Strongly Focused Vortex Beams by using Flat Fresnel-Spiral Lenses

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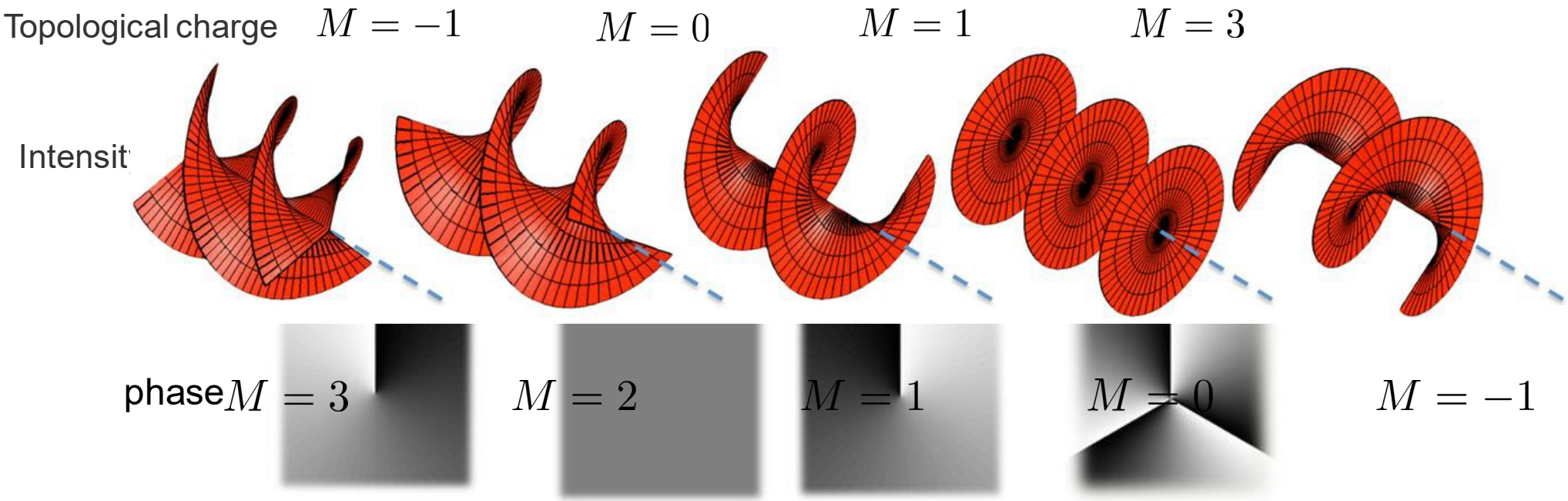
176th Meeting of the Acoustical Society of America, 2018 Acoustics Week in Canada, Victoria, BC, Canada, 5-9 November 2018

Introduction

Acoustic vortex beams

- Beams presenting a phase dislocation at the axis
- The phase of the field rotates l times with each turn
- Example, Gaussian beam in cylindrical coordinates:

$$p(\theta, r, z) = P_0 \exp\left(-\frac{r^2}{a^2}\right) \exp\left(-ik_z z\right) \exp\left(-ik_z \frac{r^2}{2F}\right) \exp\left(-iM\theta\right)$$



Introduction

Applications

Acoustic communications

- Robust and fast information coding
- Underwater communications / modems

[1] C. Shi, *et. al*. Proc. Nat. Ac. Sci., 114(28), 7250-7253 (2017)

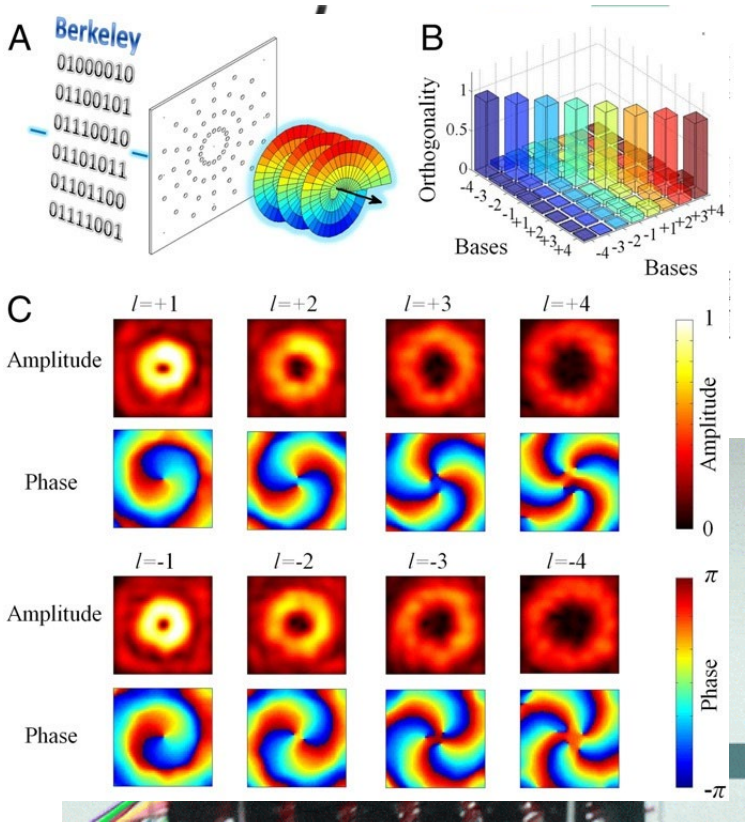
Particle manipulation

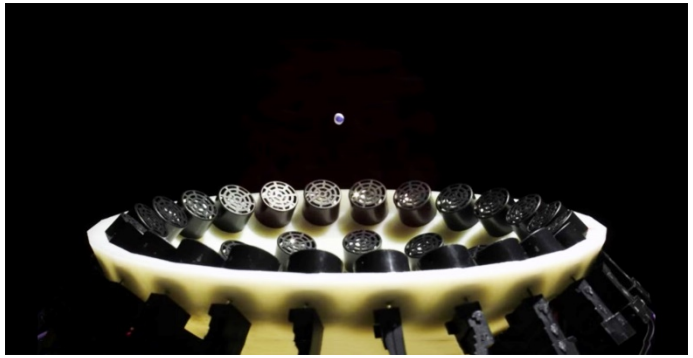
- Trapping, manipulation, rotation
- Levitation
- Acoustic tweezers

[2] K. Volke-Sepúlveda, *et al.* PRL. 100, 024302 (2008)

[3] A. Marzo, *et al.* Nat. Commun. 6, 8661 (2015)

[4] D. Baresch, *et al.* PRL, 116(2), 024301 (2016)





Vortex beam generation methods

Active methods

Active arrays

Passive methods

Helicoidal surfaces

- Helicoidal sources
- Photo-acoustics

Phase-engineered surfaces

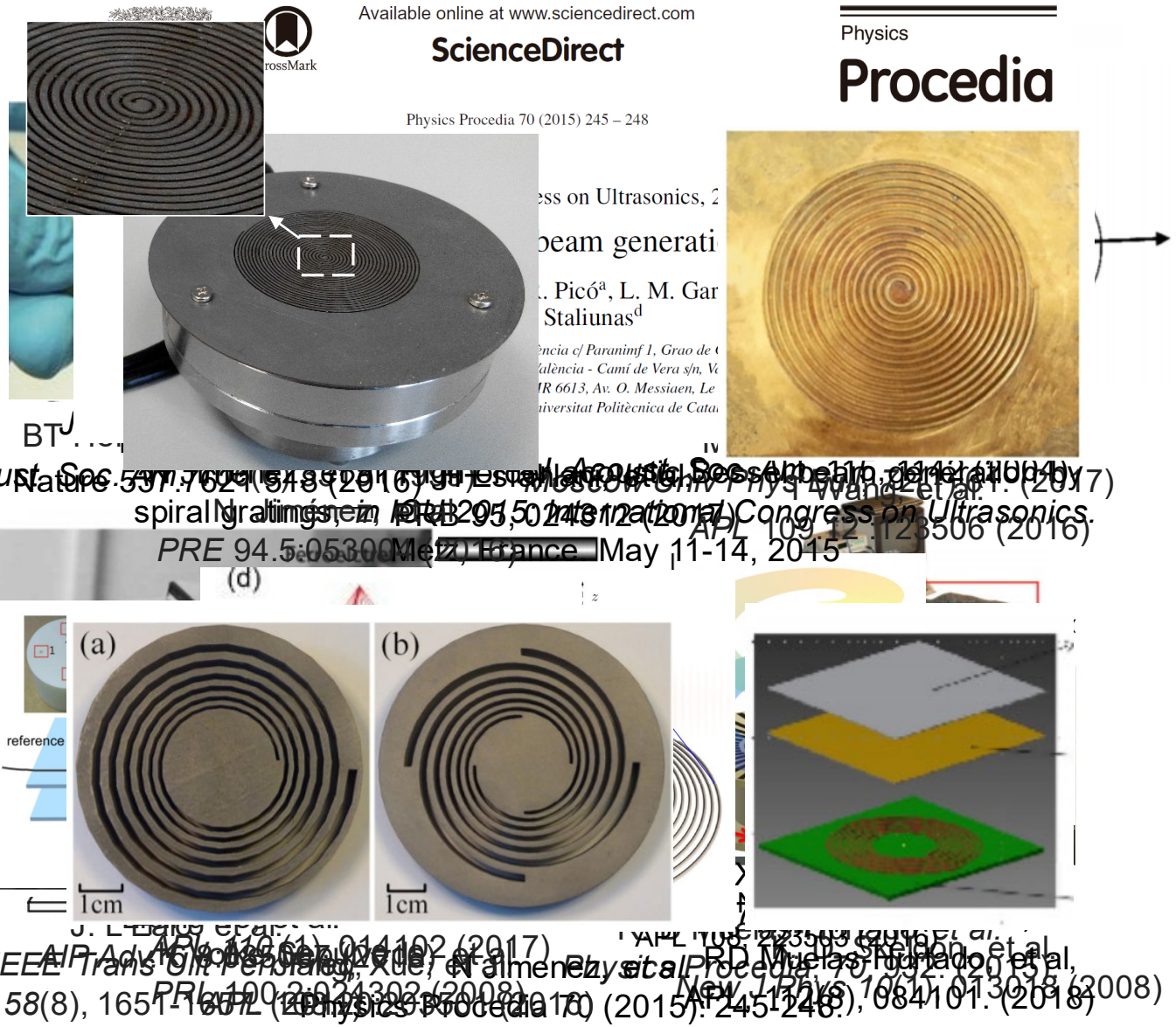
- Phase plates / holograms
- Metamaterials

Space coiled paths

- Helicoidal / delay lines / labyrinthine structur
- Leaky antennas

Chiral diffraction gratings

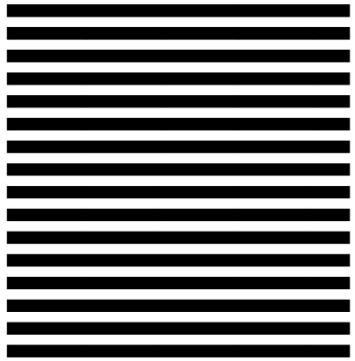
- Spiral diffraction gratings / active gratings



Wavefront control with diffraction gratings

Diffraction grating

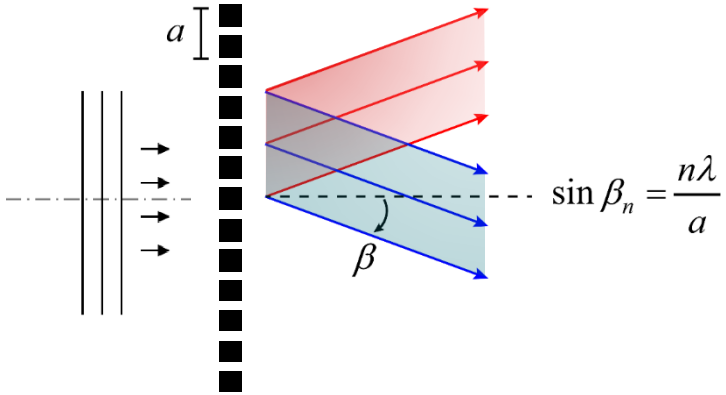
- Waves are diffracted by a periodic distribution of scatters / slits
- Waves diffract at angles given by $\pm\beta$

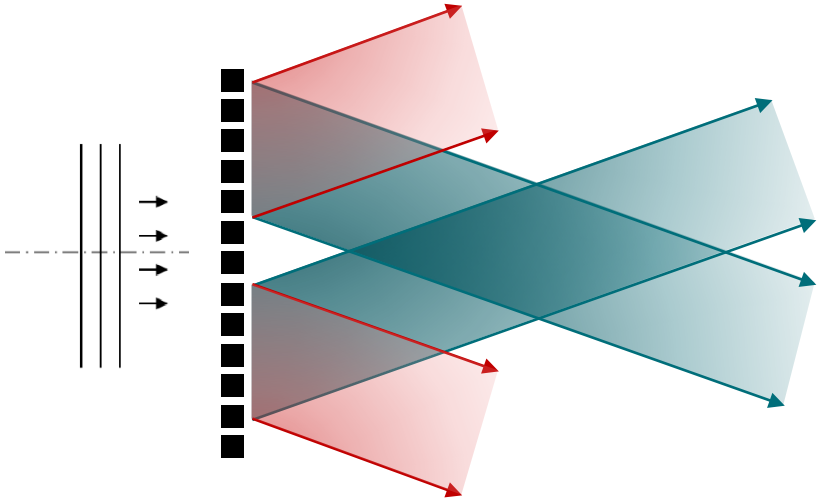


Diffraction grating



Axisymmetric diffraction grating

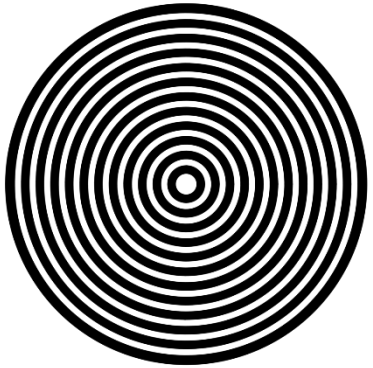


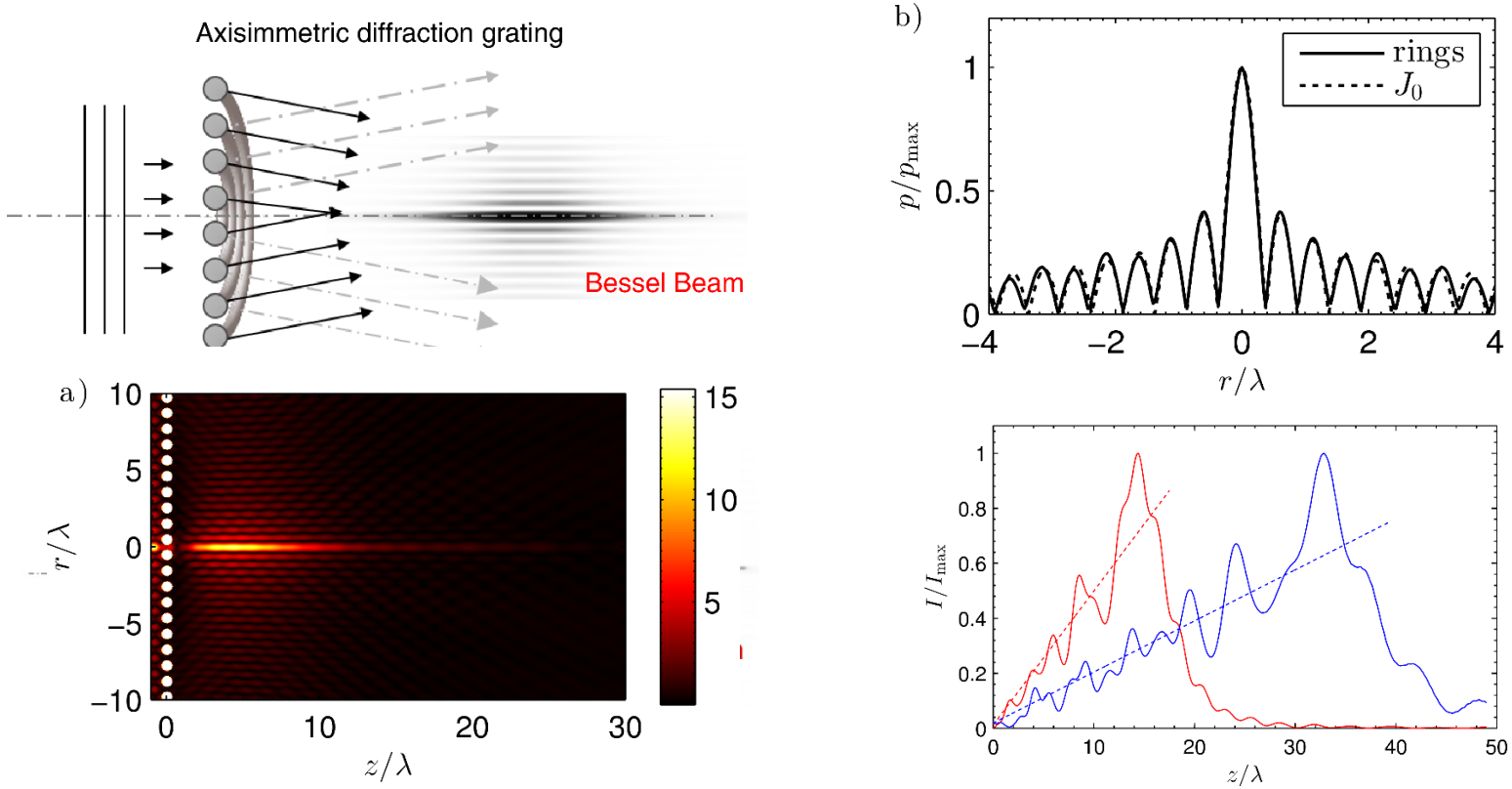


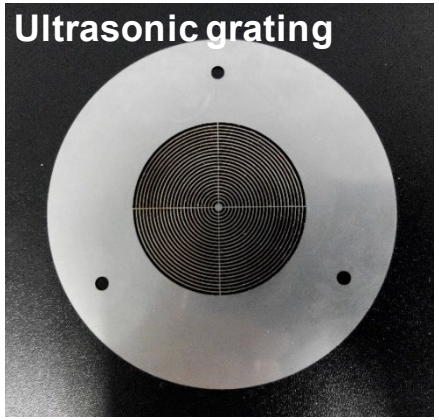
Wavefront control with diffraction gratings

Axisymmetric diffraction grating

- Waves diffract at angles given by $\pm\beta$
- Conical wavefront is produced: one diverging, one converging to axis
- Bessel beam of order zero are produced







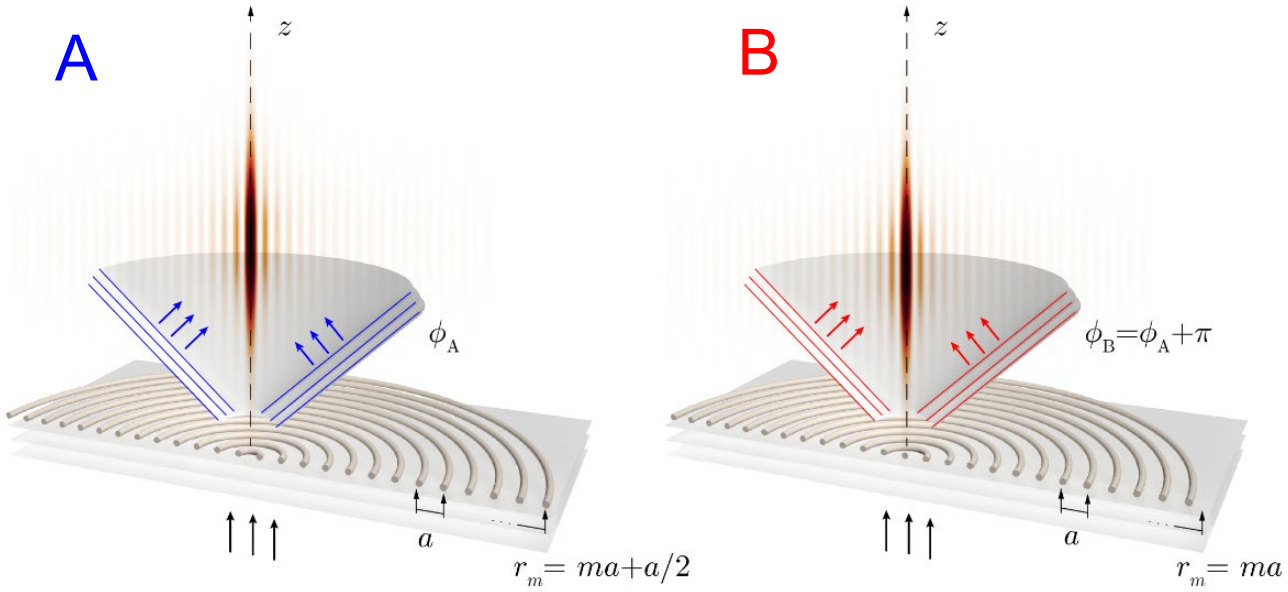


N Jiménez et al. *Acoustic Bessel-like beam formation by an axisymmetric grating*. EPL 106 24005 (2014)

Wavefront control with diffraction gratings

From axisymmetric to chiral-symmetry beams

- Consider two staggered lenses



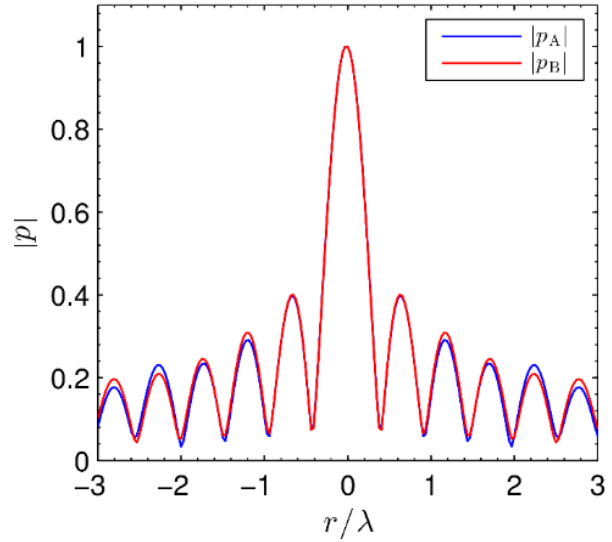
- The phase of the diffracted waves is given by $\Delta\phi = \dfrac{2\pi\Delta r}{a}$

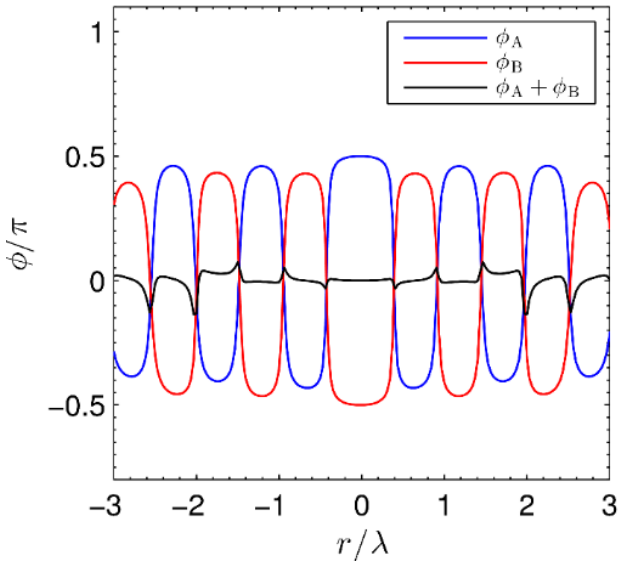
- **What grating inverts phase for each half turn?**

$$\phi(\theta) = \phi(\theta+\pi) \pm \pi, \longrightarrow \quad \Delta\phi = \Delta\theta, \longrightarrow \quad \frac{\Delta r}{\Delta\theta} = \frac{a}{2\pi}, \longrightarrow$$

Archimedes spiral

$$r(\theta) = \frac{a}{2\pi}\theta.$$





Chiral diffraction gratings

Archimedes spiral

- Polar curve is given by:

$$r(\theta) = \pm \frac{a}{2\pi}\theta$$

- Bessel beam of order M :

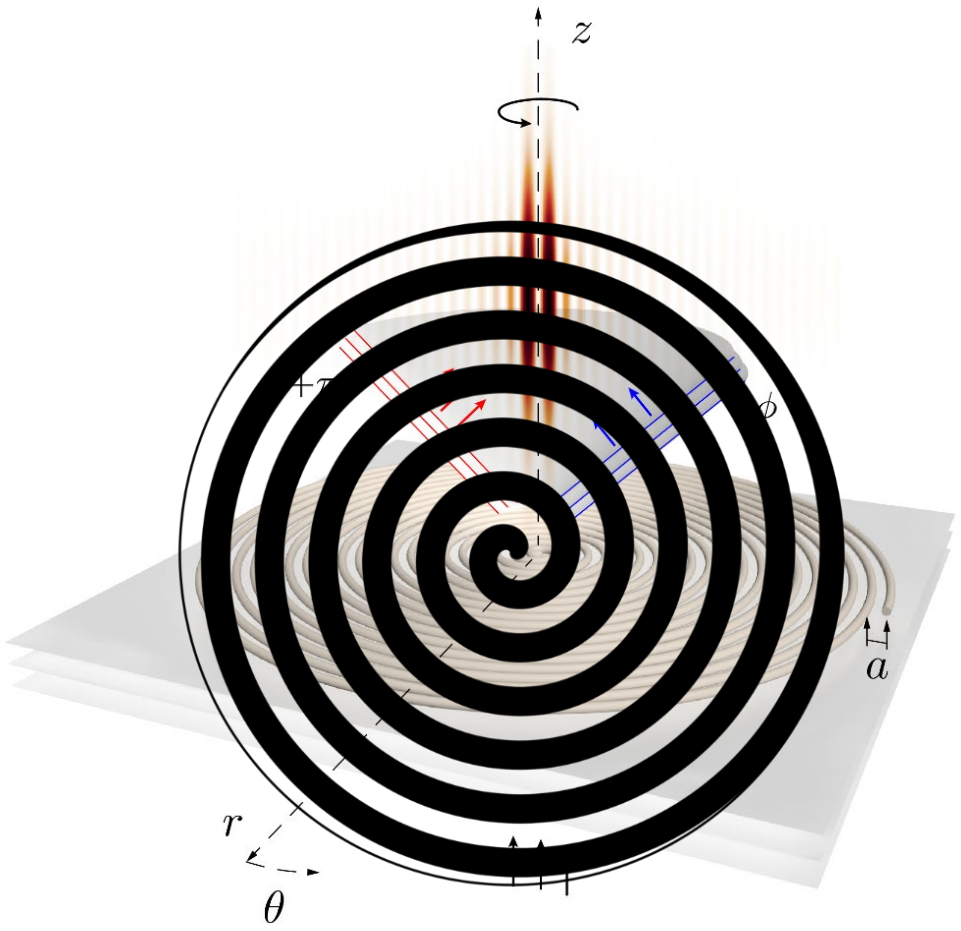
$$\psi(r, \theta, z) = \psi_0 J_M(k_r r) \exp(ik_z z \pm iM\theta)$$

- Diffraction grating angles:

$$\sin \beta_n = \frac{n\lambda}{a} \quad \begin{cases} \cos\beta = k_z/k \\ \sin\beta = k_r/k \end{cases}$$

- Field as a function of grating parameters

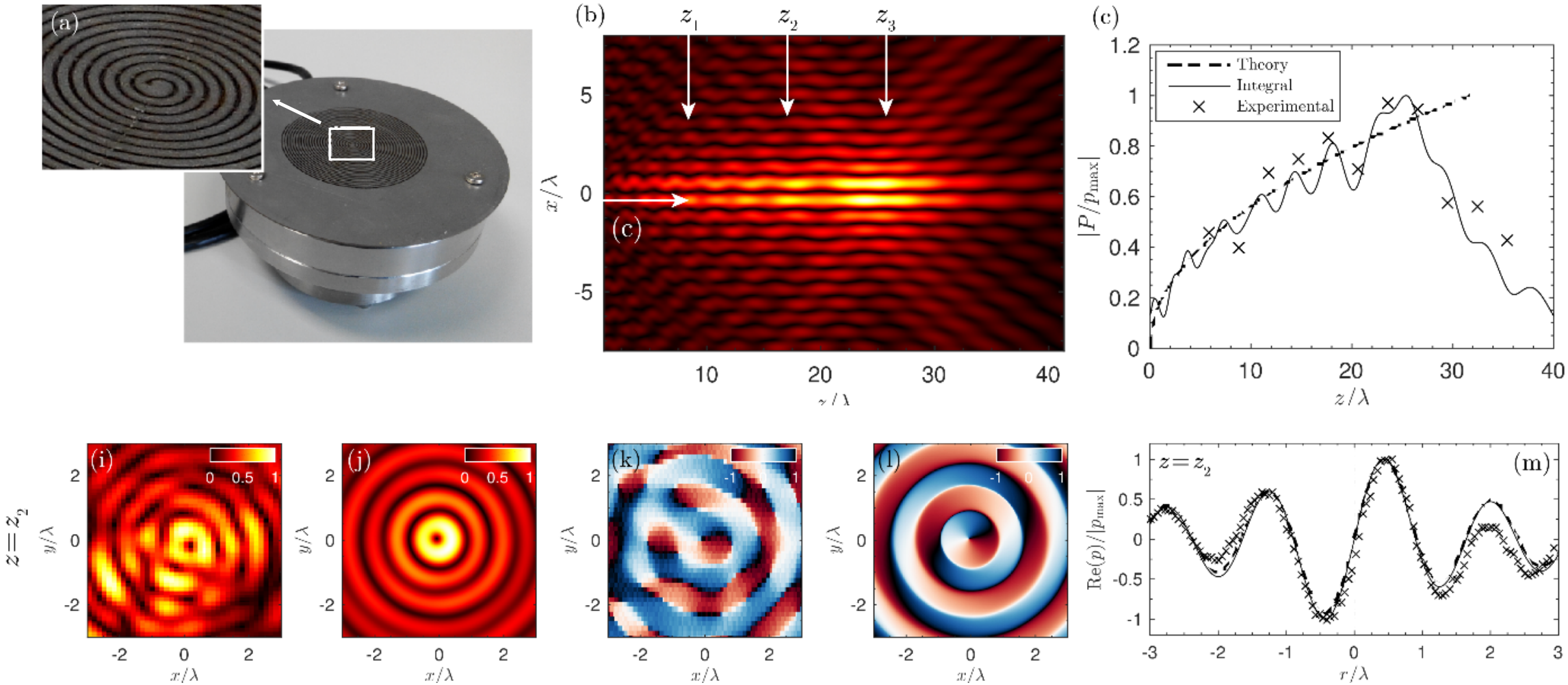
$$\psi(r, \theta, z) = \psi_0 J_M\left(\frac{2\pi}{a} r\right) \exp\left(i \frac{2\pi n}{a \tan\beta} z \pm iM\theta\right)$$



N Jiménez, *Formation of high-order Bessel beams by spiral diffraction gratings*. PRE 94 (5)053004 (2016)

Chiral diffraction gratings

Experimental validation



N Jiménez, *Formation of high-order Bessel beams by spiral diffraction gratings*. PRE 94 (5)053004 (2016)

High order Bessel beams

Control of the topological charge

- Multiple arms

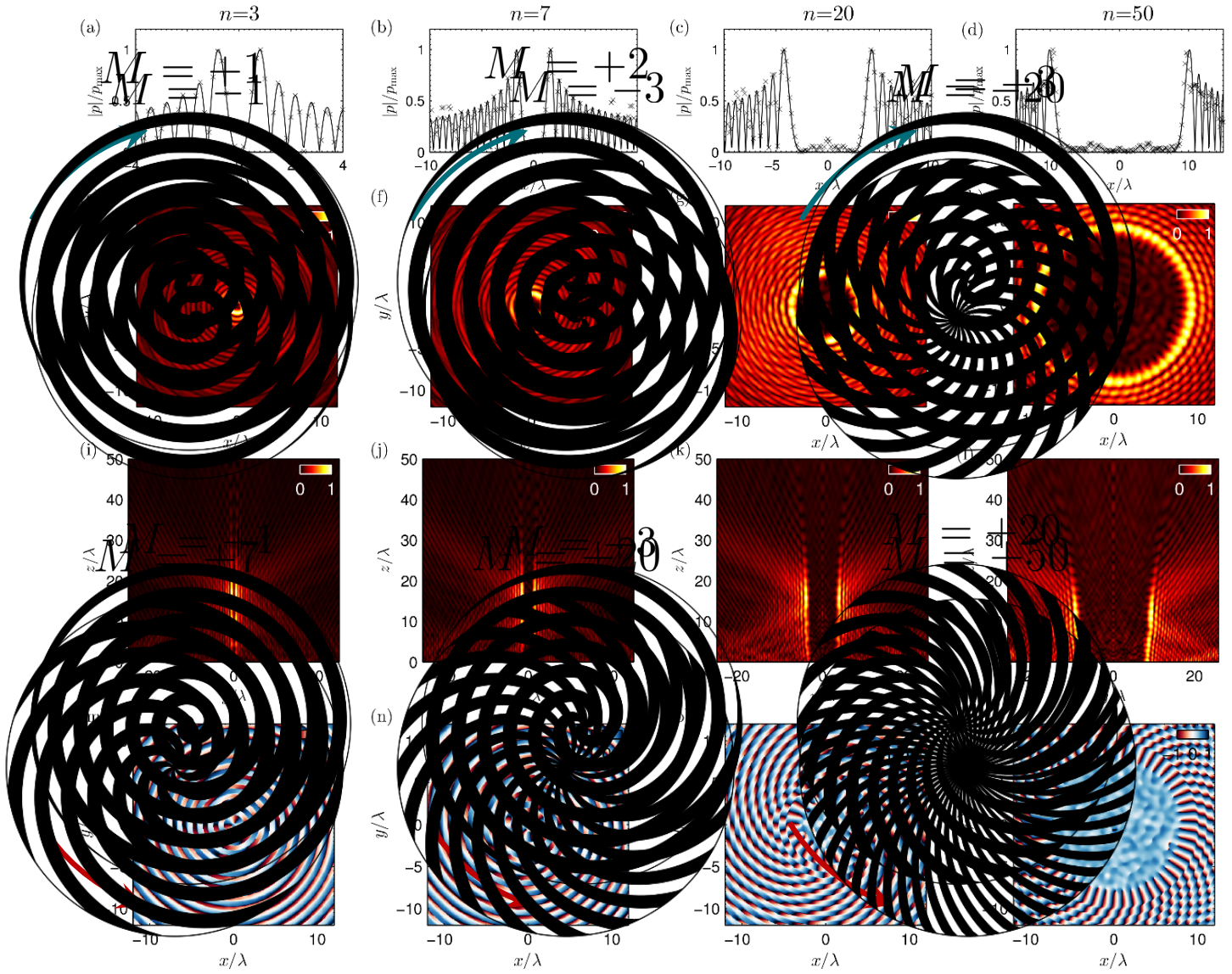
$$r_m(\theta) = \pm M \frac{a}{2\pi}\theta + ma$$

- Diffraction order

$$\sin(\beta_n) = \pm n \frac{\lambda}{a}$$

- Polarity of the charge:

  Rotation direction $\pm M$



N Jiménez, *Formation of high-order Bessel beams by spiral diffraction gratings*. PRE 94 (5)053004 (2016)

Focused vortex beams

Focusing control

- Generalized Archimedean spiral:

$$r\left(\theta\right) = \pm\frac{a}{2\pi}\theta^{\gamma}$$

- Focus of the locally-diffracted waves

$$F(\theta)^2 = \left[\frac{r\left(\theta+2\pi\right)^2 - r(\theta)^2}{2\lambda} - \frac{\lambda}{2}\right]^2 - r(\theta)^2,$$

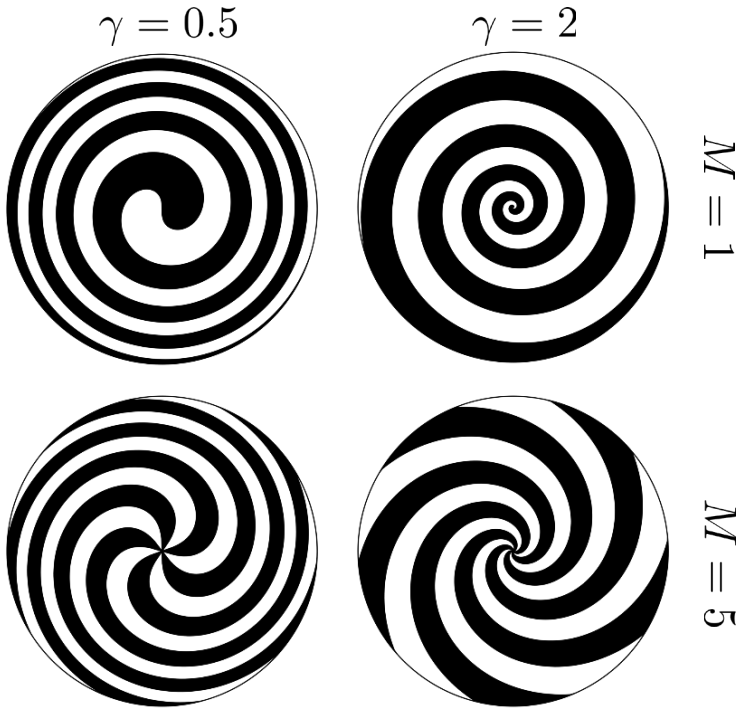
- Considering an aperture of

$$R_F = (a/2\pi)\theta_F^{\gamma}$$

- The grating step to focus at a distance *F* is:

$$a = 2\pi\sqrt{\frac{\lambda\left(2\sqrt{F^2\left(\theta_F^{2\gamma} - (\theta_F+2\pi)^{2\gamma}\right)^2 + \lambda^2\theta_F^{2\gamma}(\theta_F+2\pi)^{2\gamma}} + \lambda\left(\theta_F^{2\gamma} + (\theta_F+2\pi)^{2\gamma}\right)\right)}{\left(\theta_F^{2\gamma} - (\theta_F+2\pi)^{2\gamma}\right)^2}}$$

Fermat's spiral Lottus curve

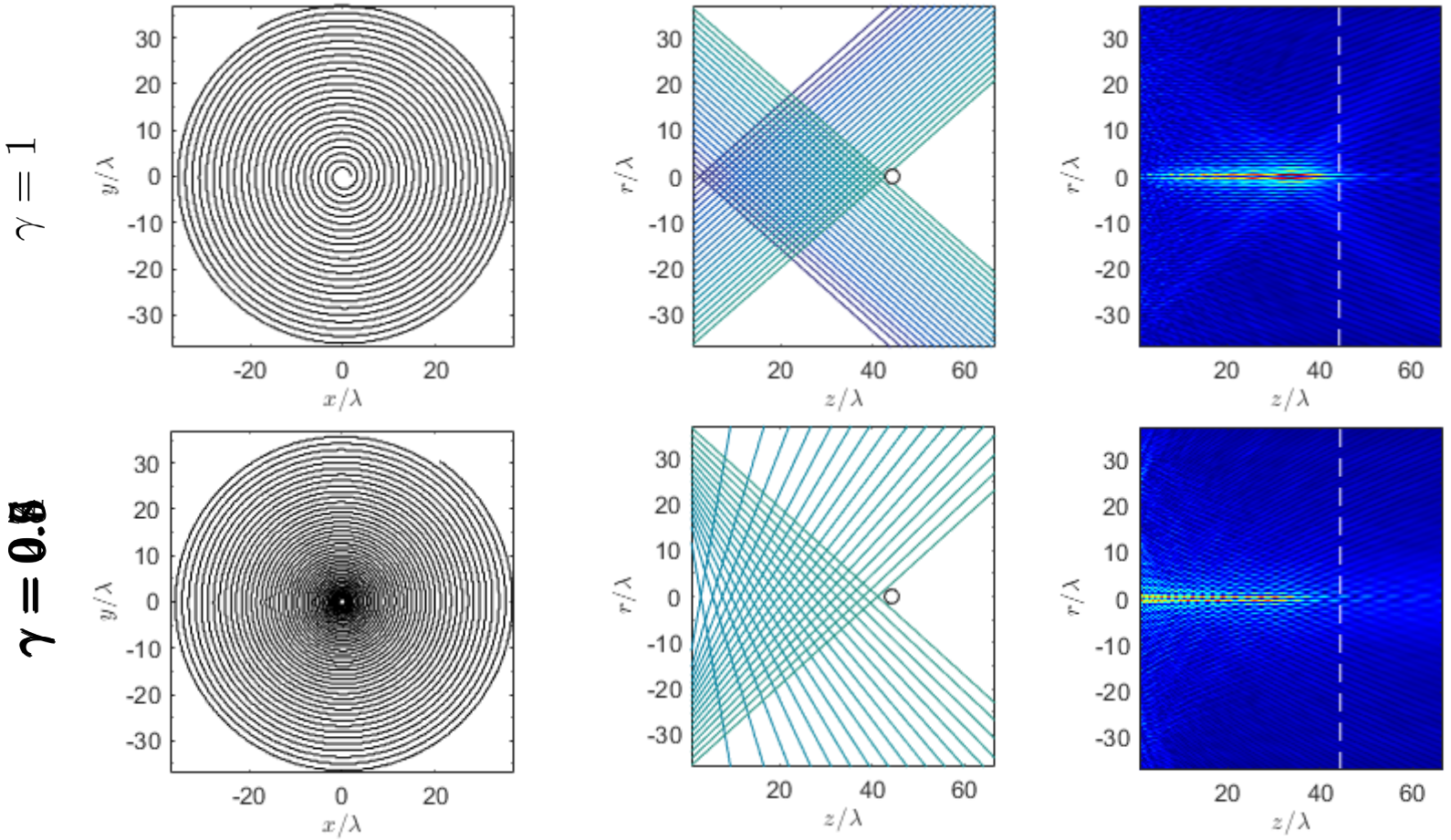


N Jiménez, *Sharp acoustic vortex focusing by Fresnel-spiral zone plates*. APL 112 (20) 204101 (2018)

Focused vortex beams

Focusing control

- Generalized Archimedean spiral: $r(\theta) = \pm \frac{a}{2\pi} \theta^{\gamma}$



N Jiménez, *Sharp acoustic vortex focusing by Fresnel-spiral zone plates*. APL 112 (20) 204101 (2018)

Geometrically-optimal focused vortex beams

Fres

- F

r

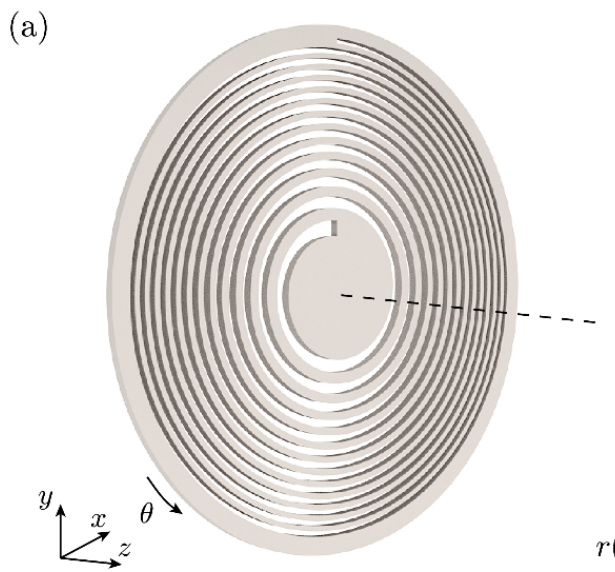
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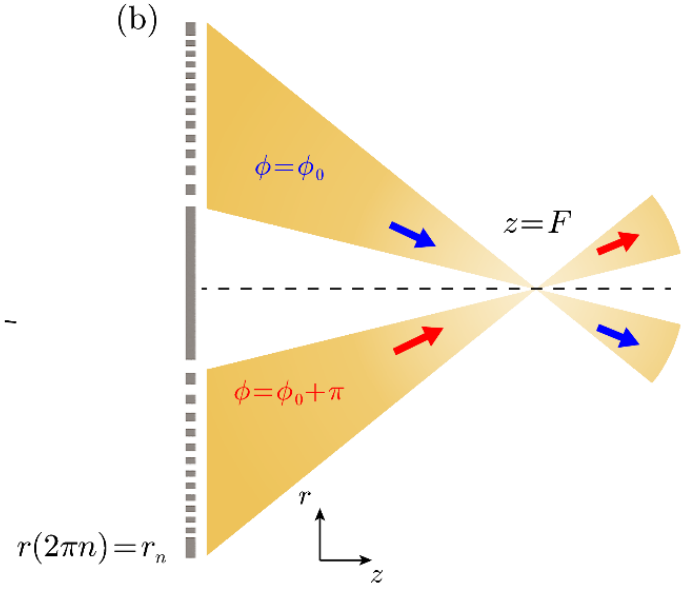
- F

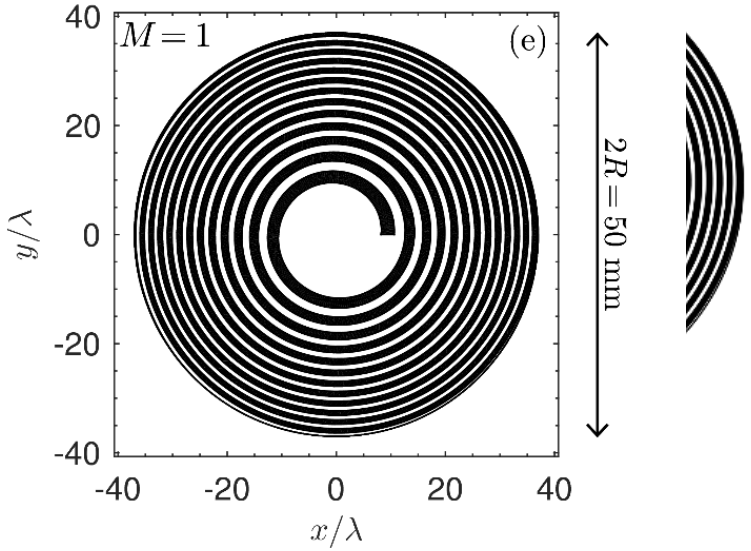
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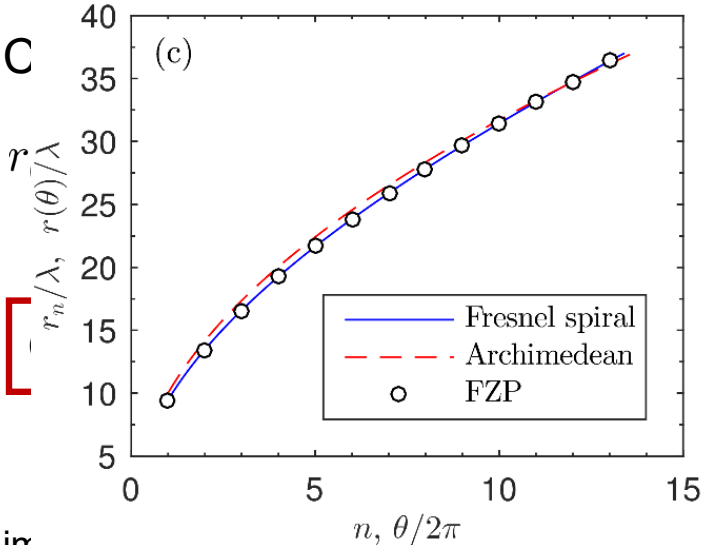
- C

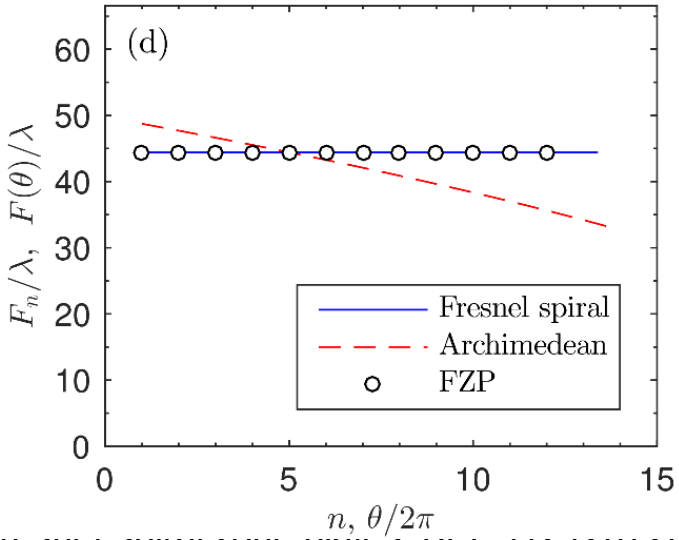
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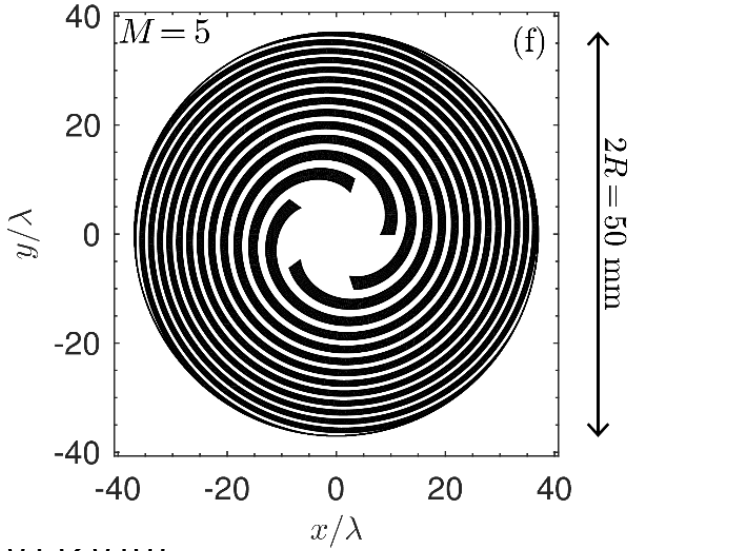










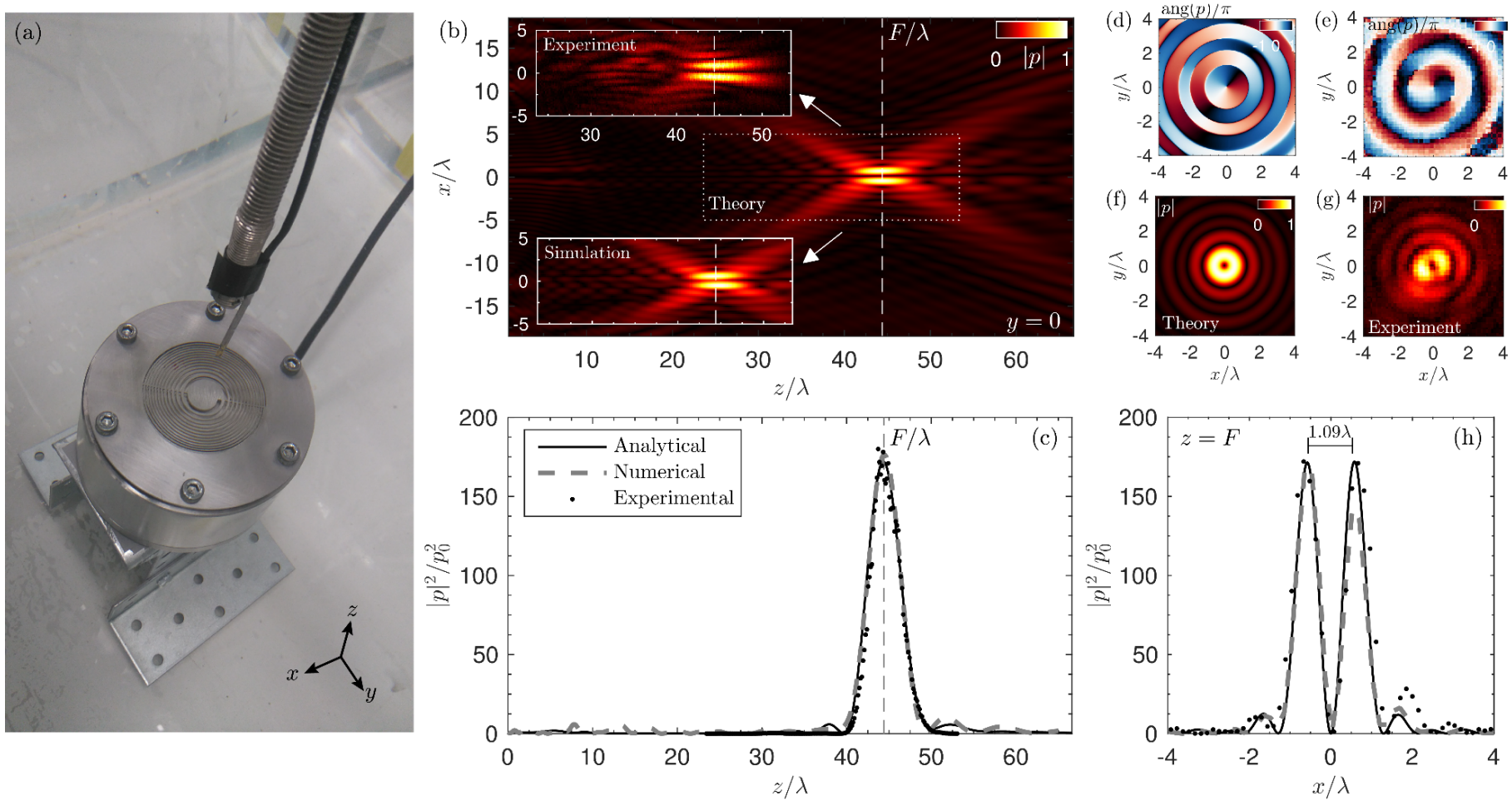


N Jim[illegible], *Sharp acoustic vortex focusing by Fresnel spiral zone plates*. APL 112 (20) 204101 (2018)

Fresnel-spiral lenses

Validation

- Simple topological charge: $M = 1$
- Ultrasound in water, $f_0 = 2$ MHz, $R = 25$ mm, $F = 30$ mm.



Fresnel-spiral lenses

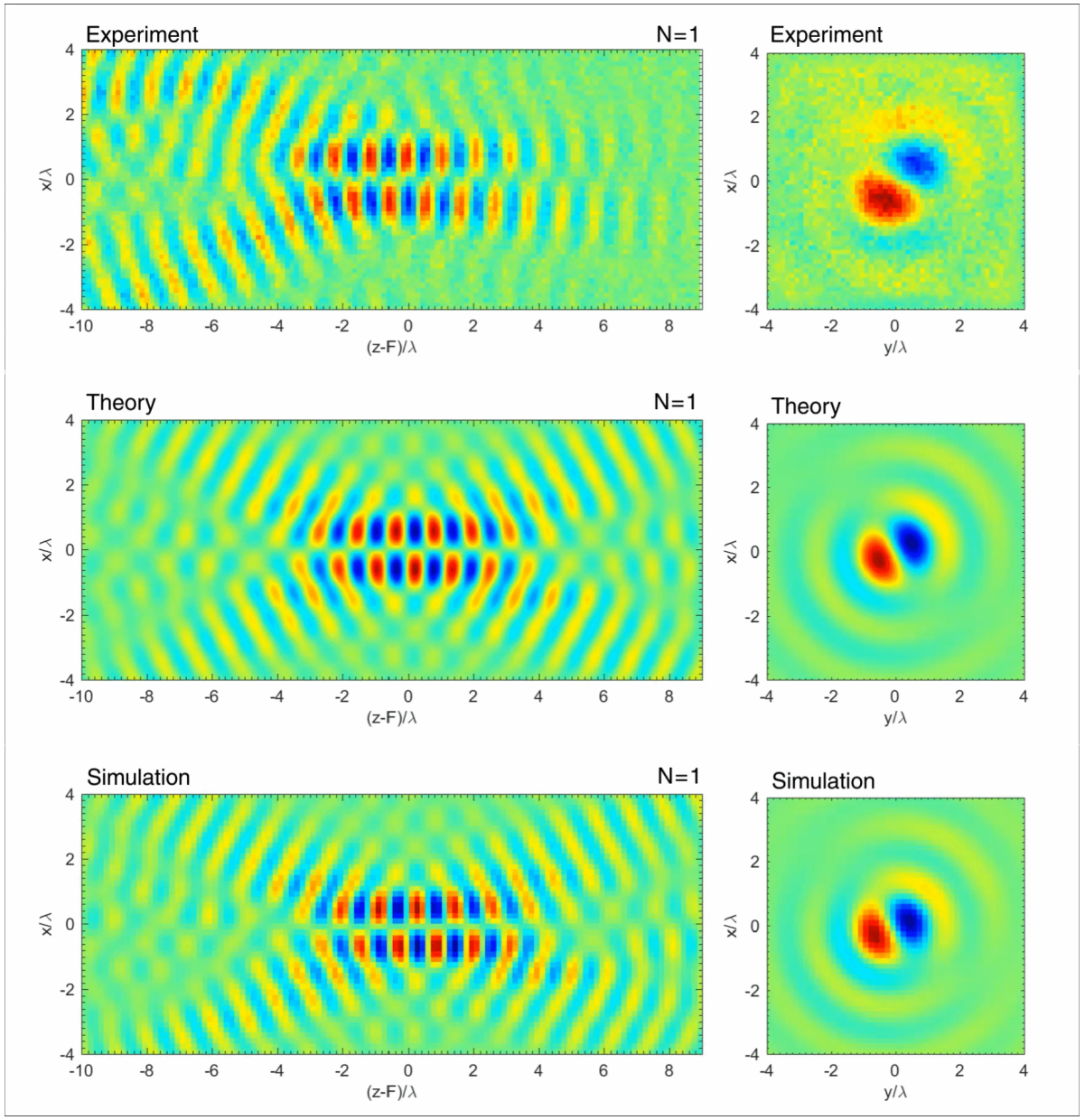
Validation

- Simple topological charge

  $M = 1$

- Ultrasound in water

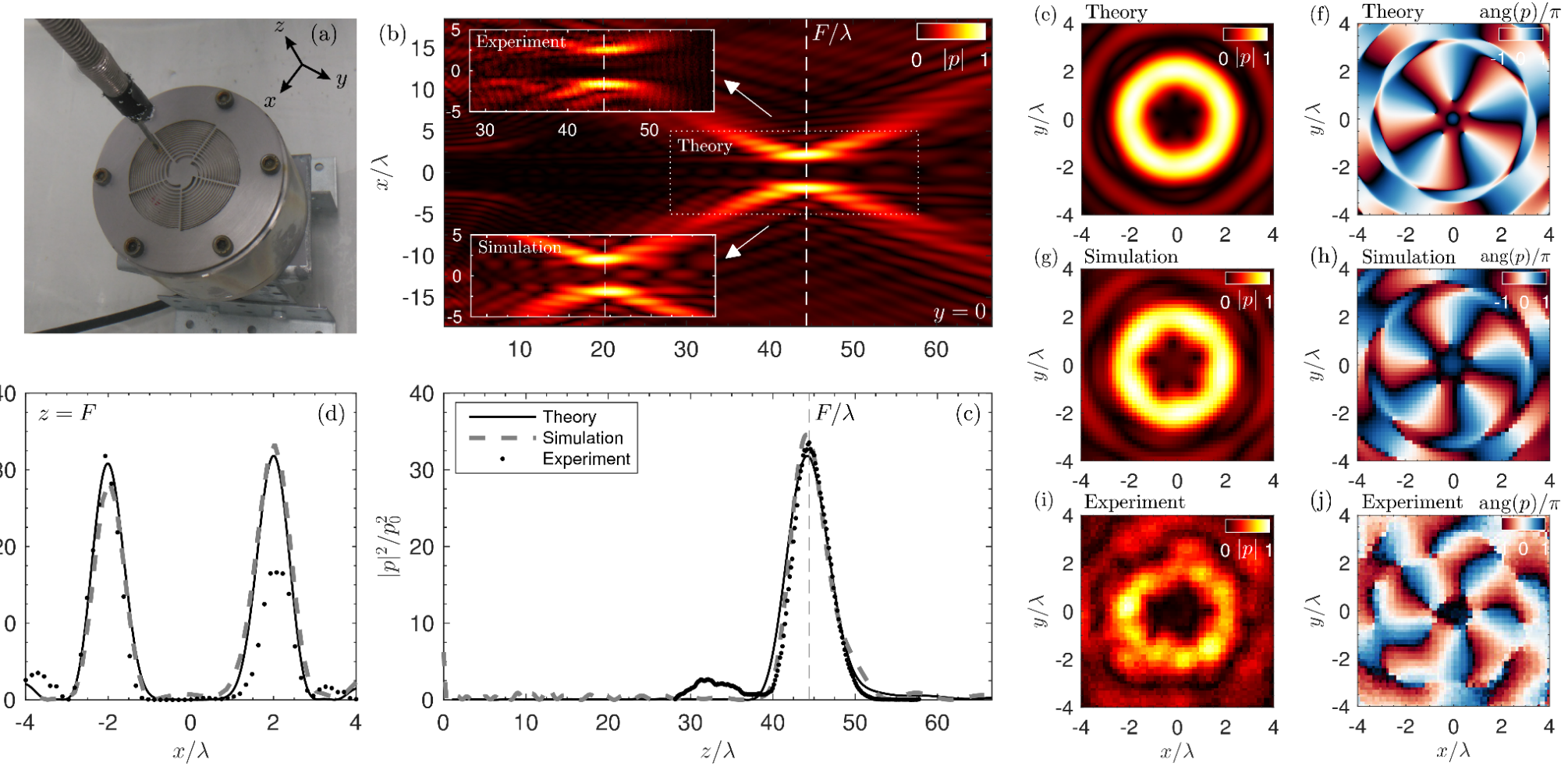
  $f_0 = 2$ MHz,
  $R = 25$ mm,
  $F = 30$ mm.



Fresnel-spiral lenses

Validation

- Higher topological charge: $M = 5$
- Ultrasound in water, $f_0 = 2$ MHz, $R = 25$ mm, $F = 30$ mm.



Fresnel-spiral lenses

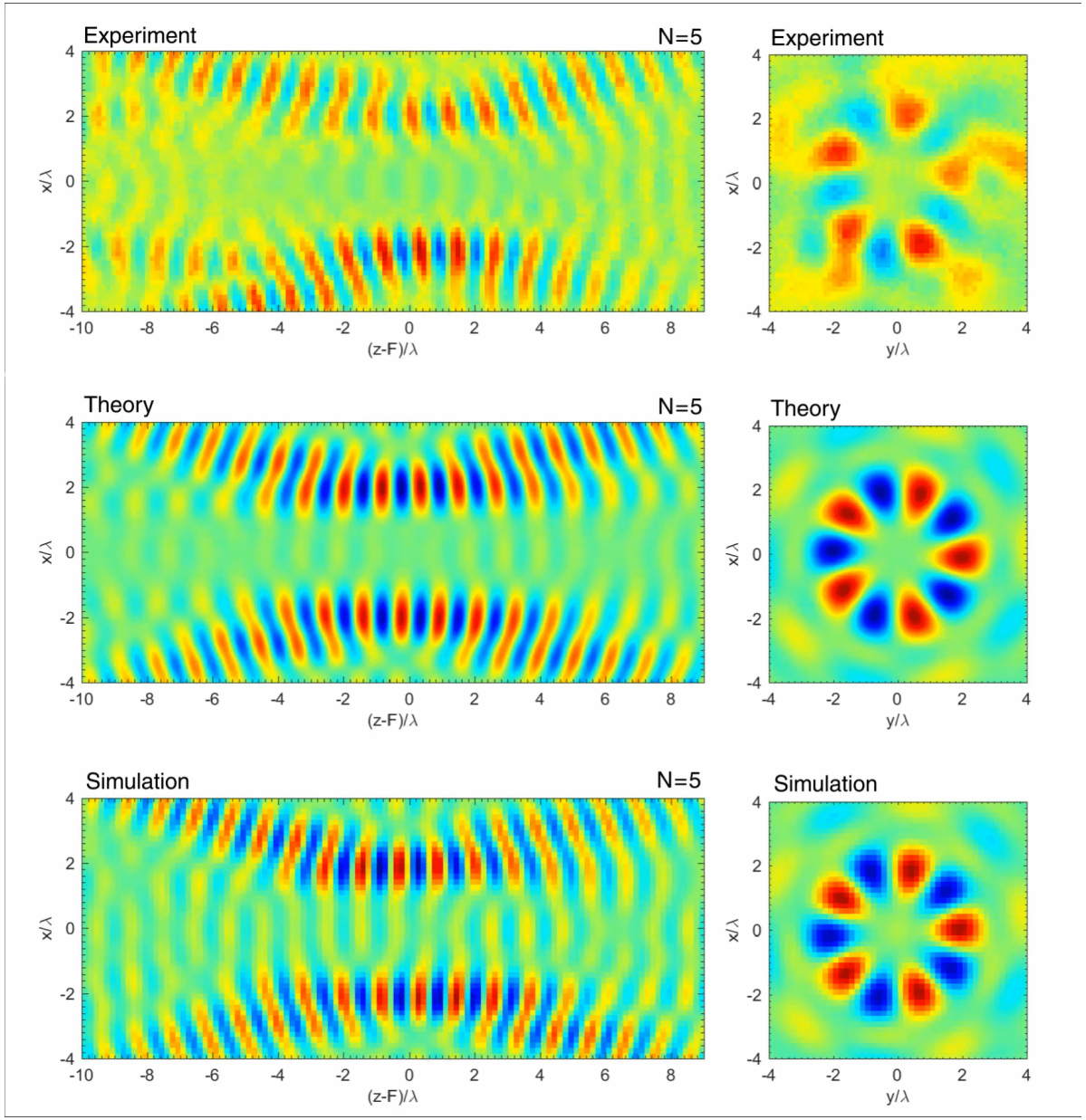
Validation

- Simple topological charge

  $M = 1$

- Ultrasound in water

  $f_0 = 2\ \mathrm{MHz},$
  $R = 25\ \mathrm{mm},$
  $F = 30\ \mathrm{mm}.$



Conclusions

Circular apertures / toroidal

- Zeroth order Bessel beam
- Intensity linear with distance

N. Jiménez et al. *Europhysics Letters* 106 24005 (2014)

Archimedes' spirals

- Higher order Bessel beams
- Topological charge : number of arms / diffraction order / harmonics

N Jiménez et al. *Phys. Rev. E* 94 (5) 053004 (2016)

Archimedean's spirals (Fermat, Lottus, …)

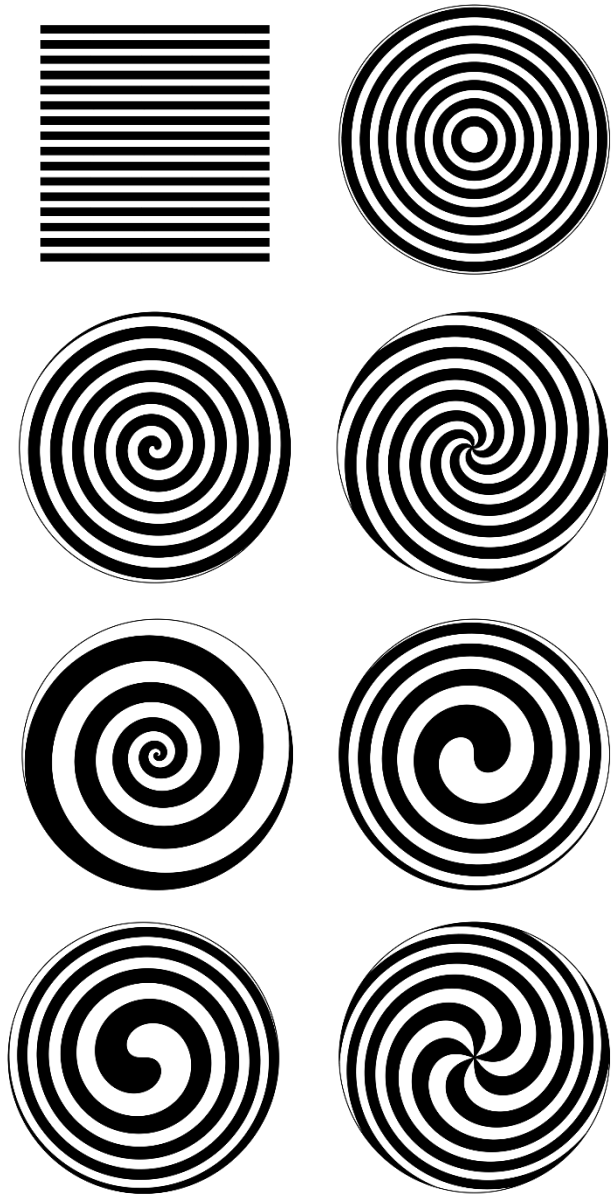
- Focused beams for $\gamma \approx 1/2$
- Focal aberrations

N Jiménez et al. *Appl. Phys. Lett.* 112 (20) 204101 (2018) [Suppl. Mat.]

Fresnel spirals

- Geometrically-optimal focusing
- Optimal focusing control

N Jiménez et al. *Appl. Phys. Lett.* 112 (20) 204101 (2018)



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