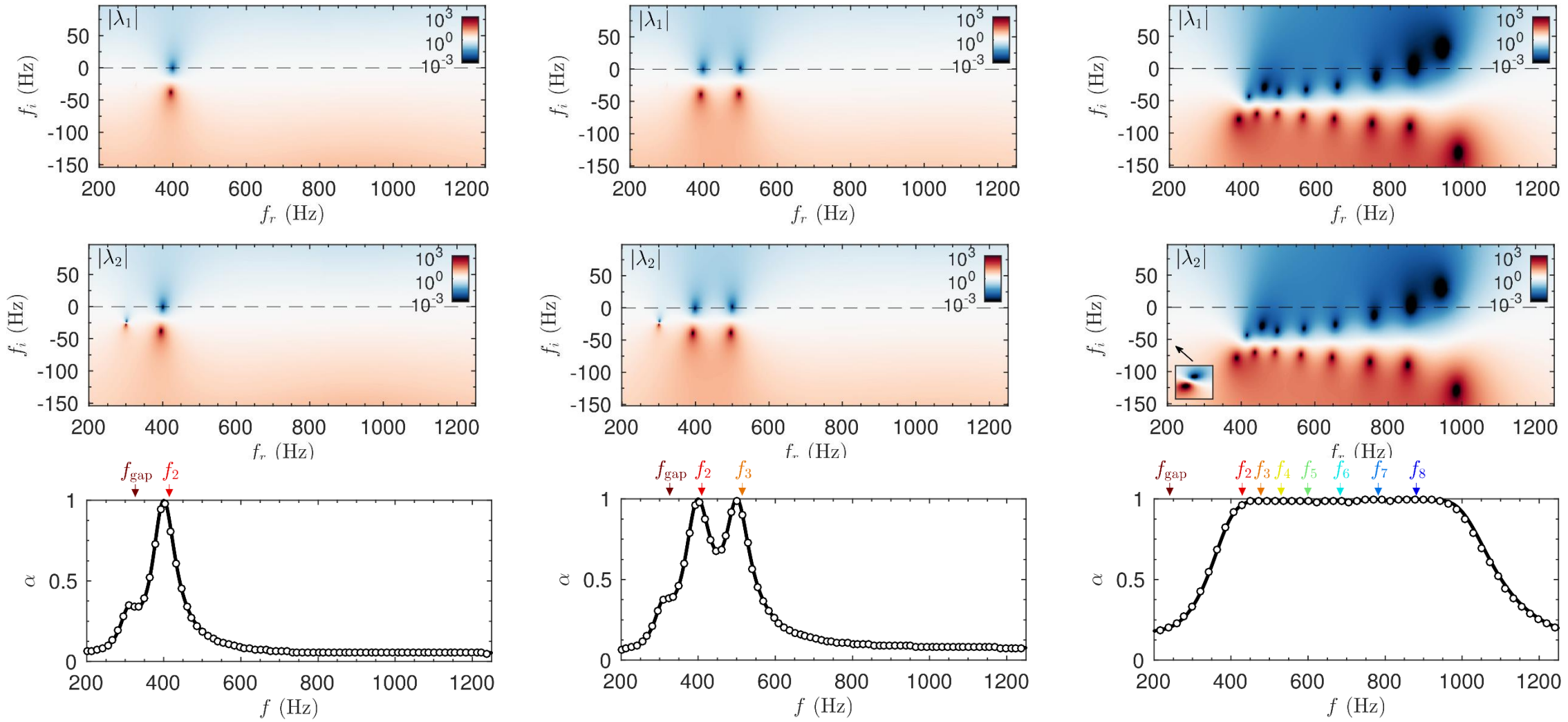
1. Resonator n
   - Lower resonance frequency: $\omega_n < \omega_{n-1}$
   - Creates a bandgap above its resonance
   - Transmission drops
2. Resonator $n-1$
   - Slightly higher resonance frequency: $\omega_n < \omega_{n-1}$
   - Critically coupled the structure at frequency ω_{n-1}
   - It creates a new band-gap at ω_n



[1] Jimenez, et. al. Scientific Reports, 7, pp 13595, (2017)

Rainbow trapping absorber

Asymmetric broadband and perfect absorption:

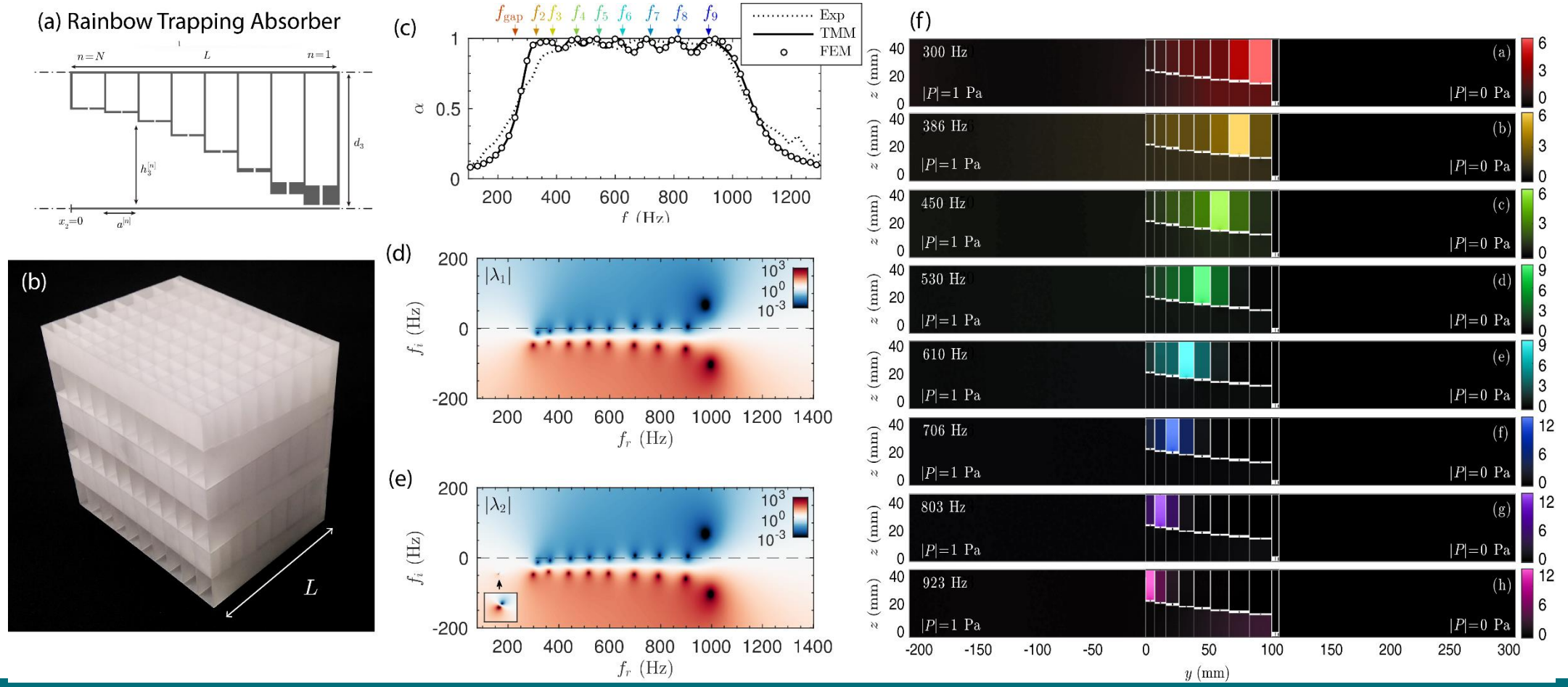


Rainbow trapping absorber

Experimental verification

[1] Jimenez, et. al. Scientific Reports, 7, pp 13595, (2017)

- Overlap of N = 8 peaks of perfect absorption, L = λ/12 (L = 10 cm)
- Experimental limitation: 3D printing device step 0.1 mm



Conclusions

1 Subwavelength Perfect Absorbers

- *Reflection problems*: Ultra-thin panel: L = λ/86
- *Transmission problems*: Subwavelength asymmetric panel: L = λ/40
- *Broadband problem in transmission*: Rainbow trapping absorber: L = λ/12

2 Metamaterials based on Helmholtz resonators

- *Versatile*: broad range of problems
- *Helmholtz resonators*: age, harsh environments
- *Open and void materials*: lightweight, transparent to light, air flows
- *High degree of freedom*: multifunctional optimization, structural, aesthetics
- *Manufacture*: complex 3D structures ...

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