Control of the acoustic waves in the subwavelength regime: metamaterials for perfect absorption and metadiffusers

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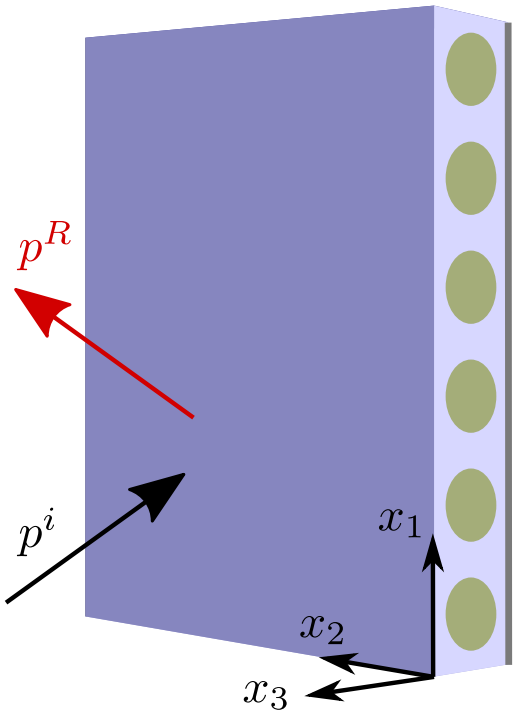
DENORMS Industrial days on new acoustic treatments, acoustic metamaterials and sonic crystals
1st March 2019

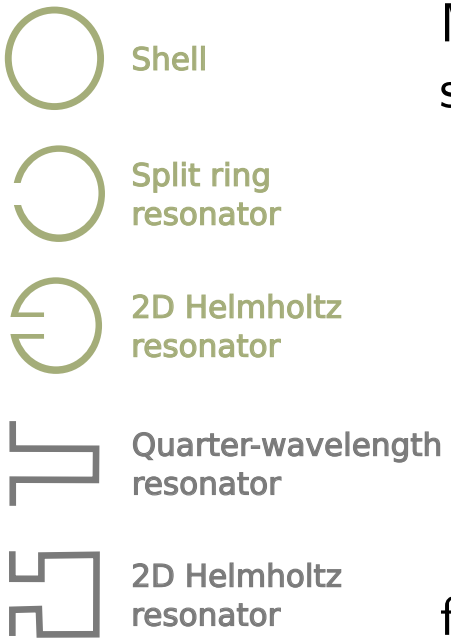






Metaporous surfaces/interfaces



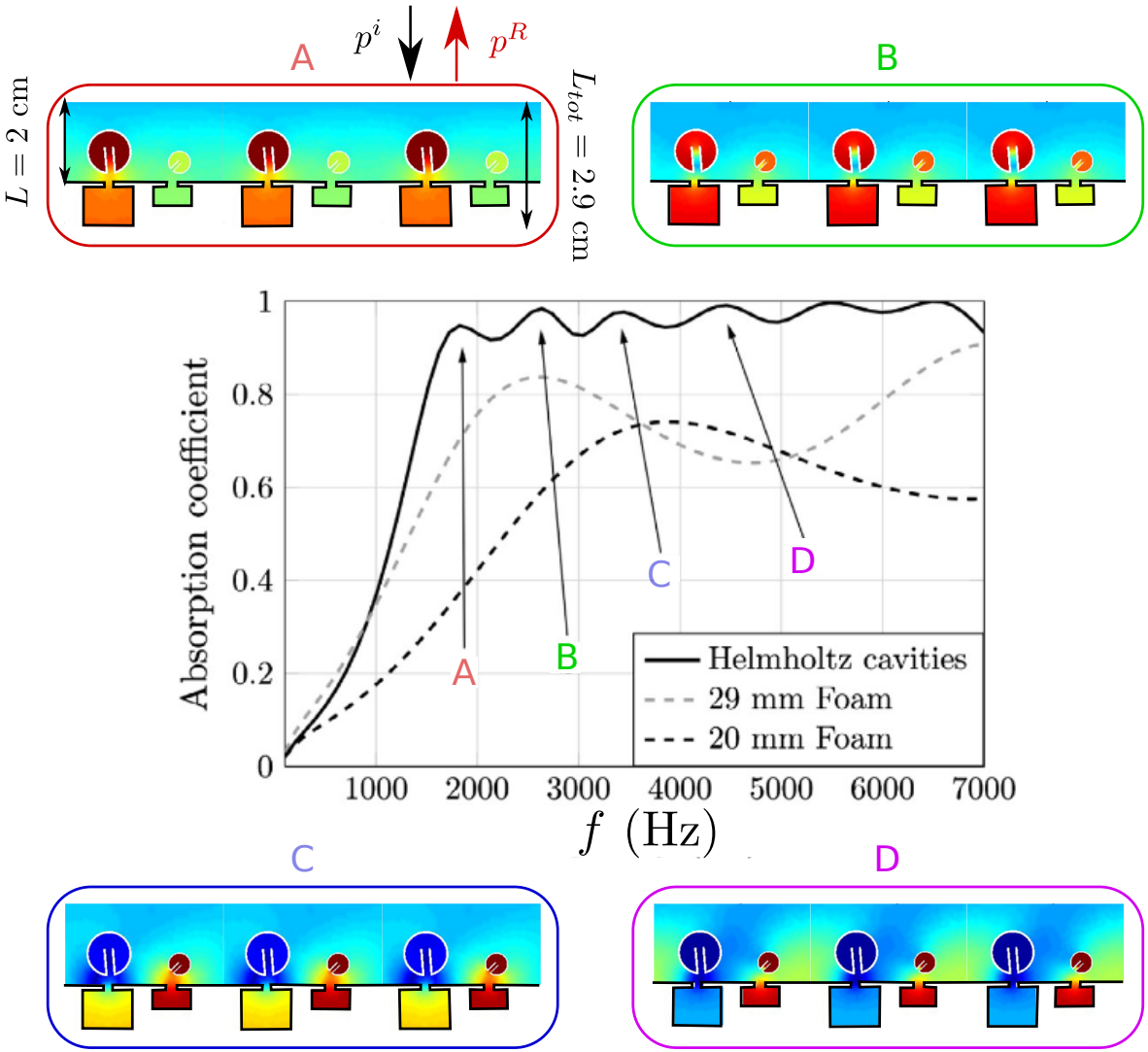


Metaporous surfaces are 2D/3D thin structures that make use of

- Viscothermal losses of porous materials to attenuate sound
- Periodic resonant elements to trap the acoustic energy
- Viscothermal losses inside the resonant elements

for the broadband absorption of sound

Example of optimized metaporous surface



No absorption decrease due to Bragg interferences

Optimization by genetic alogrithm

(Appl. Acoust. 102:49, 2016)

Broadband large absorption for $\lambda_0 \leq 10L$

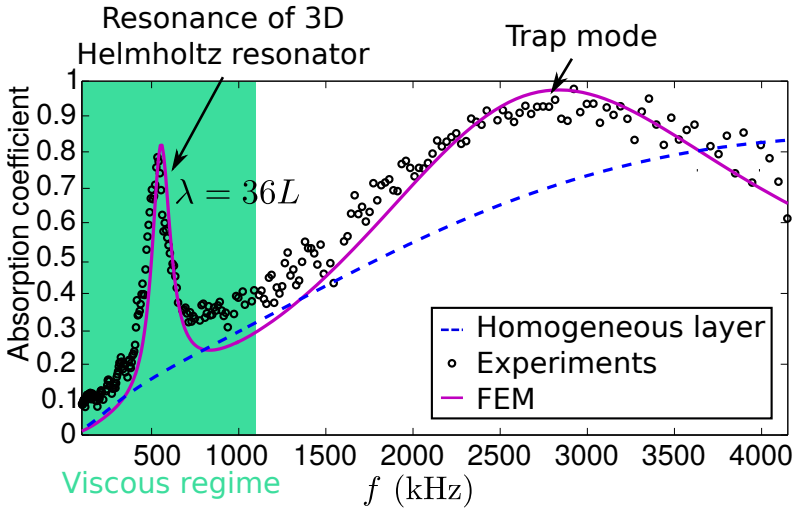
Perspectives

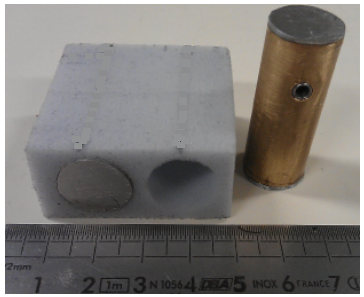
- Metaporous surfaces: enhance absorption properties of porous layer

✓ broadband large absorption for $\lambda_0 \leq 10L$

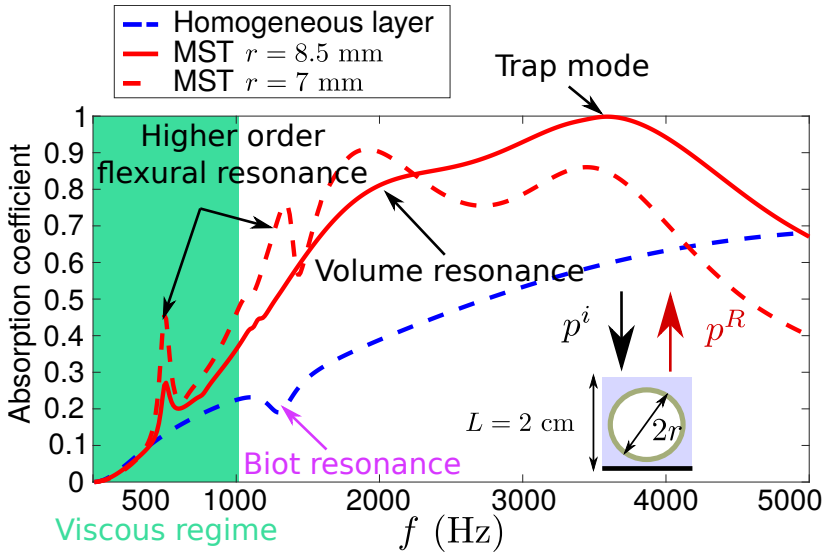
✗ limited to inertial regime

(J. Acoust. Soc. Am. 137:273, 2015)





Solution patented and commercialized by Metacoustic

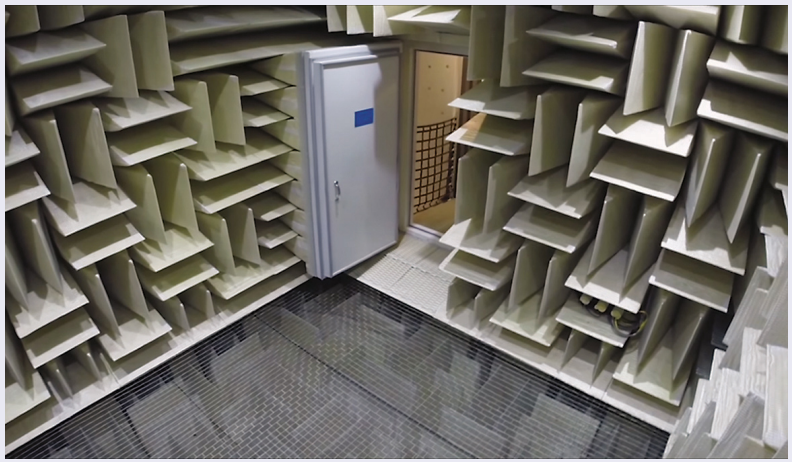


⇒ Metaporoelastic surfaces: elastic resonators

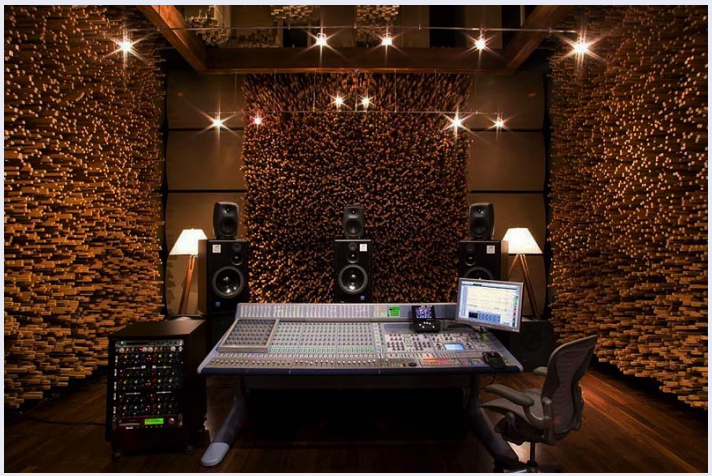
(J. Acoust. Soc. Am. 139:617, 2016)

General motivation

Absorption



Diffusion

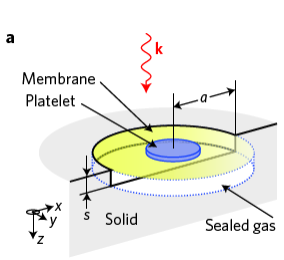


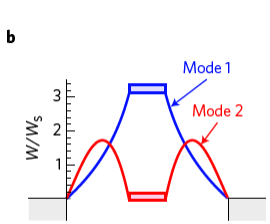
- Not efficient at low frequency!

- Can we design subwavelength structures to improve the behavior of existing systems for absorption and diffusion?
- Can we efficiently control both the absorption and the diffusion of sound at low frequencies by using metamaterials?

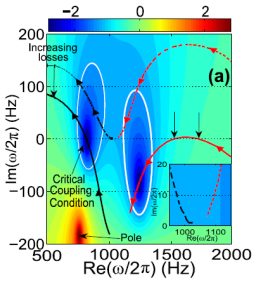
More specific motivation

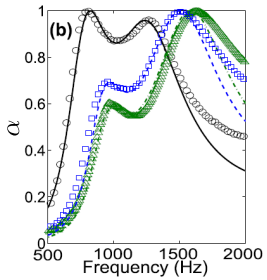
- Several solutions were proposed for perfect absorption in reflection problems



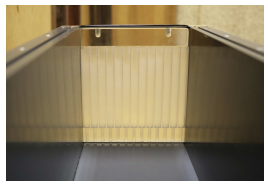


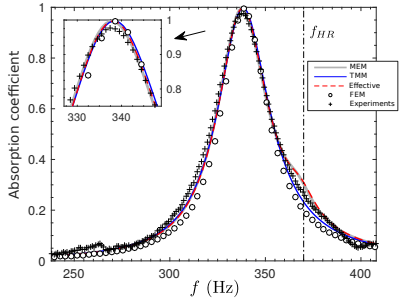
Nature Mat. 13, 813 (2014)





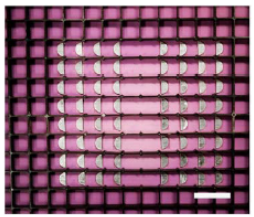
Scientific Rep. 15519 (2016)

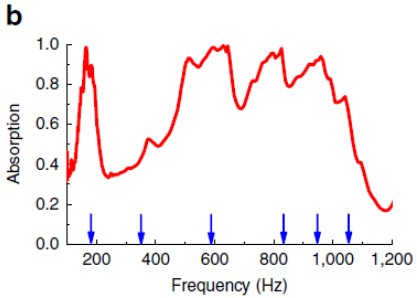




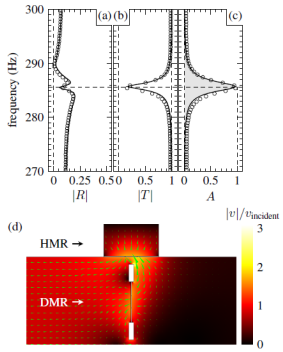
Appl. Phys. Lett., 121902 (2016)

- Only a few solutions were proposed for transmition problems

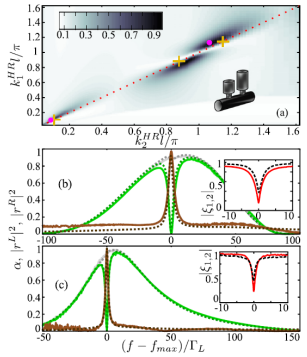




Nat. Commun., 1758 (2012)

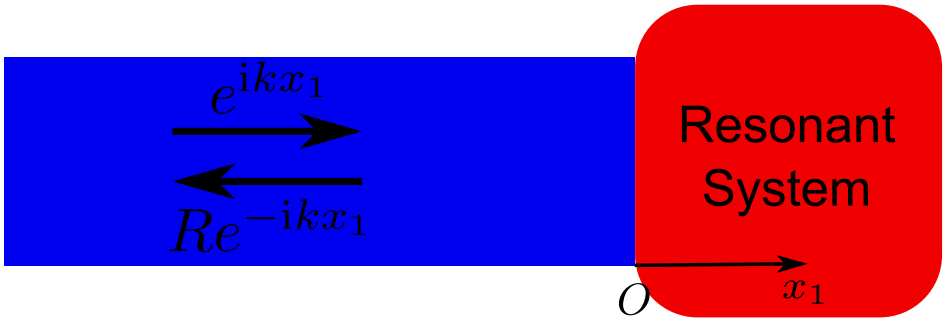


Appl. Phys. Lett., 104104 (2015)



Appl. Phys. Lett., 244102 (2015)

1D resonant system: critical coupling condition



1D resonant system

- time dependance $e^{-i\omega t}$
- $p = e^{ikx_1} + Re^{-ikx_1}$

$\Rightarrow$ $\mathcal{S}$ reduces to R

Properties in the lossless case:

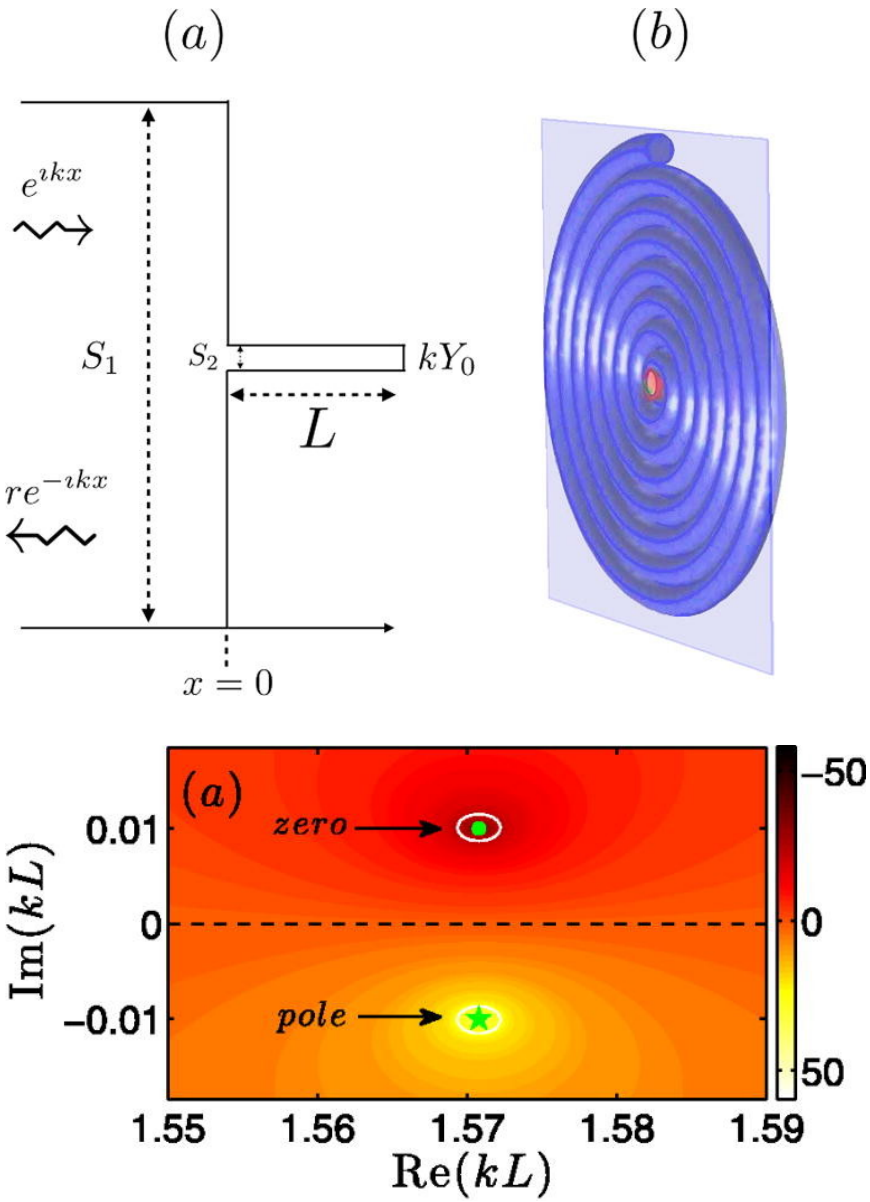
- $\mathcal{S}$ is unitary: $\mathcal{S}\mathcal{S}^\dagger = \mathbf{Id}$, i.e. $|R|^2 = 1$
- time reversal symmetry: $\mathcal{S}\mathcal{S}^\star = 1$

In the complex frequency plane $\omega = \omega_r + i\omega_i$:

- temporal invariance implies $R(\omega) = \dfrac{1}{R(\omega^\star)}$

$\Rightarrow$ Poles and zeros of $R(\omega)$ are complex conjuguate: $\omega_P = \omega_Z^\star$

Simple example: coiled Quarter-Wavelength resonator



Lossless case:

$$\left.\frac{\partial p}{\partial x}\right|_{x=L} = 0$$

$$r = \frac{\cot(kL) + \mathrm{i}S_2/S_1}{\cot(kL) - \mathrm{i}S_2/S_1}.$$

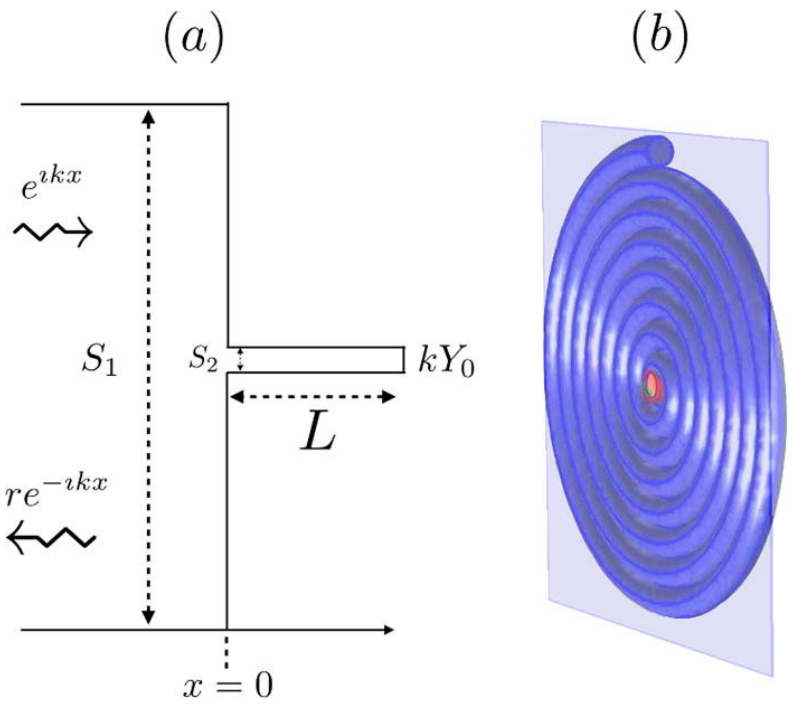
If $S_2/S_1 \ll 1$:

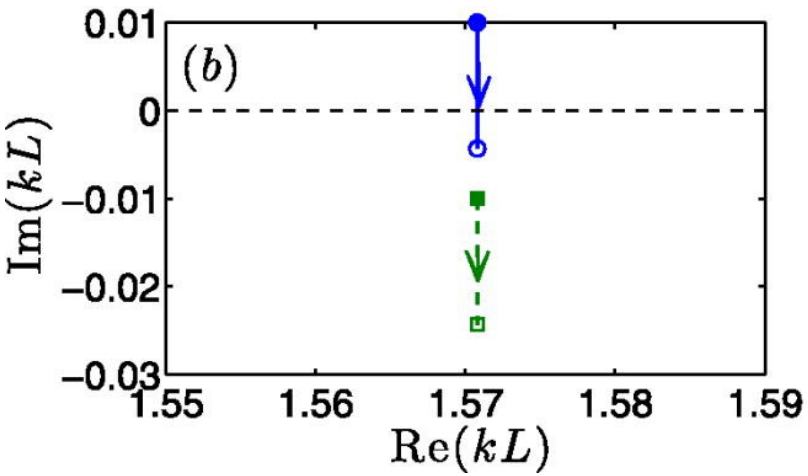
- $(kL)_P = \frac{\pi}{2} - \mathrm{i}S_2/S_1$
- $(kL)_Z = \frac{\pi}{2} + \mathrm{i}S_2/S_1$

Quality factor: $Q_{leak} = \frac{\mathrm{Re}(\omega_P)}{2\mathrm{Im}(\omega_P)}$

(J. Acoust. Soc. Am. 139:3395, 2016)

Simple example: coiled Quarter-Wavelength resonator





Lossy case:

$$\left.\frac{\partial p}{\partial x}\right|_{x=L} = kY_0 p(L), \text{ with } \mathrm{Im}\,(Y_0) > 0$$

$$r = \frac{\cot(kL) - Y_0 + \mathrm{i}S_2/S_1\,(1 + Y_0\cot(kL))}{\cot(kL) - Y_0 - \mathrm{i}S_2/S_1\,(1 + Y_0\cot(kL))}$$

If $S_2/S_1 \ll 1$:

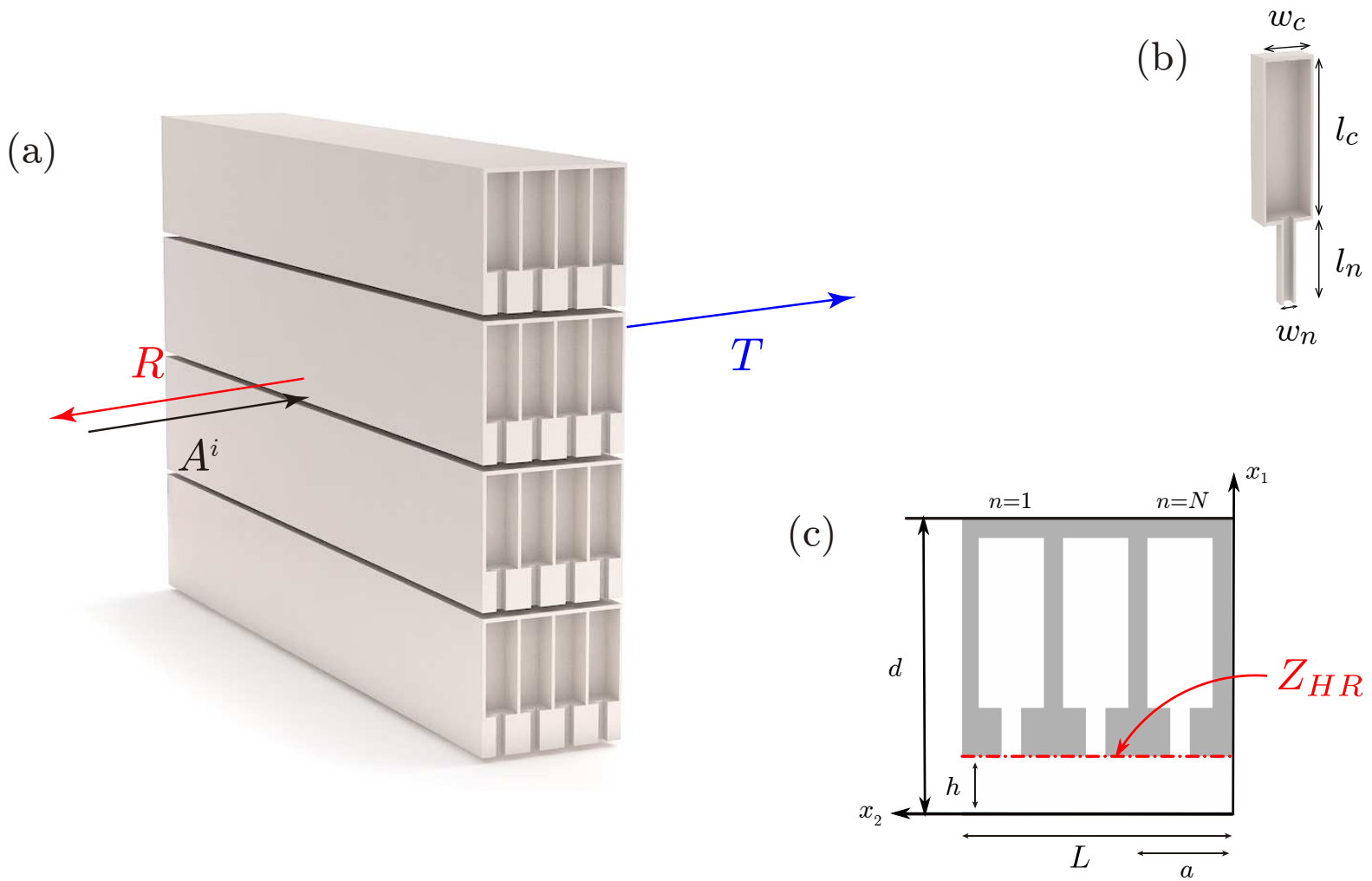
- $(kL)_P = \frac{\pi}{2} - \mathrm{i}S_2/S_1 - Y_0$
- $(kL)_Z = \frac{\pi}{2} + \mathrm{i}S_2/S_1 - Y_0$

Critical coupling condition is reached when $\mathrm{Im}\,((kL)_Z) = 0$

(J. Acoust. Soc. Am. 139:3395, 2016)

Description of the configuration

Periodic arrangement of slits loaded by Helmholtz resonators (HR)



$\Rightarrow$ Losses are accounted for in the slits and HR elements (Stinson, 1991)

$\Rightarrow$ Correction lengths are accounted for in HRs and for slit radiations

Solution of the problem

- Modal expansion based on effective impedance as well as effective parameter description cannot always be used, but it shows that in each duct element slow wave propagates

$$\rho_{eff} = \frac{\rho_d}{\phi} \quad , \quad K_{eff} = \frac{K_d}{\phi}\left[\left(1 + \frac{K_d\phi\left(V_c K_n + V_n K_c\right)\phi_n}{K_n h\left(S_n K_c - V_c \rho_n \omega^2 l_n\right)}\right)\right]^{-1} \quad ,$$

  where $\phi = h_3 h_1 / d_3 d_1$ and $\phi_n = w_n^2 / a^2$.

- Transfert Matrix Method (TMM) may be prefered, because it accounts for the finite number of resonators

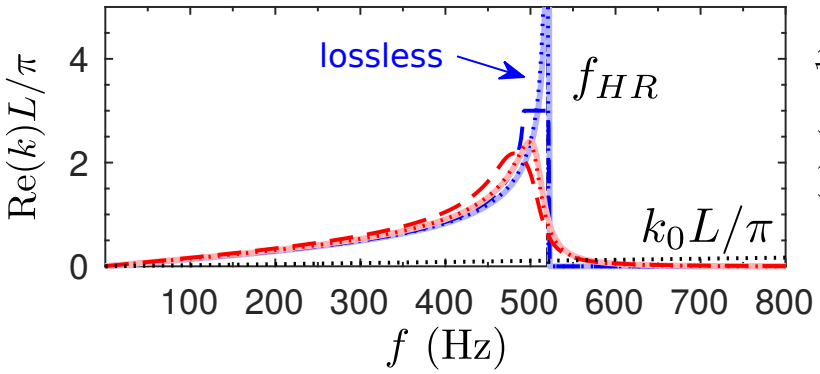
$$\mathcal{T} = M^{[1]}_{\Delta l_{\text{duct}}} \prod_{n=1}^{N} M^{[n]}_{\mathbf{s}} M^{[n]}_{\text{HR}} M^{[n]}_{\mathbf{s}} M^{[n+1]}_{\Delta l_{\text{duct}}}, \text{ with } M^{[n]}_{HR} = \begin{pmatrix} 1 & 0 \\ 1/Z^{[n]}_{HR} & 1 \end{pmatrix},$$

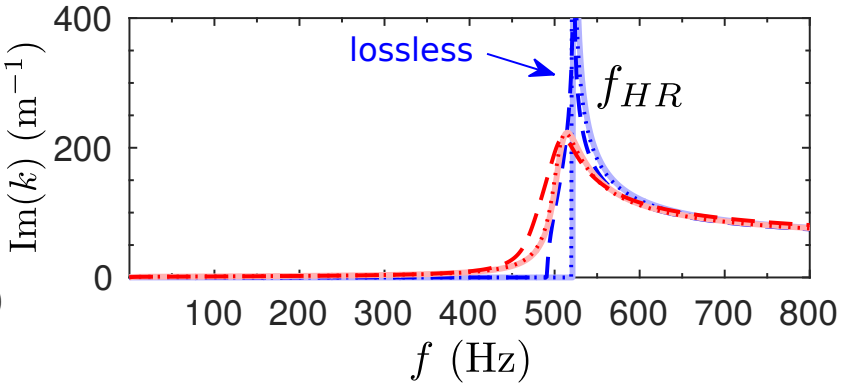
  $M_s^{[n]}$ the propagation matrix, and $M^{[n]}_{\Delta l_{\text{duct}}}$ accounts for the radiation correction of the n-th waveguide segment.

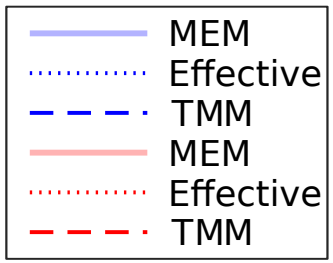
- Finite-Element method (Comsol ®Multiphysics)

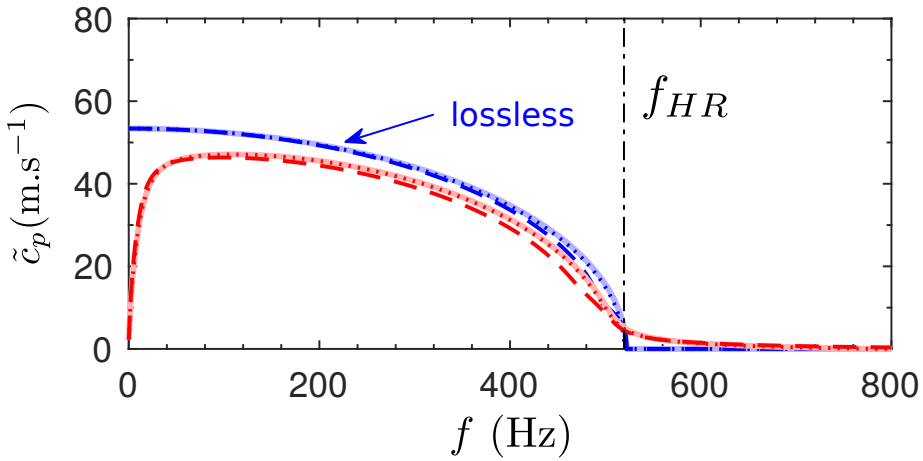
Evidence of slow slound

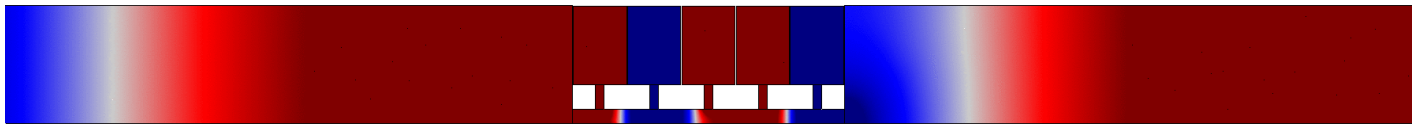
$h = 1.2$ mm, $a = 1.2$ cm, $w_n = a/6$, $w_c = a/2$, $d = 7$ cm, $l_n = d/3$, and $l_c = d - h - l_n$.







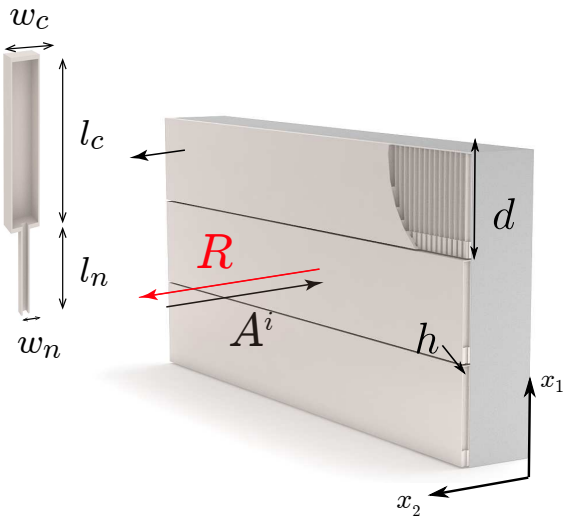


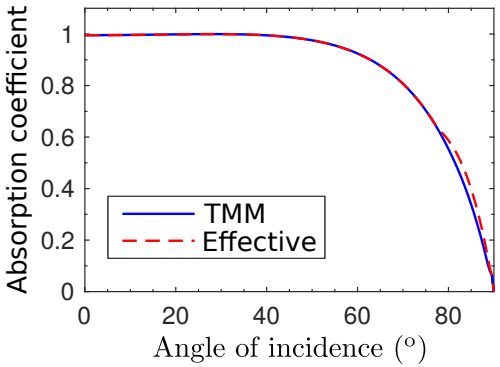


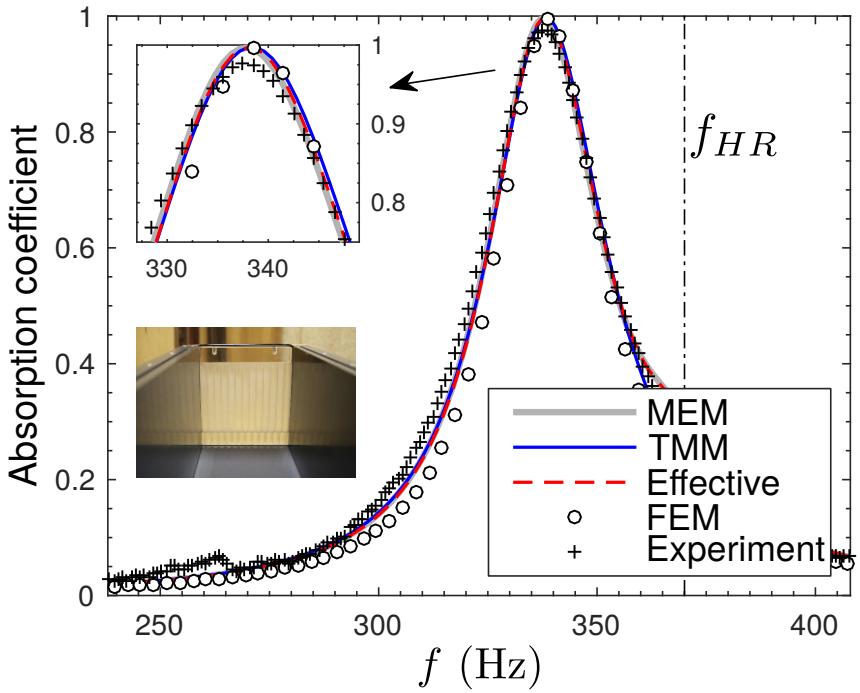
Reflection problem

Reflection problem

$N = 1$: $h = 2.63$ mm, $a = L = d/13 = 1.1$ cm, $w_n = 2.25$ cm, $w_c = 4.98$ mm, $d = 7$ cm, $l_n = 2.31$ cm, and $l_c = 12.33$ cm.







(Appl. Phys. Lett. 109: 121902, 2016)

Perfect absorption for $\lambda_0 = 88L$

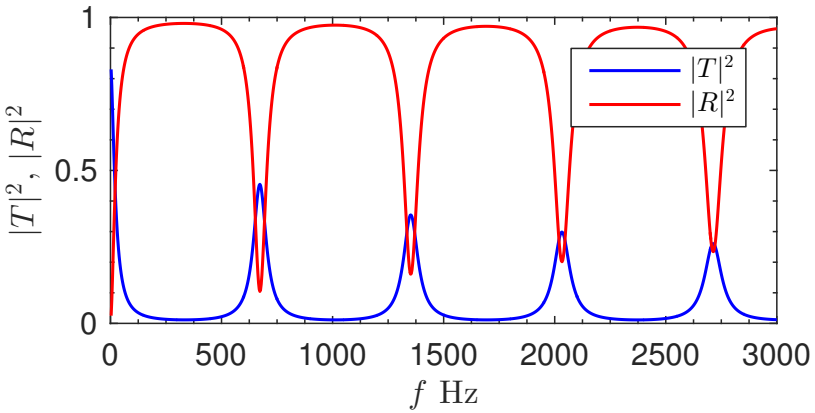
Transmission problem (with identical HRs)

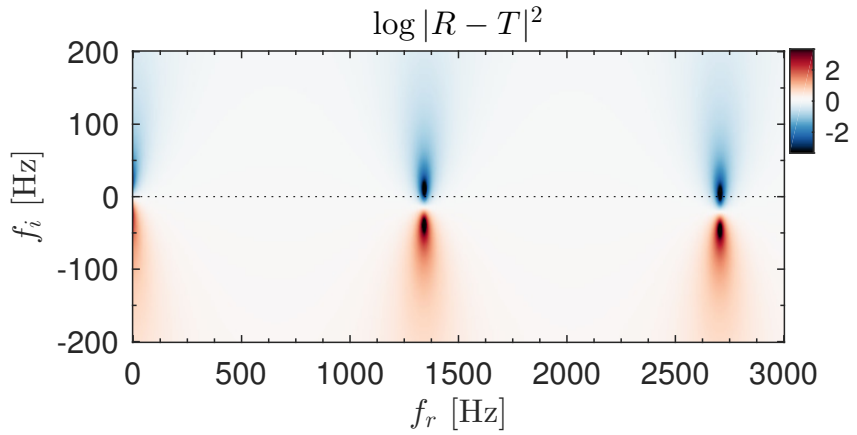
The main difficulty in ventillation problem

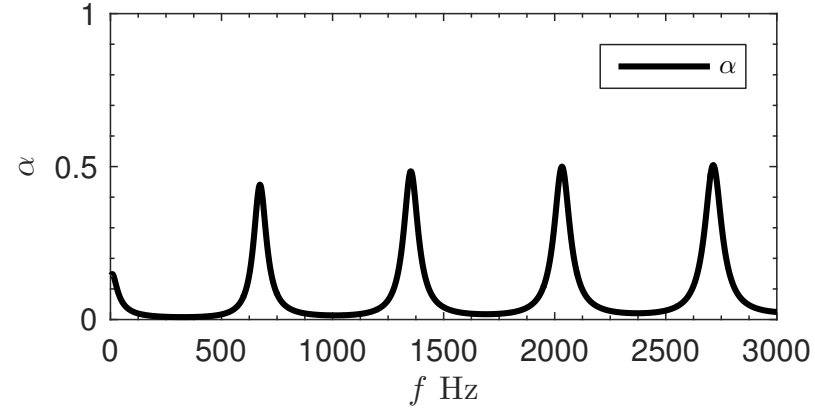
Slit: $h = 3.8$ mm, $L = 25$ cm, $d = 15$ cm

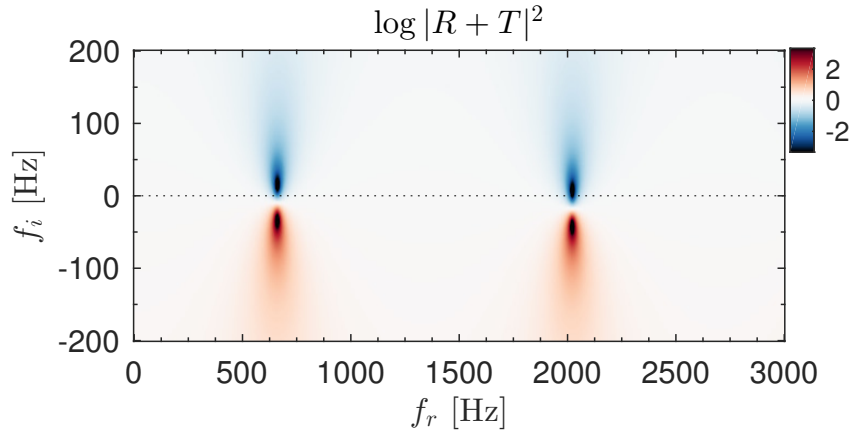
Zeros of the $|R_{sym}| = |R - T|$ and $|R_{asym}| = |R + T|$ are staggered

$\alpha = (\alpha_{sym} + \alpha_{asym})/2$, so $\alpha_{max} \geq 0.5$: the goal is $\alpha_{max} = 1$





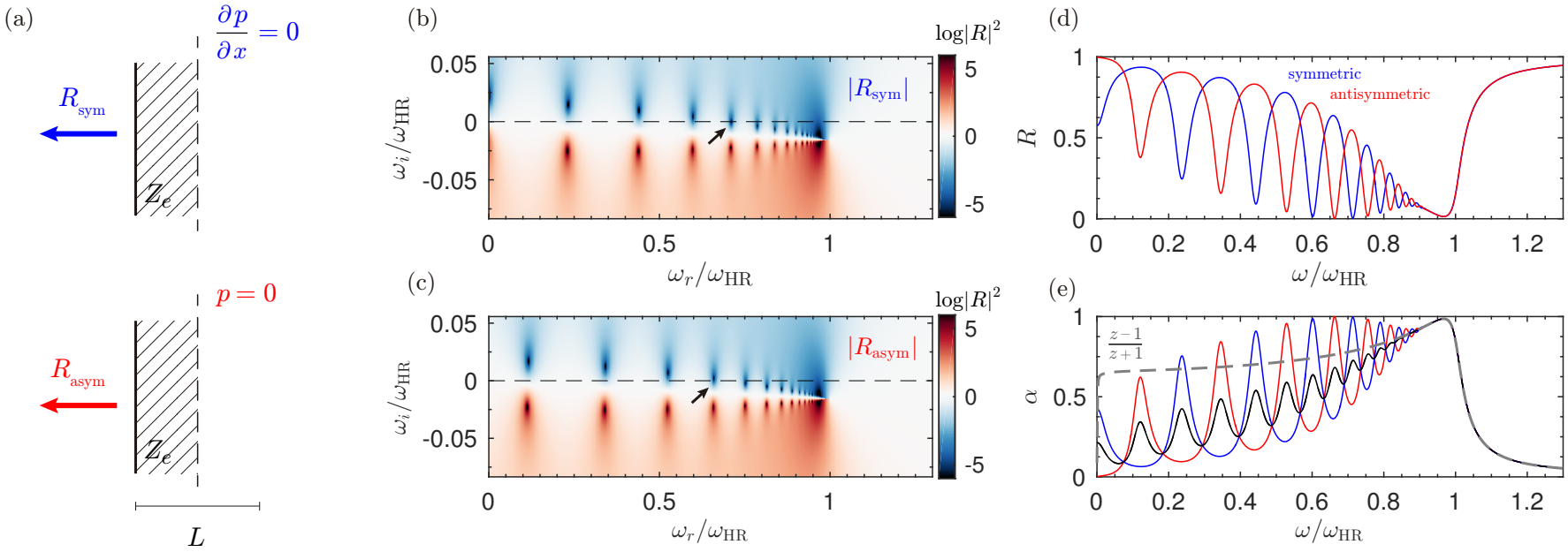




$\Rightarrow$ Use slow sound to accumulate the resonances for sound absorption

Evidence of the accumulation point

$N = 30$: $h = 1.2$ mm, $a = 4.95$ cm, $w_n = 7.1$ cm, $w_c = 4.9$ mm, $l_n = 7.3$ cm, and $l_c = 12.33$ cm, $d = 5$ cm, $L = 30a$.

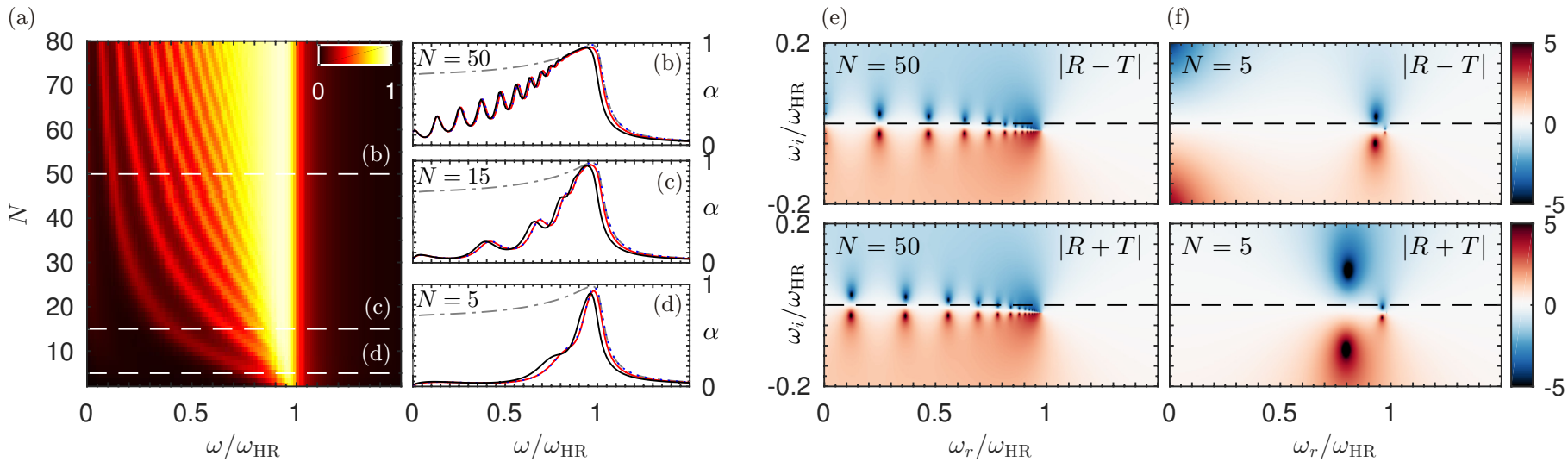


$\Rightarrow$ Accumulation point corresponds to $Z_{eff} = Z^a$, which means that perfect absorption is impossible because $\mathrm{Im}\,(Z_{eff}) \neq 0$

$\Rightarrow$ Accumulation point: $T \approx 0$ (entrance of the band gap) and $R \approx 0$ (Fabry-Perot)

Quasi-perfect absorption

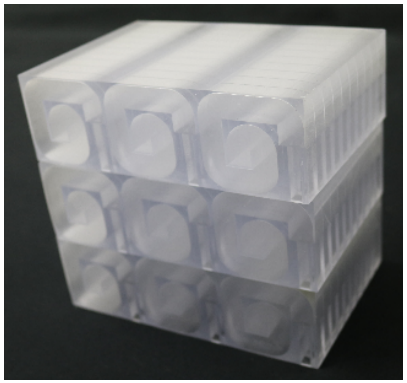
$h = 1.2$ mm, $a = 4.95$ cm, $w_n = 7.1$ cm, $w_c = 4.9$ mm, $d = 5$ cm, $l_n = 7.3$ cm, and $l_c = 12.33$ cm, $L = Na$.

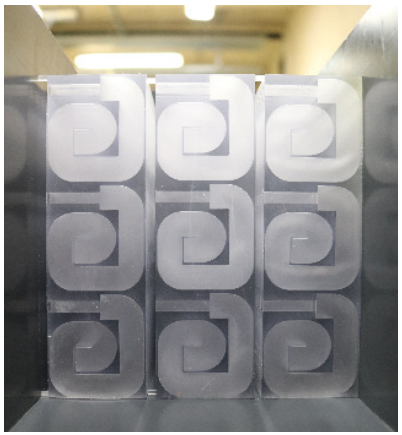


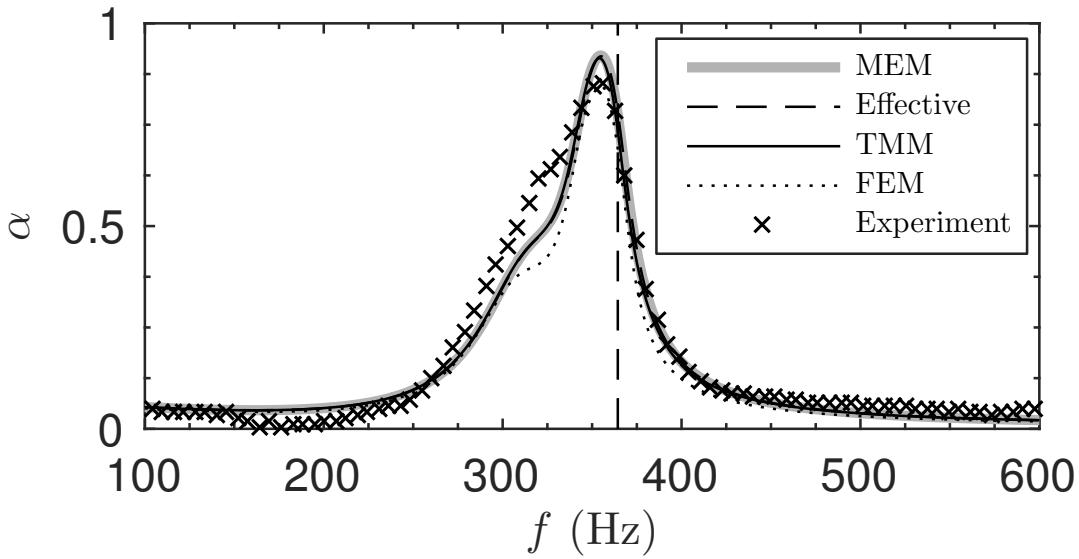
⇒ Quasi-perfect absorption is possible but cannot be deep sub-wavelength

Experimental validation

$N = 10$: $h = 4.4$ mm, $a = 9.8$ mm, $w_n = 5.3$ cm, $w_c = 9.3$ mm, $l_n = 2.5$ cm, and $l_c = 13.9$ cm, $d = 4.96$ cm, $L = 9.8$ cm.







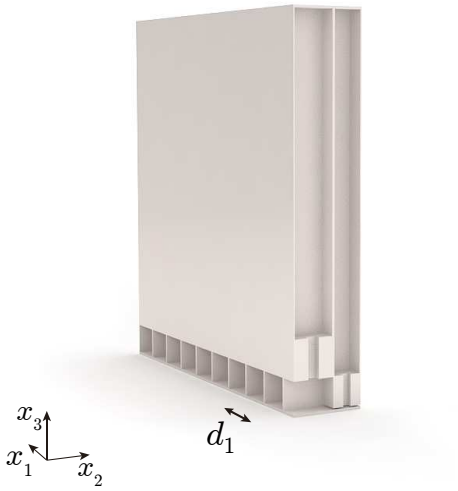
$\Rightarrow \alpha = 0.9$ for $L \approx \lambda_a/10$, with coiled HR

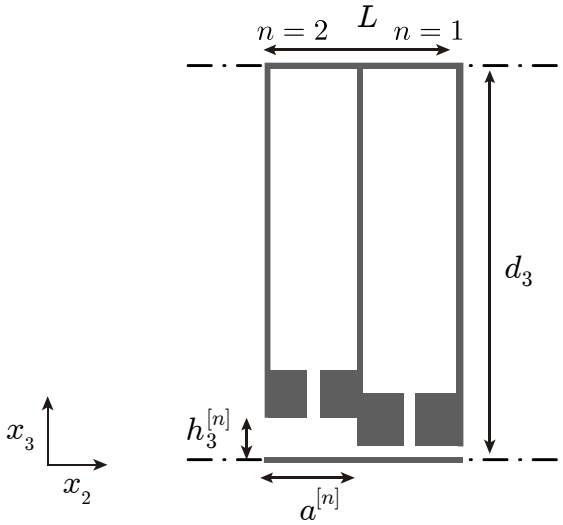
(Phys. Rev. B, 95: 014205, 2017)

Asymmetric configuration for transmission problem: rainbow trapping absorber

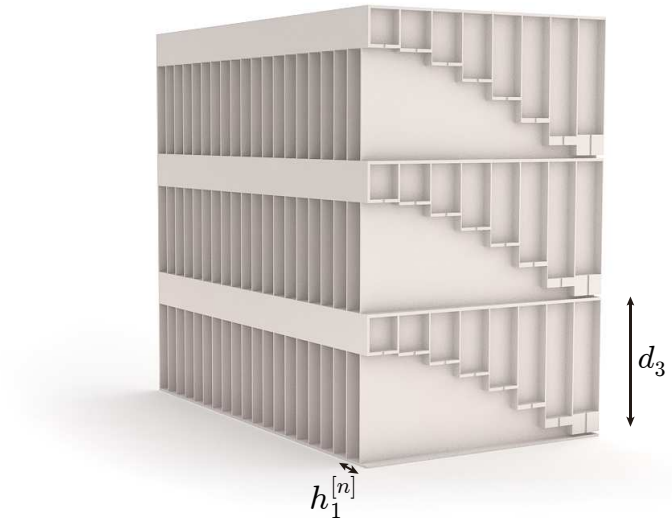
Description of the configuration

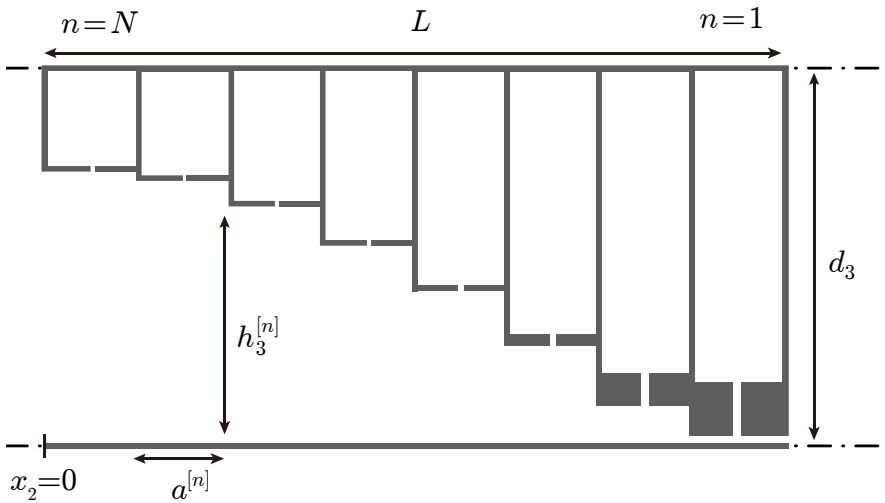
(a) Subwavelength Asymmetric panel (SAP)





(b) Rainbow-trapping absorber (RTA)

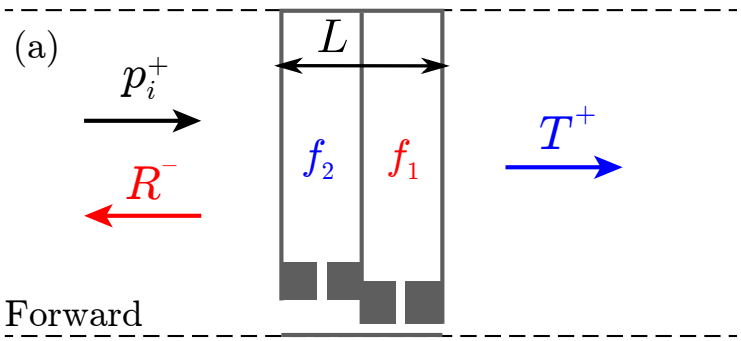


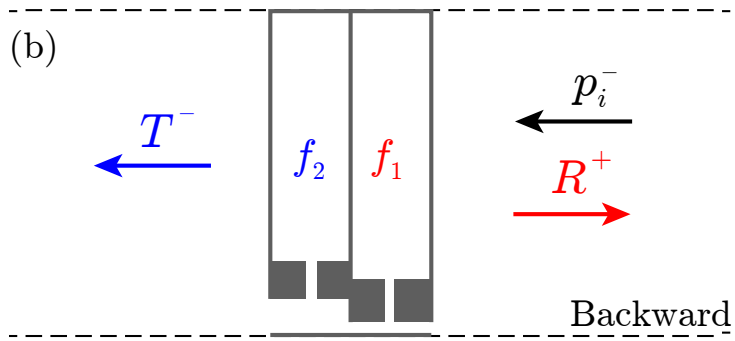


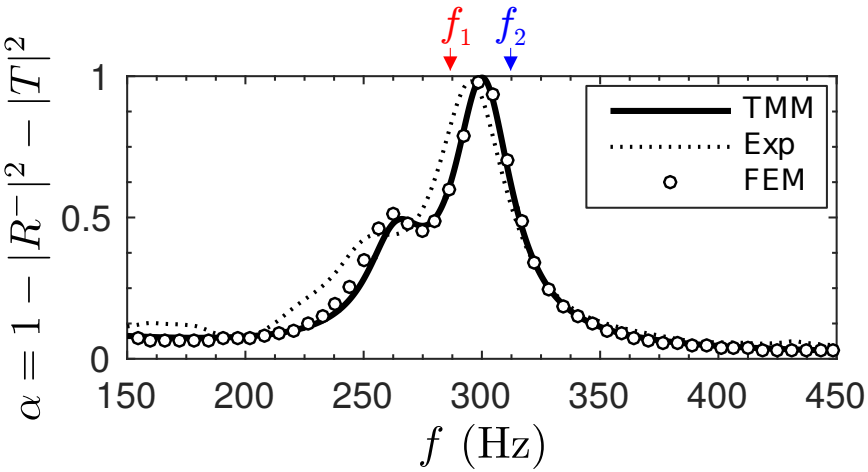
$\Rightarrow$ Losses are accounted for in the duct and HR elements (Stinson, 1991)

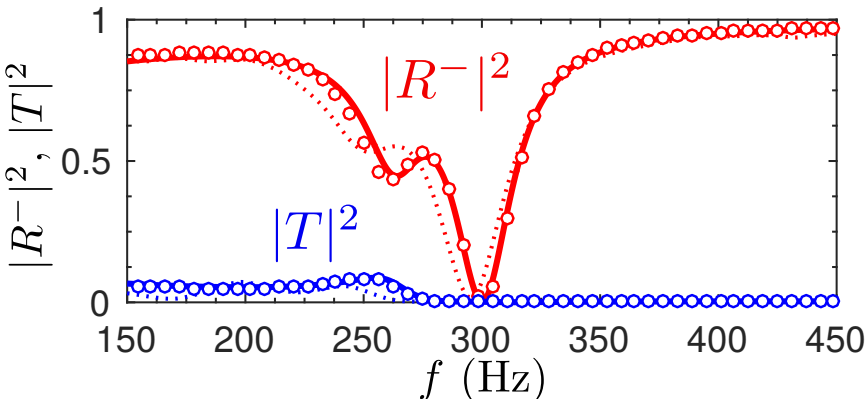
$\Rightarrow$ Correction lengths are accounted for in the HR and duct elements

Design of the Sub-wavelength Asymmetric Panel







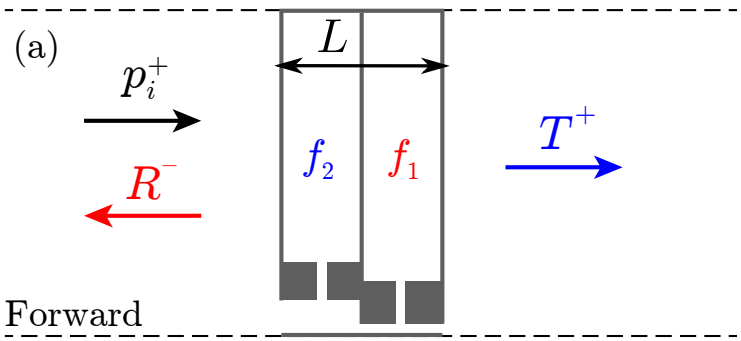


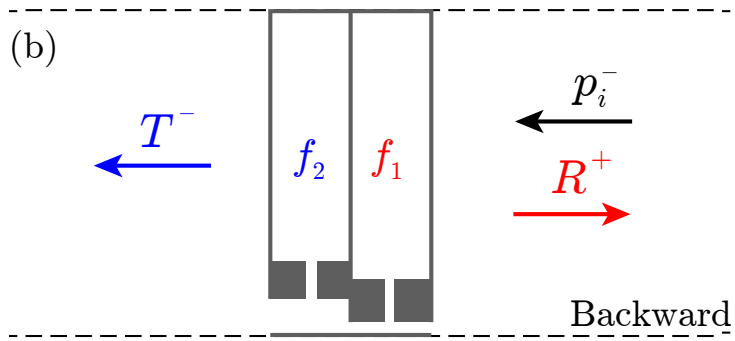
$f_1 < f_2$

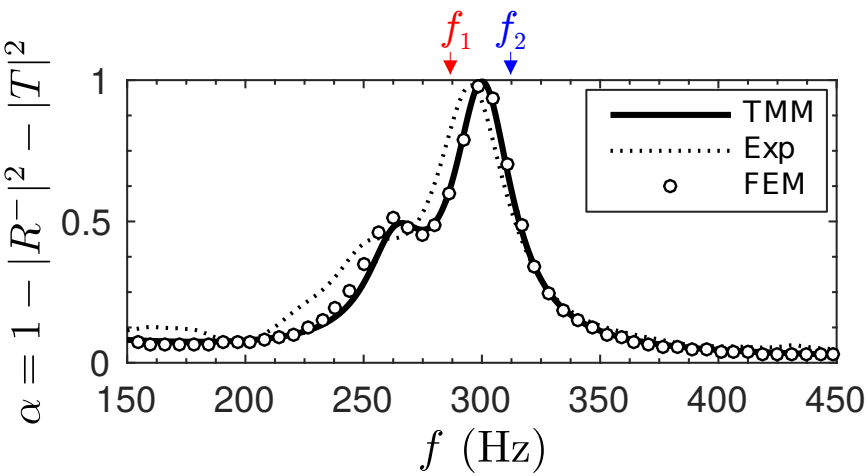
- when 1 resonates, a bandgap opens
- 2 is tuned to critically couple the system
- the duct section is also modified

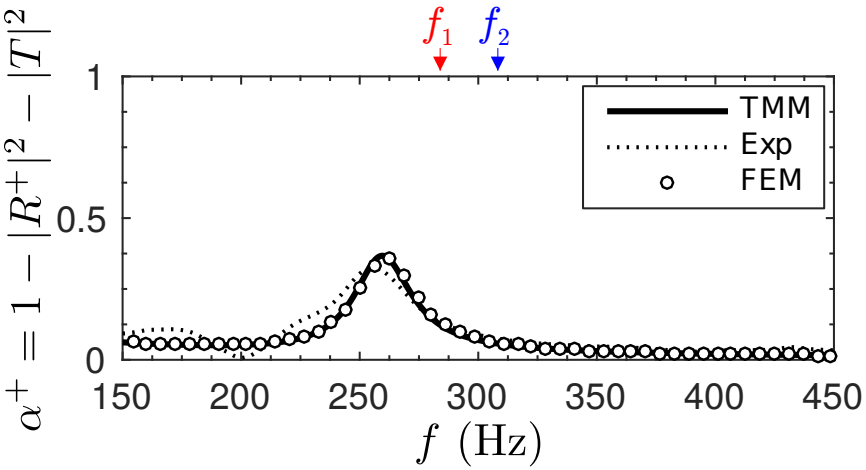
Perfect absorption is achieved for $L = 2.64$ cm, i.e., $\lambda_a \approx 40L$.

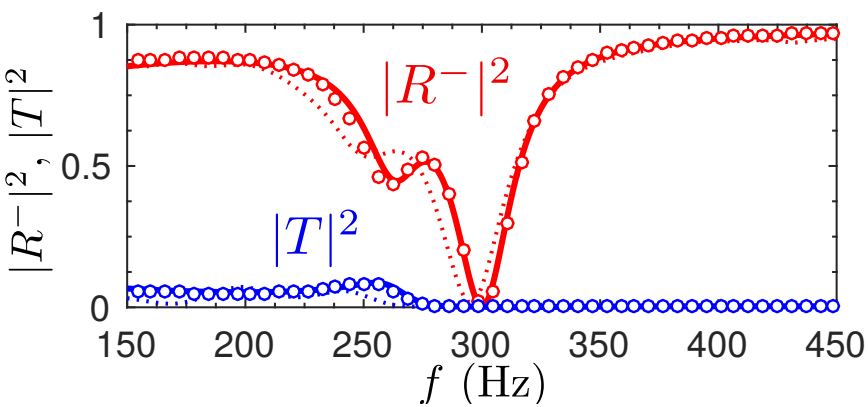
Design of the Sub-wavelength Asymmetric Panel

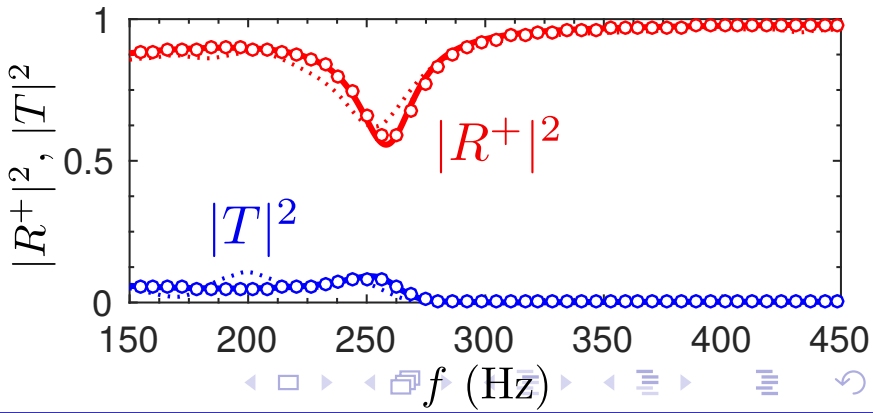




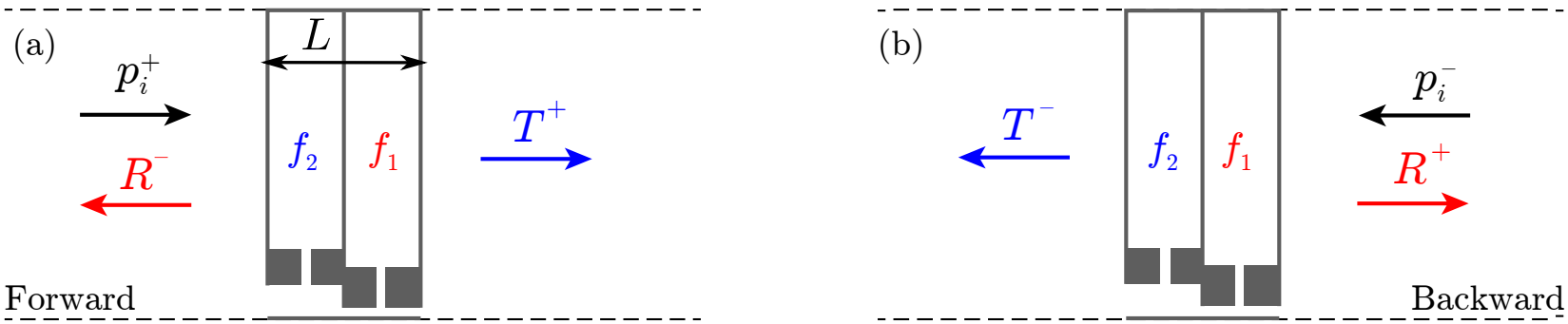








Analysis of the SAP



The system is analyzed through the scattering matrix $\mathcal{S}$.
The system is reciprocal ($T^+ = T^- = T$) but asymmetric:

$$\mathcal{S} = \begin{pmatrix} T & R^- \\ R^+ & T \end{pmatrix}.$$

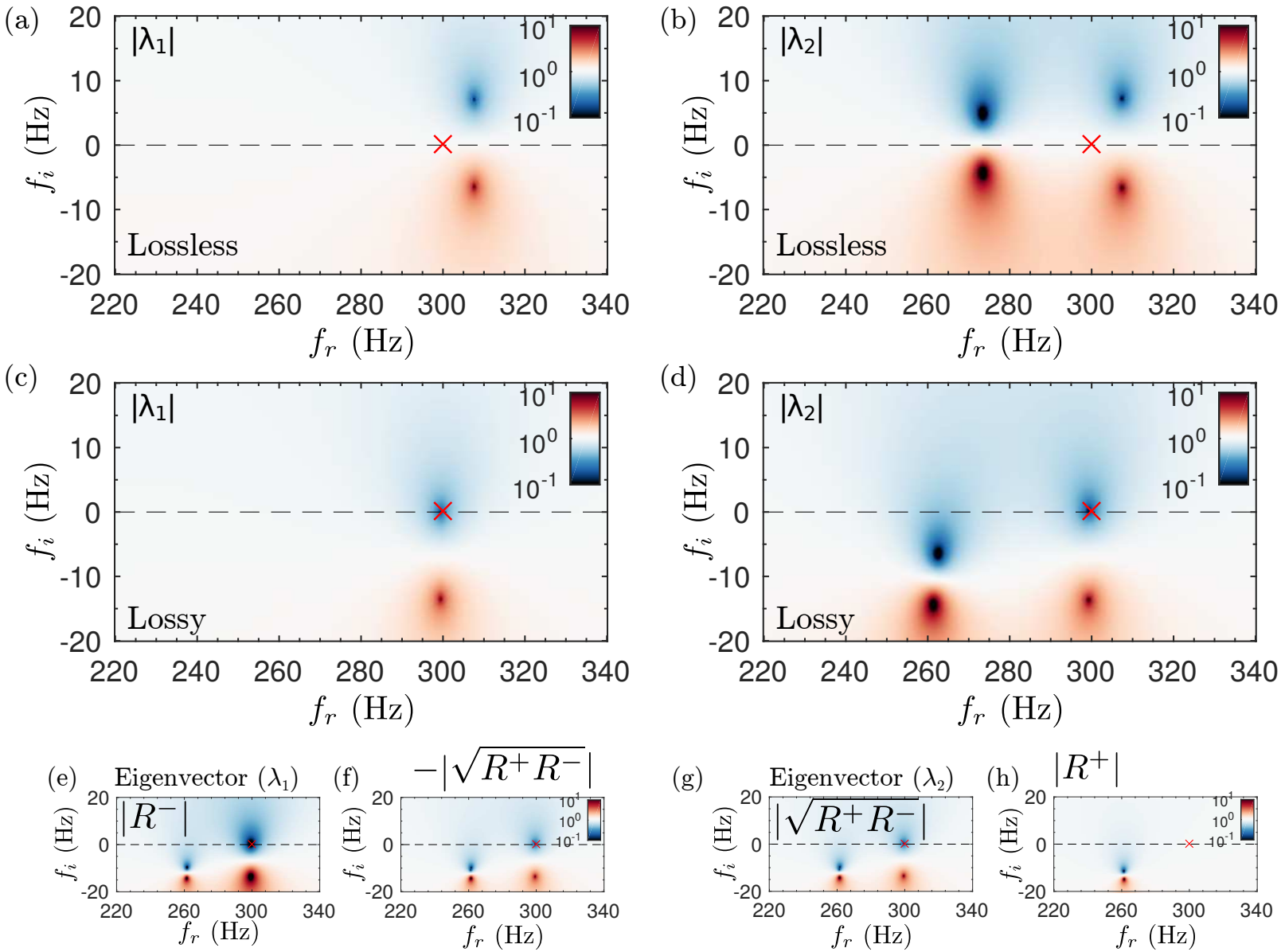
The scattering matrix possesses two eigenvalues

$$\lambda_{1,2} = T \mp \sqrt{R^+ R^-},$$

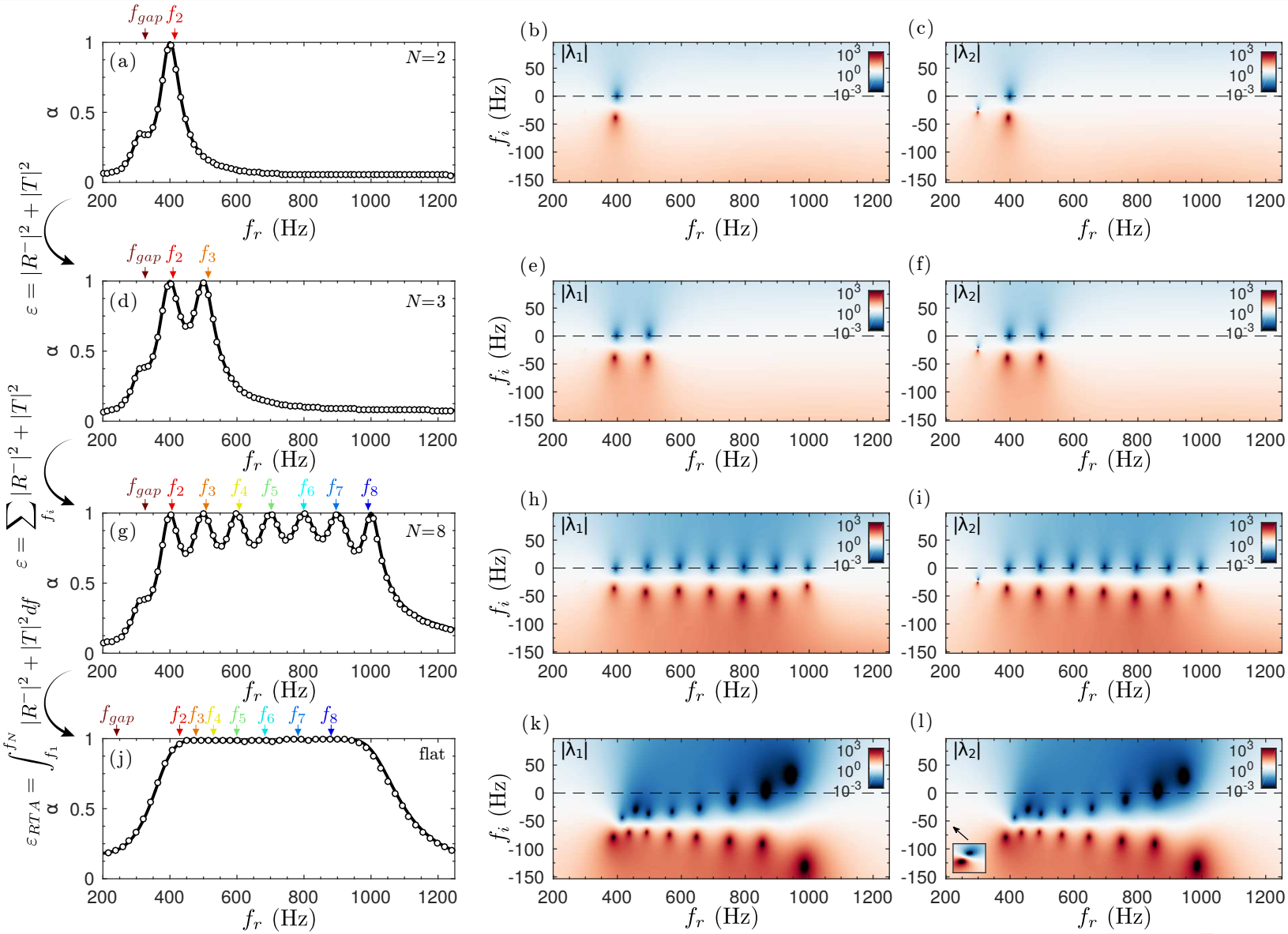
whose associated eigenvectors are

$$\vec{v}_1 = \left[R^-, -\sqrt{R^+ R^-}\right], \quad \vec{v}_2 = \left[\sqrt{R^+ R^-}, R^+\right].$$

Analysis of the eigenvalues and eigenvectors

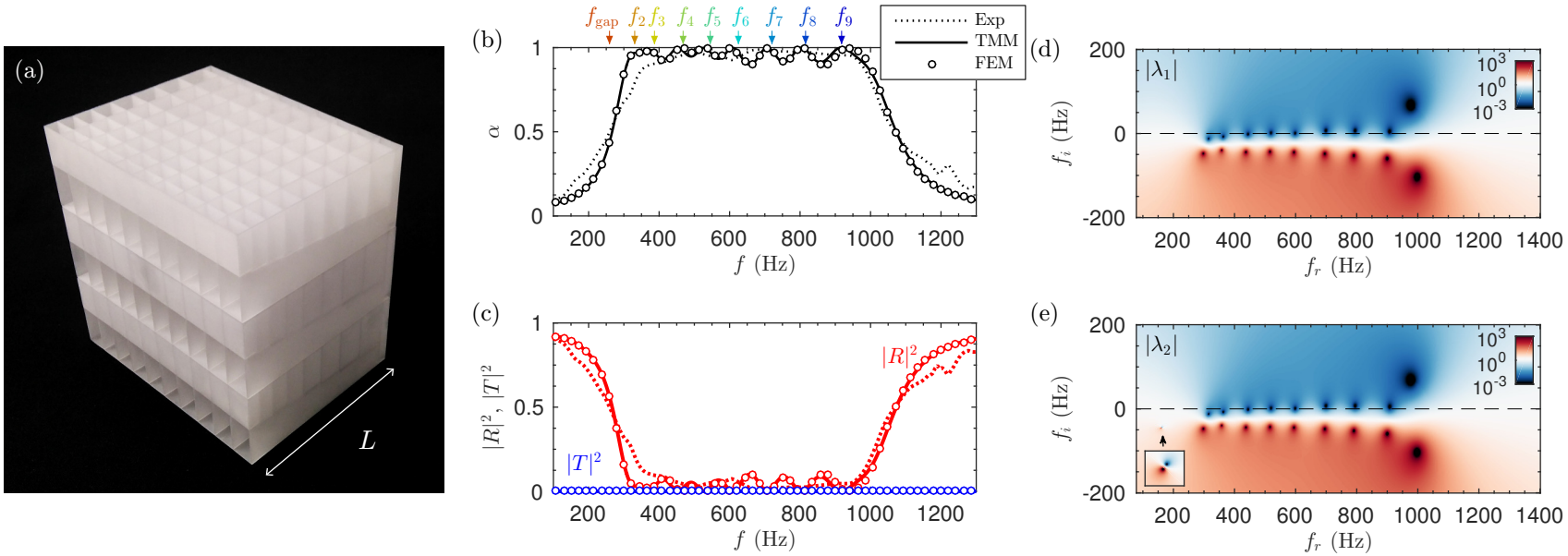


How to design a broadband perfect absorbers in ventilation problem?



Experimental validation of the RTA

Due to precision of the available 3D printing system (the minimum step was 0.1 mm), the RTA presented previously cannot be easily manufactured.



Perfect absorption is achieved over $[350\ \text{Hz}, 1000\ \text{Hz}]$ for $L = 11.3\ \text{cm}$, i.e., $\lambda_a > 10L$.

(Sci. Rep., 7: 13595, 2017)

Metadiffusers

Implementation of diffuser

- **Fraunhofer-Fourier approximation**: in the far field, the scattered pressure by a $2b$ width panel is

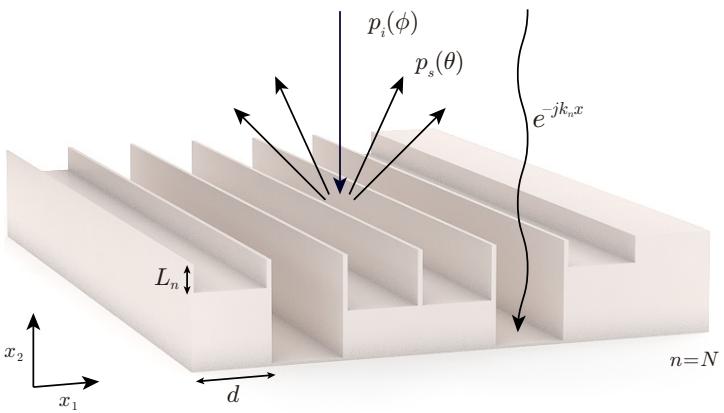
$$p_s(\theta) = \int_{-b}^{b} R(x) e^{ik_0 x \sin\theta} dx$$

where θ is the polar angle and k_0 is the wavenumber in air.

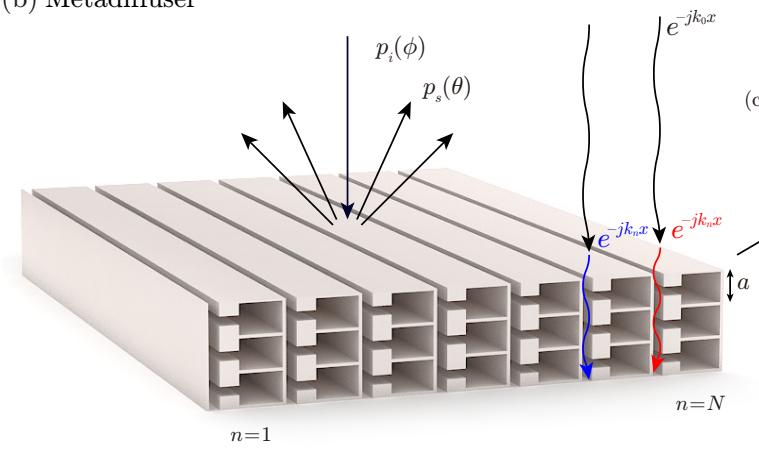
- Structures the reflection coefficient of which shows a **flat spatial Fourier transform** present good sound diffusion **in the far field**

- Schroeder: reflected energy into each diffraction lobe is the same by chosing a quadratic residue (QR) sequence, $s_n = n^2 \mathrm{mod}(N)$
- Classical QRD $L_n = s_n \lambda_0 / 2N$ where λ_0 is the design frequency

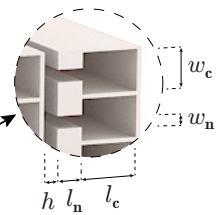
(a) Phase grating diffuser



(b) Metadiffuser

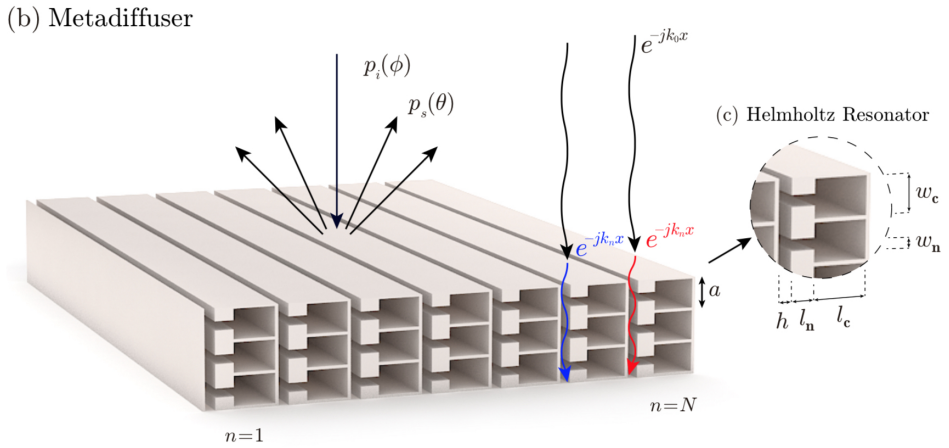


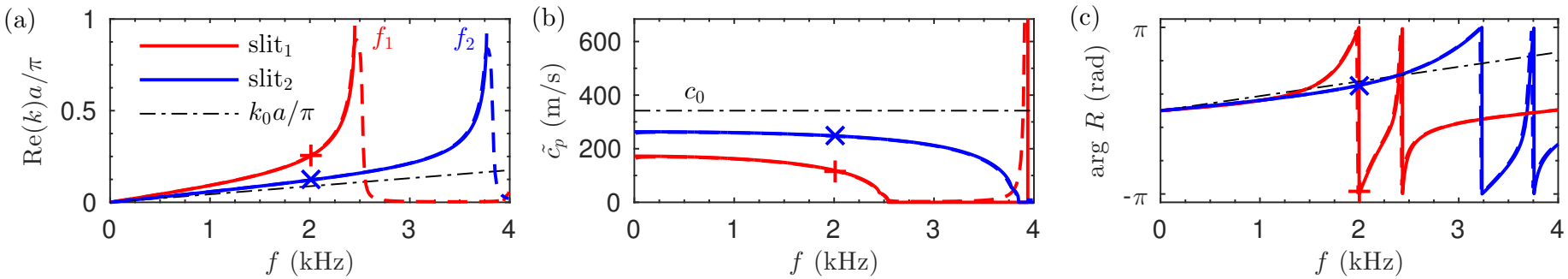
(c) Helmholtz Resonator



Spatially dependent reflection coefficient

2D flat panel: periodic distribution of unit cells made of two different slits, $n = 1$ and $n = 2$, loaded by $M = 3$ HR with different geometric parameters

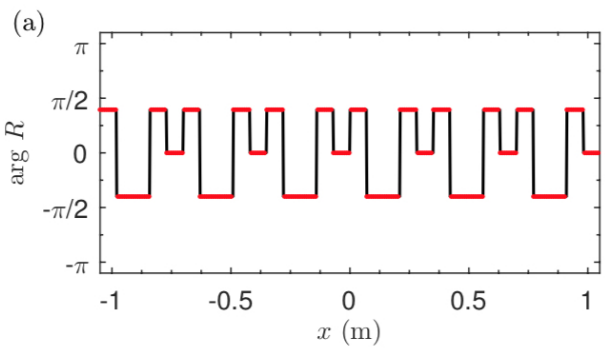


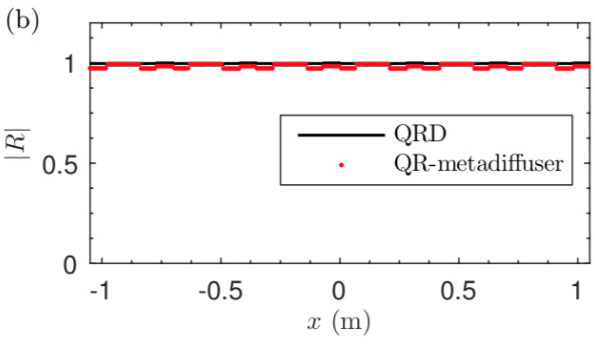


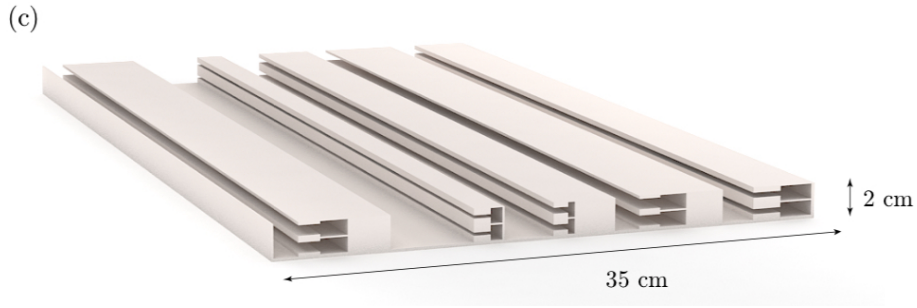
The phase profile of classical diffusers can be mimicked by a sub-wavelength metadiffuser over a given frequency band

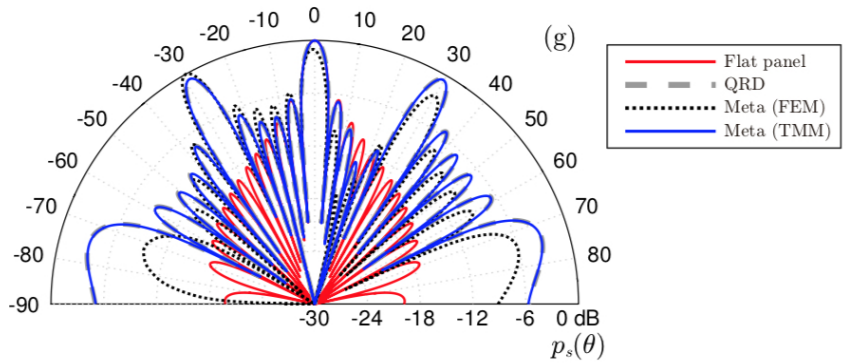
Quadractic residue

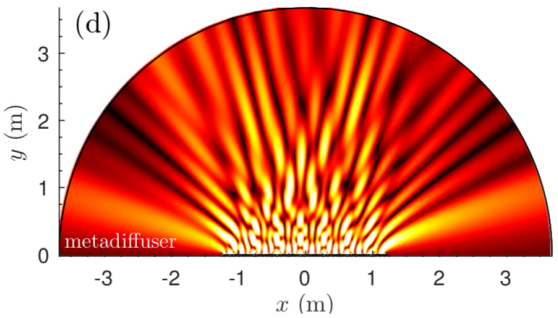
QRD, $N = 5$, $L = 27.4$ cm and side $d = 35$ cm, @ 2000 Hz
$L = 2$ cm thick **QR-metadiffuser** and $M = 2$ HRs of same lateral dimensions

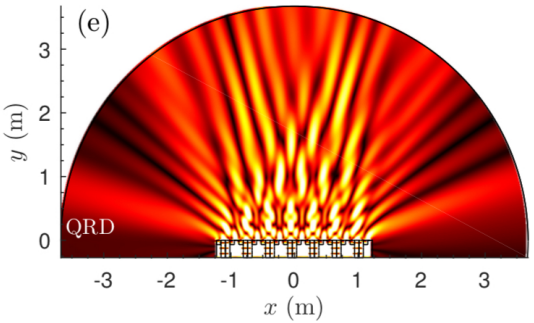


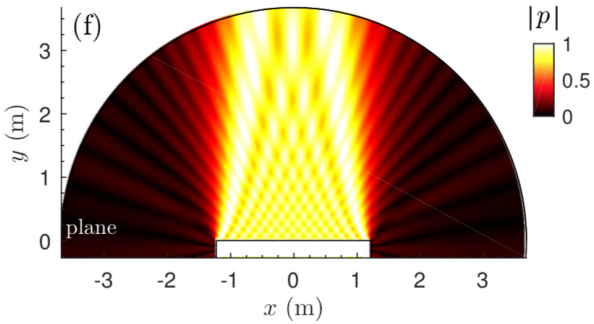






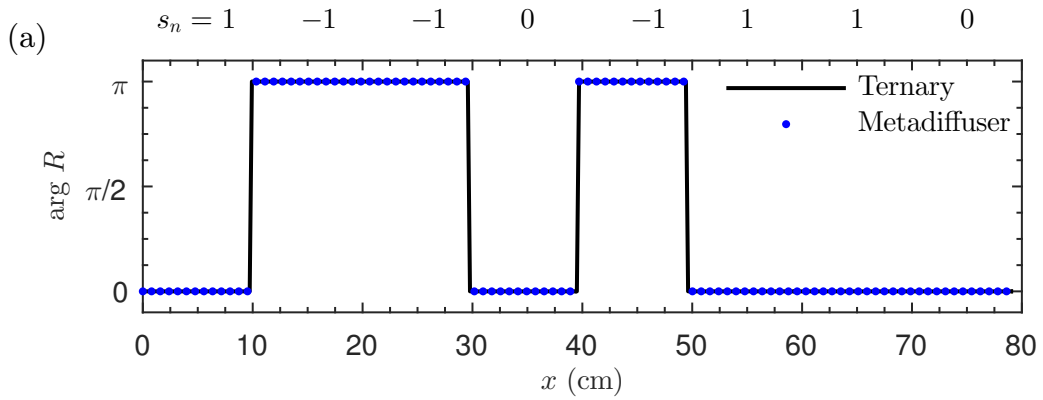


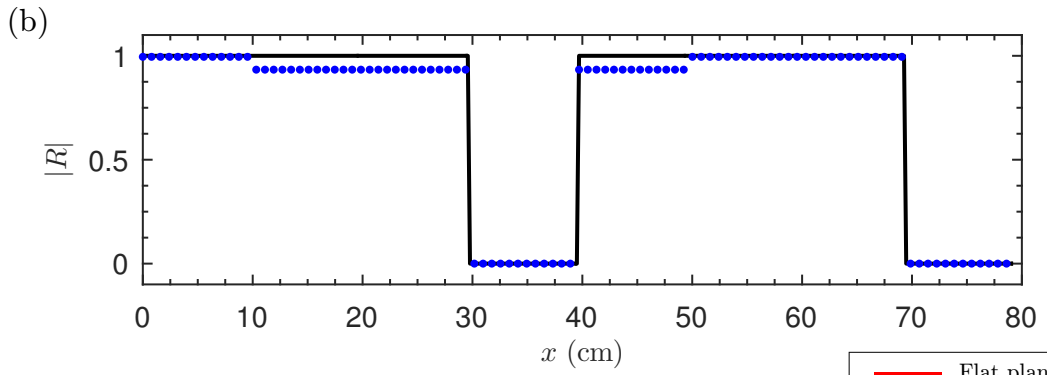


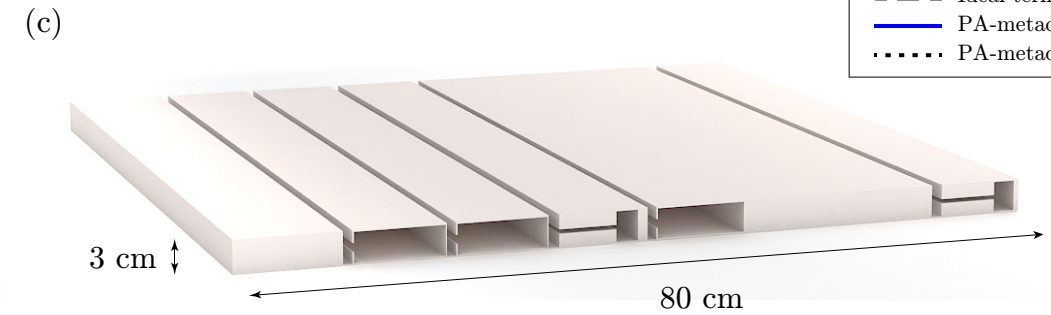


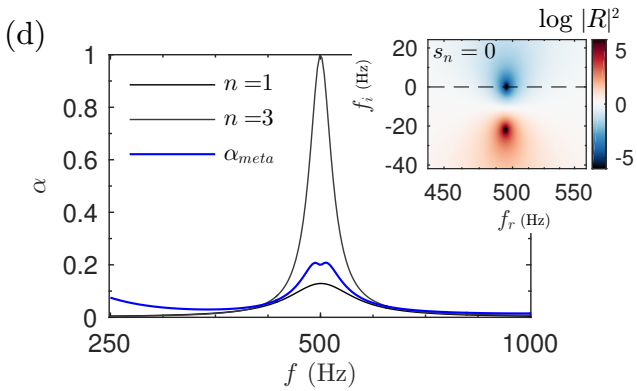
Hybrid metadiffusers

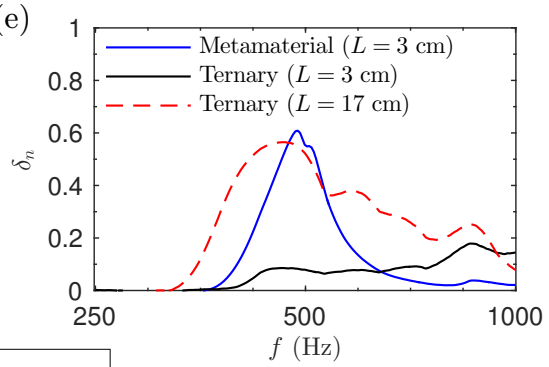
Design @ 500 Hz ($N = 8$, $M = 1$)

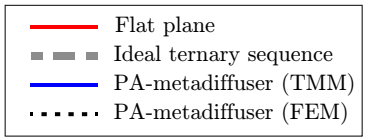


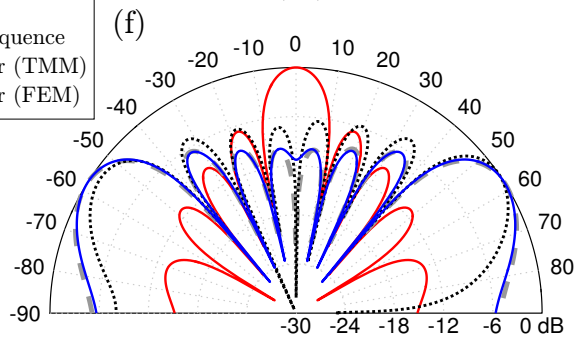






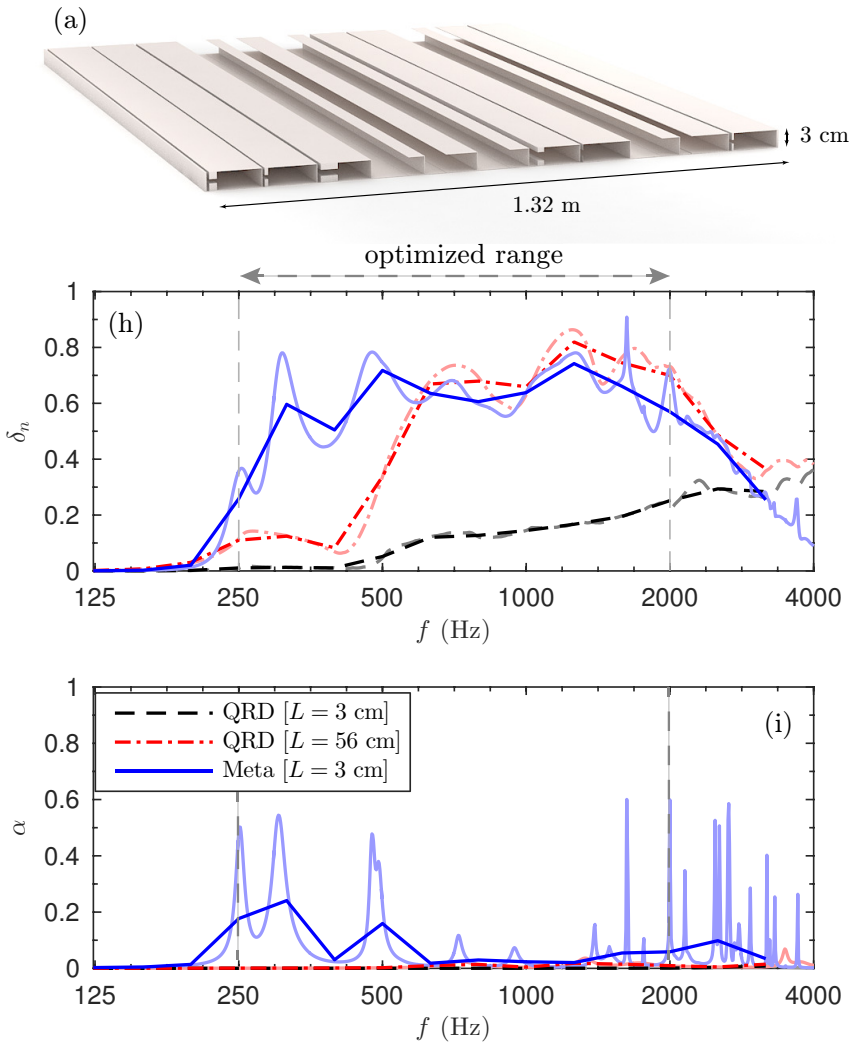


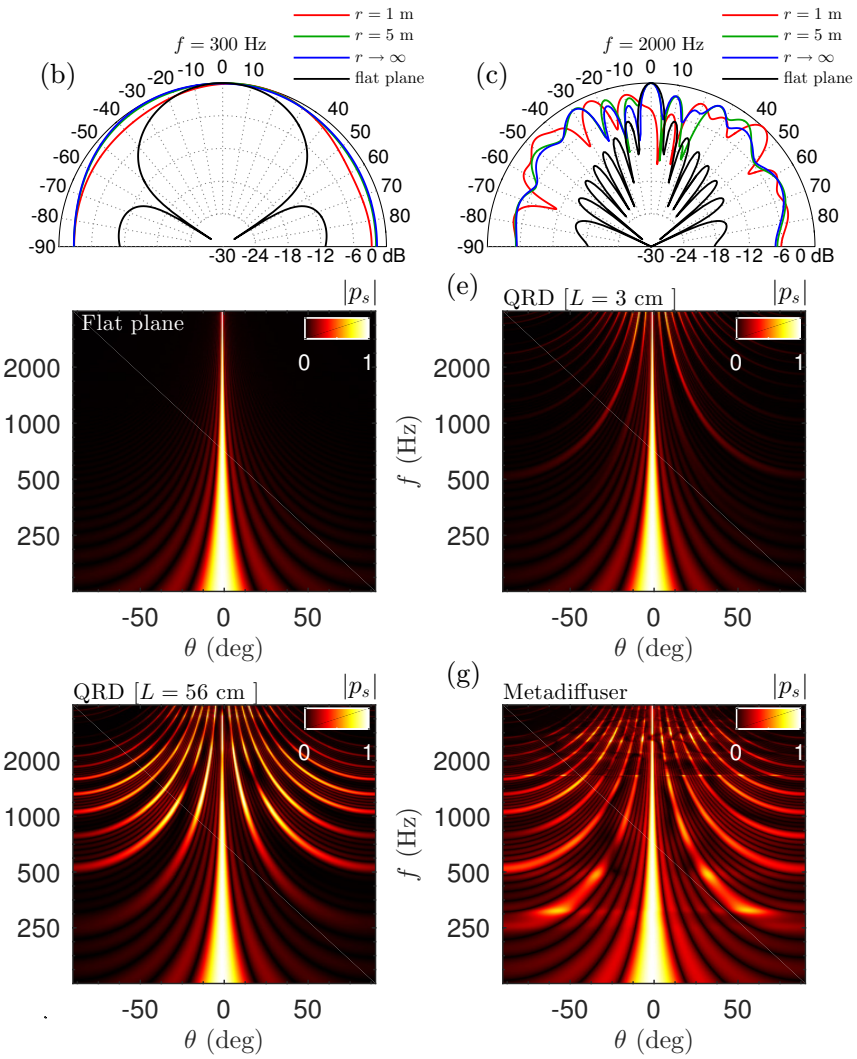




Broadband optimal metadiffuser

Optimization: $\epsilon = 1 - \int_{f_{low}}^{f_{high}} \delta_n df$, with $f_{low} = 250$ Hz and $f_{high} = 2000$ Hz





(Sci. Rep. 7:5389, 2017)

Conclusions and perspectives

- Slow sound enables the design of sub-wavelength resonators
- Resonators may be critically coupled to the exterior medium to design perfect subwavelength absorbers
  - Reflection problem: $\lambda_0 = 88L$ which has to be compared to $\lambda_0 = 4L$
  - Transmission problem: broadband $\lambda_0 = 10L$ which has to be compared to $\lambda_0 = 2L$
- ⊗ Quasi-omnidirectional: locally reacting material
- ⊗ Finite depth structures incorporate a finite number of modes!

$\Rightarrow$ Problem of causality pointed out in *Optimal sound-absorbing structures*, Materials Horizon, 2017.

- *Metadiffusers*: novel acoustic diffusers with tailored scattering distributions
  - based on metamaterials comprising slotted panel loaded by a set of Helmholtz resonators
  - propagation in each slit presents strong dispersion and the sound speed can be significantly reduced so that each slit effectively behaves as a deep-subwavelength resonator
- efficient in a broadband frequency range covering 3 octaves. The thickness of the panel was 1/46 times the impinging structure wavelength, i.e., about a twentieth of the thickness of traditional designs.

Thank you for your attention!



This work has been funded by the METAUDIBLE Project (ANR-13-BS09-0003), co-funded by ANR and FRAE.





This work is based upon work from COST Action DENORMS - CA15125, supported by COST (European Cooperation in Science and Technology).